WAVE FORCES ON SUBMERGED HORIZONTAL CYLINDERS

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Abstract This paper presents the results of an experimental investigation of the wave-induced forces acting on a horizontal cylinder submerged beneath water waves with its axis parallel to the wave crests. The experiments are carried out in a towing tank. A flap-type wave generator is used to produce regular sinusoidal waves. Two cylindrical models, one being circular and the other rectangular, are separately tested. The diameter of the circular cylinder is equal to the width of the rectangular cylinder. Each model is tested at eight different depths of submergence. A multi-component load cell is used for the measurement of forces acting on the submerged cylinder. A Personal Computer is installed on the carriage of the towing tank in order to compute the mean forces right at the time of performing the experiments. The time-averaged mean horizontal as well as vertical forces acting on the cylinder at various depths of submergence are plotted and then physically interpreted.

Keywords: Towing tank, sinusoidal waves, drifting force, wave breaking.

INTRODUCTION

At present a wide variety of offshore structures are being used, often under severe environmental conditions. These are predominantly related to the exploration, recovery and production of oil and gas, but they are also used in ocean energy extraction, harbour engineering etc. Difficulties in the design and construction of such structures are enormous, particularly as they are being installed in ever-increasing depths and are subject to extremely hostile environmental conditions. The potential of major catastrophic failures underlines the paramount importance of reliable design of these structures.

It is well known that, according to linear theory, surface waves in deep water suffer a phase shift in passing over a submerged cylinder with its axis parallel to the crests. This was first proved by Dean (1948), using the conformal mapping technique. He showed also that to the same approximation there are no waves reflected from the cylinder. This result was confirmed by Ursell (1950), using a series of multi-pole potentials, from which complete details of the flow may be inferred.

From the first-order velocity potential for the flow around a horizontal cylinder submerged beneath waves in deep water, Ogilvie (1963) derived expressions for the first-order force on the cylinder (i.e. the force component that is proportional to the wave amplitude when other independent parameters remain constant) and the time-independent part of the second-order force (proportional to $a^2$). More recently, alternative and more general solutions to the linearized irrotational flow problem of the effects of the cylinder on the waves have been given by Leppington and Siew (1980), Mehlum (1980) and Grue and Palm (1984). For the case of the circular cylinder, their conclusions are identical with those of Dean, and, in developing the velocity potential only to first order, they are unable to make any advance in calculating the nonlinear forces beyond those derived by Ogilvie.

However, in addition to the nonlinearities of the inviscid flow, there remain those that arise from the action of viscosity. In a series of papers, Chaplin has studied by both experimental and theoretical means the flow of a real fluid around a horizontal circular cylinder submerged beneath waves with its axis parallel to the wave crests. In the first paper, Chaplin (1984a) investigates the mass transport flow around the cylinder. For conditions in which the wave amplitude is sufficiently small for the effects of separation to be unimportant, and with the cylinder submerged at a depth about 2.5 diameters below mean water level, the results are in good agreement with predictions. In a second paper, Chaplin (1984b) has observed that the forces experienced by the cylinder reveal nonlinear components with frequencies up to three times the fundamental wave frequency. More recently Chaplin (1992) has considered the general orbital flow about a circular cylinder, using boundary-layer techniques introduced by Stuart (1966). In his most recent paper, Chaplin (1993) attacks the problem of uniform circular orbital flow in the presence of a circular cylinder at finite Reynolds number using the full Navier-Stokes
equation. The same problem is addressed by Stansby and Smith (1991), using the random vortex method.

The purpose of this paper is to determine experimentally the wave-induced forces acting on submerged horizontal cylinders located at different depths of submergence. Experiments are performed with a circular cylinder and a rectangular cylinder with a view to predicting the effect of geometry of the cylinder on the wave forces. The study relates to the practical areas of flow about the horizontal pontoons of semi-submersibles and tension-leg platforms, and wave energy devices.

EXPERIMENTAL SET UP AND PROCEDURE

The experiments are conducted in a towing tank of the University of Tokyo. The length, width and depth of the tank are 86 m, 3.5 m and 2.5 m respectively. A flap-type regular wave generator is used to produce regular sinusoidal waves. A wave-board of size 3.455 m x 1.530 m is installed at the end of the towing tank. A rectangular channel having two side walls made of transparent plexiglas with sharp leading edges is used for generating a two-dimensional flow field. The length, width and depth of the channel are 3.0 m, 0.53 m and 0.80 m respectively (see Fig. 1).

Two cylindrical models, one being circular and the other rectangular, are separately used for the experiments. Both the models are made of plexiglas of thickness 10 mm, and four circular holes of diameter 10 mm each are bored at their top and bottom ends which let water in and out of the models and thereby neutralize the buoyancy effects. Each model is 0.52 m long which leaves a clearance of 5 mm at each end in order to avoid any contact with the wall of the channel. This ensures unhindered motion of the cylinder under the action of wave forces. The cross-sectional area of the rectangular model is 0.4 m x 0.4 m whereas the diameter of the circular model is 0.4 m. The model is rigidly fixed to a carriage through a strut and a box of gauge system (a multi-component load cell) for the measurement of forces.

Each model is tested at eight different depths of submergence viz., \( D/B = 1.000, 1.125, 1.250, 1.375, 1.500, 1.750, 2.000 \) and 2.250 where \( B \) is the radius of the circular cylinder (or half-width of the rectangular cylinder) and \( D \) is the depth of the centre of the model from the still water level. It is of significance to note that the top surface of the cylinder just coincides with the still water surface when \( D/B = 1.00 \). Regular waves of length 1.57 m and 3.14 m are generated by the wave-maker. The amplitude \( a \) is 6 cm for both of the waves.

One of the important parameters of the flow is the amplitude of fluid motion relative to the cylinder size, which is usually expressed as the Keulegan-Carpenter number \( K_c \)

\[
K_c = \frac{U_m T}{2B}
\]

where

\[
U_m = n a e^{-nD}
\]

\[
n = \frac{2\pi}{T}
\]

\[
m = \frac{2\pi}{\lambda}
\]

Here \( \lambda, m, n, T \) and \( U_m \) denote the wave length, wave number, wave frequency, wave period and velocity amplitude of the flow respectively. In the present investigation, the values of \( K_c \) at the shallowest immersion of the models are 0.42 and 0.63 when the wave length is 1.57 m and 3.14 m respectively. Again, \( K_c \) attains a value of 0.16 and 0.38 corresponding to wave lengths of 1.57 m and 3.14 m respectively at the deepest submergence. Since the Keulegan-Carpenter number never exceeds unity in this investigation, the viscous effects are assumed to be very small.

The horizontal and vertical components of wave forces acting on the submerged cylinder are measured with the multi-component load cell. It may be noted that the down-wave and downward forces are considered positive as shown in Fig. 2. The forces are made dimensionless with respect to the length and width of the model as follows:

\[
F_X = \frac{FX}{\rho g a B L}
\]

\[
F_Z = \frac{FZ}{\rho g a B L}
\]

where \( FX \) denotes the horizontal component of wave forces, \( FZ \) the vertical component of wave forces, \( g \) the acceleration due to gravity, \( L \) the length of the cylinder and \( \rho \) the density of water.

The measured forces are instantaneously digitized and stored in a transient memory. Then they are copied onto a floppy magnetic diskette which is subsequently conveyed to the Computer Centre of the University of Tokyo where the analysis of the measured data is executed on the supercomputer HITAC M 680/682 H. Moreover, a Personal Computer is installed on the carriage of the towing tank in order to compute the mean forces right at the time of performing the experiments.
RESULTS AND DISCUSSION

During the course of this investigation, two distinct cases are studied viz., (i) a circular cylinder horizontally submerged beneath waves with its axis parallel to the wave-crests; and (ii) a rectangular cylinder horizontally submerged beneath waves with its axis parallel to the wave-crests. The diameter of the circular model is the same as the width of the rectangular model so that the wave forces acting on the two models can be compared with each other.

Fig. 3 shows the variation of the time-averaged mean horizontal force acting on the circular cylinder as well as the rectangular cylinder at various depths of submergence under a wave of length 1.57 m. The salient features of the force curves are as follows. The drifting force is found to be zero when \( D/B = 1.5 \) in case of the circular cylinder, and \( D/B = 1.75 \) in case of the rectangular cylinder. At larger depths also, this drifting force is more or less zero for both the models. (Slight departures from the zero value may be due to the experimental errors which are inherent in the measuring system). However, the force curves clearly indicate that the nonlinear effects are absent in case of deeply submerged cylinders irrespective of their shape. But as the depth of submergence decreases, a negative drifting force is developed on both the models. This negative drifting force (i.e., the upwave force) attains a peak value when \( D/B = 1.25 \) in case of the rectangular cylinder, and \( D/B = 1.125 \) in case of the circular cylinder. This is physically significant and merits explanation. The model is completely submerged beneath the wave only if \( D/B \) exceeds 1.30. But the top surface of the model pierces the free water surface and a part of it is exposed to air when \( D/B < 1.30 \). It is but easy to imagine that the depth of submergence, which corresponds to a value of \( D/B = 1.125 \) or 1.25, lies at such a critical juncture when the top surface of the model is about to pierce the free water surface and thereby induces strong nonlinear interactions with the incident water waves. However, as the depth of submergence decreases further, the negative drifting force rapidly decreases. This negative drifting force is a direct consequence of the nonlinear interactions of waves with the cylinder, and cannot be explained in the light of classical theoretical hydrodynamics. The present author would like to attribute this phenomenon to the fact that the waves pass over the shallowly submerged cylinder and then break behind it. As a consequence, the fluid particles in that region are naturally decelerated. These decelerated fluid particles exert a high static pressure on the rear side of the model. Hence it experiences an up-wave force.

Fig. 4 shows the variation of the measured mean horizontal force acting on the circular cylinder as well as the rectangular cylinder submerged under a wave of length 3.14 m. The circular cylinder is found to experience a zero drifting force when \( D/B = 2.15 \) whereas the rectangular cylinder experiences the same when \( D/B = 1.80 \). Both the models are subjected to a maximum negative drifting force at a depth of submergence, \( D/B = 1.25 \). Apart from these, the force curves exhibit similar characteristics to those corresponding to a wave of length 1.57 m. It is clearly observed from both Figs 3 and 4 that the maximum negative drifting force on the rectangular cylinder is greater than that on the circular cylinder. It may, therefore, be concluded that the nonlinear effect is less severe in case of a circular cylinder compared to that of a rectangular cylinder.

The time-averaged mean vertical forces acting on the circular model as well as the rectangular model under waves of length 1.57 m and 3.14 m are shown in Figs 5 and 6 respectively. The force curves exhibit approximately similar characteristics. The general trend of the variation of the force due to the change of the depth of submergence is that the upward mean force steadily increases with the decreasing depth of submergence, and then abruptly decreases when a part of the model emerges above the free water surface. However, the upward forces tend to vanish when \( D/B > 2 \). This remark holds good for both of the models and under both of the waves.

CONCLUSIONS

The principal conclusion of this experimental investigation is that the breaking of waves behind a shallowly submerged cylinder is primarily responsible for the generation of nonlinear wave forces. The negative drifting force (or the up-wave force) which acts on the submerged cylinder is a direct consequence of wave-breaking. When a wave passes over a shallowly submerged cylinder and breaks behind it, the water particles in that region are naturally decelerated. These decelerated fluid particles exert a high static pressure on the rear side of the cylinder, and as a consequence, it experiences a negative drifting force. This force attains a peak value when the top surface of the cylindrical model pierces the free water surface. On the other hand, this drifting force tends to vanish when the cylindrical model is deeply submerged (viz., at depths given by \( D/B > 2 \)).

The geometry of the cylindrical structure also plays a vital role in the wave-structure interaction. The negative drifting force acting on the rectangular model is greater than that on the circular one. It is primarily because of the difference in their shape. It is only the horizontal component of the static pressure acting on the rear side of the circular model which contributes to the negative drifting force. On the other hand, the entire static pressure (and not merely the horizontal component of it) acting normal to the flat vertical rear wall of the
A rectangular model is responsible for the large up-wave force.

REFERENCES


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**Fig. 1**: Arrangement of the cylindrical model in the channel

**Fig. 2**: Definition sketch for notations used in the study
Fig. 3 Measured mean horizontal force acting on the cylinders at various depths of submergence under a wave of length 1.57 m

Fig. 4 Measured mean horizontal force acting on the cylinders at various depths of submergence under a wave of length 3.14 m
Fig. 5 Measured mean vertical force acting on the cylinders at various depths of submergence under a wave of length 1.57 m.

Fig. 6 Measured mean vertical force acting on the cylinders at