NATURAL CONVECTION AND RADIATION IN CIRCULAR AND ARC CAVITY

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ABSTRACT
The radiation effect of gray surfaces on multiple steady-state solutions obtained in circular and arc-square inclined enclosure filled with air has been investigated numerically, by a finite-volume procedure. The left and right surfaces of the cavity are, respectively, heated and cooled at constant temperatures, while its horizontal walls are adiabatic. Parameters of the problem are the Rayleigh number $Ra(10^2 > Ra > 10^6)$, the inclination angle $\gamma$ (0 to 90°), aspect ratio equal to 1 and the surface emissivity (0 < $\varepsilon$ < 1).

Keywords: Constant Temperature, Circular And Arc Cavity, Nusselt Number, Radiation

1. INTRODUCTION
Heat transfer by combination of natural convection, conduction and radiation occurs when simulating building components, in particular passive heating and cooling systems, cooling of electronic components, design of solar collectors etc. Natural convection, coupled with surface radiation in a square cavity, heated from below [1], the conjugate heat transfer in an inclined square enclosure bounded by a solid wall [2], mixed convection and thermal radiation in ventilated cavities with gray surfaces [3] have been studied numerically. In the past, a great number of studies have focused on a variety of cavity configurations formed with straight walls, but the effects of curving the walls inward to form a derived circular cavity are beneficial because the fluid follows a less convoluted path. The flow and temperature field alters with the shape and orientation of the cavity.

2. PHYSICAL MODEL
The Physical diagram of the problem and the boundary conditions are shown in the fig. 1 (a) and (b). They are circular and arc cavities filled with air. Diameter of the cavity is $H$, where adiabatic horizontal sides are 0.57 $H$. The left wall is at a constant temperature $T_h$ and it is heated. The cold wall is also at a constant temperature $T_c$. It is inclined at an angle $\gamma$ with the horizontal axis. Laminar fluid flow and constant fluid properties are assumed here. The gravitational acceleration is considered in the normal vertical direction. The fluid outside and the horizontal walls are maintained to an ambient temperature $T_a$. In the present study, Rayleigh number is varied from $10^7$ to $10^8$; emissivities from 0 to 1; inclination angles from 0 to 90°. The aspect ratio and Prandtl number are 1 and 0.7 respectively throughout the problem.

3. MATHEMATICLAL MODELING
It is assumed that the flow is two-dimensional, steady state, laminar and the fluid is incompressible. The thermo-physical properties of the fluid are assumed to be constant except in the buoyancy term of the momentum equation, i.e., the Boussinesq approximation. Radiation heat transfer is taken into consideration along with natural convection mode. The viscous energy dissipation term in the energy equation can be neglected because of the small flow velocities associated with free convection.

Continuity Equation:
\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  

X-Momentum Equation:
\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + Ra \theta \sin(\gamma)
\]

Y-Momentum Equation:
\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ri \theta \cos(\gamma)
\]

Energy Equation:
\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re \cdot Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

At the surface:
\[
Nr_s = \left( \frac{\partial \theta_f}{\partial X} - k_r \frac{\partial \theta_r}{\partial X} \right)
\]
where $N_r = \sigma T^4_w / q^*$ is the radiation number, $\zeta$ is dimensionless radiative heat flux, $\zeta = q_r / \sigma T^4_w$.

$\Delta R f / \Delta X$ and $\Delta R s / \Delta X$ are the heat flux from the wall surface to the fluid on the right and to the solid on the left, respectively. The governing parameters are $Ra$, $Pr = \nu / \alpha$, $k_r$, $\varepsilon$, and $A = L / H$ and $w = l / H$.

The average convective and radiative Nusselt numbers are calculated at $X = 0$ plane as

$$Nu_c = -\int \frac{1}{\theta} \frac{\partial \theta}{\partial X} dY \quad (6)$$

$$Nu_r = -\int \frac{1}{\theta} N_r \zeta dY \quad (7)$$

The dimensionless temperature $\theta = (T - T_c) / (T_h - T_c)$

4. GRID SENSITIVITY TEST

In order to obtain grid independent solution, a grid refinement study is performed for the cavity under constant temperature considering $Ra=10^6$, $Pr = 0.71$, $\gamma = 0$ as shown in Table 1.

<table>
<thead>
<tr>
<th>No. of Nodes</th>
<th>Nu$_{av}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.99587268</td>
</tr>
<tr>
<td>30</td>
<td>1.0433495</td>
</tr>
<tr>
<td>50</td>
<td>1.0516758</td>
</tr>
<tr>
<td>70</td>
<td>1.0532773</td>
</tr>
<tr>
<td>90</td>
<td>1.0531155</td>
</tr>
</tbody>
</table>

5. CODE VALIDATION

Figure 2(a) and 2(b) reveals the streamline and isotherm obtained in this present study have excellent agreement with those obtained by El Hassan Ridouane and Antonio Campo [4]. The table 2 shows the comparative values of Nusselt numbers for different Rayleigh numbers.

<table>
<thead>
<tr>
<th>Rayleigh number</th>
<th>Circular cavity</th>
<th>Circular cavity</th>
<th>Arc cavity</th>
<th>Arc cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^2$</td>
<td>1.281</td>
<td>1.245</td>
<td>1.138</td>
<td>1.114</td>
</tr>
<tr>
<td>$10^4$</td>
<td>1.4038</td>
<td>1.32</td>
<td>1.1758</td>
<td>1.175</td>
</tr>
<tr>
<td>$10^6$</td>
<td>4.8195</td>
<td>4.94</td>
<td>4.700</td>
<td>4.71</td>
</tr>
<tr>
<td>$10^8$</td>
<td>9.1151</td>
<td>9.2</td>
<td>9.0378</td>
<td>9.05</td>
</tr>
</tbody>
</table>

6. RESULTS AND DISCUSSION

The numerical results obtained here are governed by several variable influential parameters. Those are Rayleigh numbers varied from $10^2$ to $10^8$, the inclination angles ranged from $0^\circ$ to $90^\circ$ and the emissivity of materials changed from 0 to 1.

6.1 Effect of Rayleigh Number

From the streamline plots it can be seen that all four configurations contain a single rotating vortex. As the Rayleigh number increases, the circulation evolving from buoyancy inside the cavities gets energetic. It is seen that there are two vortices instead of a single, in the lower right and upper left corners. Vortex rotation is clockwise.

The isotherms are parallel to the gravity field at low $Ra$ that indicates the typical conduction temperature distribution. Marked compression of the isotherms toward the boundary surfaces of the enclosure occurs to a greater extent with increasing $Ra$. Combined Nusselt number is a strong increasing function of $Ra$. It is almost horizontal up to critical $Ra=10^3$, then the deviation begins.

6.2 Effect of Emissivity

The flow pattern and isotherms for the surface emissivity $\varepsilon = 0.0, 0.6$ and 0.1 for circular cavities are shown in fig 3 and 4 for circular and arc cavity respectively. For the case with $\gamma = 0$ we present $Nu_t$ as a function of the Rayleigh number in Fig.5. We see the combined nusselt number is an increasing function of $\varepsilon$.

6.3 Comparison of Heat Transfer Rate for Natural Convection and Radiation

It is seen that generally the temperature field is more brilliant and the gradients much higher for $\varepsilon = 0$ than $\varepsilon = 1$. So it can be anticipated that the convective heat transfer be decreased with surface radiation in spite of enhanced circulation.

The spatial arrangement of temperature field changes creating distance within the isotherms which grows with $Ra$. The fluid temperature inside the cavity goes toward homogeneity with the existence of wall’s radiation. It is seen from Figure 6(a), $H_c$ decreases by increasing $\varepsilon$ for a given $Ra$. So, radiation affects convection negatively. $H_r$ increases quickly with $\varepsilon$ (for a given $Ra$). So, surface emissivity acts positively for $H_r$. The Heat transfer due to radiation, and therefore, the total heat transfer, increases in a drastic manner for highly emissive walls of the cavity (Figure 6(b))

6.4 Comparison of Circular and Arc Cavity

The nusselt number for total heat transfer rate considering radiation with natural convection increases at the same course as for only natural convection. $Nu_t$ is always higher for circular cavity in comparison with arc cavity.

For convection $Nu_c$ is 0.86% higher for $Ra=10^6$, $\gamma = 0$, and $Nut$ is 1.13% higher considering radiation, $\varepsilon = 1$. Heat transfer increases 1.74% for circular cavity in comparison of arc taking $Ra=10^6$, $\gamma = 0$ and $\varepsilon = 1$. 
6.5 Effect of Inclination Angles

In Figure 7(a), convective Nusselt numbers for circular cavity is presented as a function of the inclination angle, $\gamma$ with $Ra=10^2$, $10^3$, $10^5$, $10^6$. Nu is an increasing function of Ra and at Ra=10^6, it goes through a maximum at 22 degree for circular cavity and 9 degree for arc(fig 7(b)). Percent of radiation in total heat transfer is quasi constant for $\gamma$ from 0 to 20°, thereafter they increase by 6.81% from 30 to 90° in fig 8(a). Invariance of the radiative heat transfer is observed in fig 8(b) where total heat transfer is a decreasing function of $\gamma$.

7. CONCLUSION

The surface radiation modifies temperatures on all enclosure walls; the velocity and temperature gradients are decreased. Considering radiation, the percentage of radiation heat transfer decreases gradually at low Rayleigh, increases at high Rayleigh and again decreases at higher Rayleigh number. The radiative heat flux is a strong increasing function of the surface emissivity starting at its low values; as a consequence, the convective heat flux is a strongly decreasing function of it. The combined Nusselt number Nu is an increasing function of Ra as well as surface emissivity, $\varepsilon$. Nu is always higher for circular cavity in comparison with arc cavity.

The convective Nusselt number Nu goes through a broad maximum at the inclination angle of 22 degree for circular cavity and 9 degree for arc cavity at Ra=10^6 while the radiative Nusselt number Nu is unaffected to its fluctuation. At high inclination angle, Nu declines; i.e. total heat transfer rate decays. But the Radiation heat transfer is insensitive to angle variation.

The percentage of radiation heat transfer increases with higher inclination angle.

8. REFERENCES


9. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>Cp</td>
<td>specific heat capacity at constant pressure</td>
<td>J/kg.K</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity</td>
<td>W/m.K</td>
</tr>
<tr>
<td>Nu</td>
<td>average Nusselt number; $Nu= \frac{hL}{k}\Delta T/ k(\Delta T/L)$</td>
<td></td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number, $Gr=\frac{g\beta L^3}{\nu^2}(Tw-Ta)/\nu^2$</td>
<td></td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number, $Pr=\frac{\nu}{\alpha}$</td>
<td></td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number, $Ra = Gr*Pr$</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td>°K</td>
</tr>
<tr>
<td>U,V</td>
<td>non-dimensional x &amp; y velocity component, $U=\frac{uH}{\alpha f}$, $V=\frac{vH}{\alpha f}$</td>
<td></td>
</tr>
<tr>
<td>Ri</td>
<td>Richardson number, $Gr/Re^2$</td>
<td></td>
</tr>
</tbody>
</table>

Symbols
- $\alpha$ thermal diffusivity | m^2/s
- $\gamma$ inclination angle | degree
- $\varepsilon$ emissivity |
- $\theta$ non-dimensional temperature |
- $\nu$ dynamic viscosity | kg/m.s

Subscripts
- s surface |
- h hot |
- c cold |
- f fluid |

10. MAILING ADDRESS

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Fig 1. Physical Model of (a) circular (b) arc geometry

(a) present study  
(b) existed paper

Fig 2. Code Validation; Comparison of stream lines and isotherms for high $Ra=10^6$ In circular cavity

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Fig 3. Streamlines Variation of Ra at γ = 0° (circular cavity)

Fig 4. Isotherms Variation of Ra at γ = 0° (circular cavity)
Fig 5. Streamlines and isotherms Variation of $Ra$ at $\gamma=0^\circ$ (for arc cavity)

Fig 6. Variation of $Ra$ on Nusselt number at different emissivity at $\gamma=0^\circ$ (a) for circular cavity (b) for arc cavity

Fig 7. (a) Natural convection heat transfer rate for different emissivities in a circular cavity (inclination angle 0) (b) Radiation heat transfer rate for different emissivities in a circular cavity (inclination angle 0)
Fig 8. (a) Nusselt number variation for different inclination angle in a circular cavity considering natural convection; (b) Finding of the optimum inclination angles for maximum Nusselt number for arc and circular cavity considering natural convection ($Ra=10^6$).

Fig 9. (a) Percentage of radiation heat transfer in the combined heat transfer rate for different $\gamma$ in a circular cavity for $\varepsilon=1$ and $Ra=10^6$; (b) combined and radiation heat transfer rate for different $\gamma$ in a circular cavity, taking $\varepsilon=1$ and $Ra=10^6$. 