REYNOLDS NUMBER EFFECT ON FLOW-INDUCED FORCES ON TWO TANDEM CYLINDERS

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ABSTRACT
Reynolds number (Re) effects on fluctuating lift (C_L) and drag (C_D) forces and Strouhal numbers (St) of the downstream cylinder of two tandem cylinders are investigated experimentally for Re = 9.7×10^3 ~ 6.5×10^4. The center-to-center spacing ratio (L/D) between the cylinders is varied from 1.1 to 4.5 with a step of 0.1, where D is the cylinder diameter. The results show that C_L, C_D and St are highly dependent on Re in the Re range examined for L/D < 3. While C_L and C_D at L/D = 2.4 for Re = 6.5×10^4 are about 2 and 2.8 times higher, respectively, than those for a single cylinder, those for Re = 1.6×10^4 are 0.5 and 0.7 times. They are very sensitive to L/D for higher Re but less for lower Re. The associated physics are dug out.

Keywords: Forces, Flow Structures, Strouhal Numbers, Tandem Cylinders, Fluctuating Lift and Drag.

1. INTRODUCTION
Slender structures in groups have many engineering applications, for example, chimney stacks, tube bundles in heat exchangers, cooling of electronic equipments, high-rise buildings, harvesting wave and tide energy from ocean, overhead power-line bundles, bridge piers, stays, masts, chemical-reaction towers and offshore platforms. Naturally, it is important to understand the proximity effect on aerodynamics associated with multiple closely spaced cylindrical structures. Two fluid-dynamically interfering cylinders may be considered as the basic element of multiple structures and the knowledge of this flow is insightful for understanding the flow around more structures. As such, the flow around two cylinders has received a great deal of attention in the literature. There is no doubt that flow physics around two cylinders is much more complex and complicated than that around a single cylinder, because of interference between the cylinders, between the wakes, among four shear layers, etc. The alternate shedding of vortices in the near wake leads to fluctuating forces on the structures and may cause structural vibrations, acoustic noise, or resonance, which in some cases can trigger failure. The study of aerodynamics of two closely separated structures is thus of both fundamental and practical significance. Fluid forces, Strouhal numbers (St) and flow structures are the major factors considered in the design of slender structures subjected to a cross-flow.

When a cylinder is placed in-line downstream of another cylinder, it is called a tandem arrangement. When two cylinders are placed in tandem, a complex flow structure is generated as a result of mutual interactions among the wakes behind the bodies. Flow around two tandem circular cylinders may be classified into three regimes based on center-to-center spacing L/D [1]: (1) an extended-body regime, where L/D ranges from 1 to 1.5 and the two cylinders are so close to each other that the free shear layers separated from the upstream cylinder overshoot the downstream one; (2) a reattachment regime, where L/D is between 1.5 and 4 (critical L/D), and the shear layers reattach on the downstream cylinder; (3) a co-shedding regime, where L/D ≥ critical and the shear layers roll up alternately, forming a vortex street in the gap between as well as behind the cylinders. In the co-shedding regime, the frequency of vortex shedding from one cylinder is identical to that from the other [2]. Vortex shedding from the downstream cylinder is triggered by the arrival of vortices generated by the upstream cylinder.

Biermann and Herrnstein [3] measured the combined drag force acting on the two parallel circular cylinders in a tandem arrangement up to L/D = 8 (Re = 1.05×10^5). They measured only time-averaged drag (C_D) on the individual cylinders; therefore, more investigation was needed to clarify the other parameters, such as, root-mean-square (fluctuating) drag and lift (C_Df, C_Lf), Strouhal number (St), cylinder surface pressures, wakes, boundary layer characteristics around the cylinders, etc. Time-averaged pressure measurements on the surfaces of the cylinders were conducted by Zdravkovich and Pridden [4] at Re = 6×10^4, and Alam et al. [5] at Re = 6.5×10^4. The results showed that for L/D < 3.5 a negative pressure on the front surface of the downstream cylinder.
was generated instead of a positive pressure, exceeding that on the rear surface. In case of the upstream cylinder, the pressure only on rear surface was affected by the presence of the downstream cylinder. Nevertheless, beyond $L/D = 4$, the pressure distribution around the cylinders is comparable to that around a single isolated cylinder. Fluctuating surface pressure, $C_{Df}$, $C_{Lf}$ and behavior of boundary layer on the cylinders for $L/D < 9$ were investigated by Alam et al. [5] at $Re = 6.5 \times 10^3$. In the reattachment regime, they established a correlation between $C_{Lf}$ and reattachment position of the upstream-cylinder shear layers on the downstream cylinder; as the reattachment position moved toward the forward stagnation line, $C_{Lf}$ on the downstream cylinder increased and vice versa.

Previous investigations are in general focused on flow, $C_{D}$, $St$, for certain $Re$, barely giving any attention to $C_{Df}$ and $C_{Lf}$ in spite of the fact that the latter is of both fundamental and practical importance in fluid-structure interaction. The objectives of the investigation were to (i) measure $St$, $C_{Df}$ and $C_{Lf}$ on the downstream cylinder, (ii) find their dependences on $Re$, and (iii) get insight into the physics of the dependences including the ‘kink’.

Igarashi [6] examined the $Re$ effects in the wake of two tandem cylinders on $St$ and pressure fluctuations for $Re = 8.7 \times 10^3 \sim 5.2 \times 10^4$, and noted that, with increasing $Re$, the pressure fluctuation on the cylinder surfaces increased and $St$ decreased greatly between $Re = 1 \times 10^4$ and $4 \times 10^4$. Ljungkrona and Sundén’s [7] flow visualization and pressure distribution measurements in the wake of two tandem cylinders ($L/D = 1.25 \sim 4.0$ and $Re = 3.0 \times 10^3 \sim 4.0 \times 10^4$) indicated a dependence on $Re$ of the critical $L/D$, at which a bi-stable flow occurred and the shear layers separating from the upstream cylinder changed from reattachment upon the downstream cylinder to the rollup, forming vortices, in the gap of the cylinders. Zdravkovich and Pridden [4] examined $Re$ effect on $C_{D}$ at $Re = 3.1 \times 10^6 \sim 1.2 \times 10^7$. When the two cylinders were in contact, $C_{D}$ on the downstream cylinder was negative, increasing rapidly with $L/D$. An interruption of the increase rate occurred, forming a ‘kink’ at $L/D = 2.5$. The ‘kink’ was however absent at $Re \leq 3.1 \times 10^7$. The physics behind the presence and absence of the ‘kink’ was left unexplained, with a notation “further research is necessary to clarify this change at small Reynolds numbers”.

2. EXPERIMENTAL DETAILS

Measurements were done in a low-speed, closed-circuit wind tunnel with a test section of 1.20 m in height, 0.30 m in width, and 2.2 m in length (Fig 1b). In the test section side walls, two circular holes of 0.5 m diameter, one opposite to the other, were made where two circular disks, each included a slit for cylinders were placed (Fig 1b). The diameter of each cylinder was 49 mm. The cylinders spanned the horizontal 0.3 m dimension of the wind tunnel. $Re$ based on cylinder diameter and free-stream velocity was $9.7 \times 10^3$, $1.6 \times 10^4$, $3.2 \times 10^4$, and $6.5 \times 10^4$. The turbulent intensity was 0.5%. A fine-mesh honeycomb that was placed at the entrance of the test-section to provide a uniform flow was responsible for generating turbulence. In order to check the spanwise uniformity of flow as well as spanwise separation of flow over a single cylinder for fluid forces being measured by a load cell (which will be discussed next), circumferential time-averaged and fluctuating pressures on the surface of the cylinder at the mid-section, and at ±35 mm and ±80 mm (from the mid-section), were measured. The results showed that the time-averaged and fluctuating pressure distributions at
the five different sections were the same within the accuracy of measurement. The position of a point on the surface of the cylinder is denoted by \( \theta \), measured from the front stagnation point.

Fluid forces were measured over a small spanwise length of the cylinders, using load cells (Fig. 1c). The cylinder to be measured was built in with an active ('live') section of a spanwise 45 mm (0.92D) length and two dummy sections. This size was determined taking into account the cross-correlation length of fluctuating pressure in the spanwise direction of the cylinder. The active section, placed between the two dummy sections, corresponded to the midspan of the cylinder and was installed with a load cell that consisted of four semiconductor strain gauges. One of the dummy sections was also instrumented with another load cell of the same configuration. The load cell inside the active section measured a combination of fluid forces and forces due to vibration transmitted from outside through the cylinder support, whilst that inside the dummy section measured the latter forces only. Hence the fluid forces acting on the active section could be calculated by subtracting the output of the load cell inside the dummy section from that of the load cell inside the active section. See Alam et al. [9] for details of the load cell.

\( St \) was estimated from spectral analysis of the fluctuating lift and flow velocity measured at 2D downstream and D sidewise from the center of the downstream cylinder. Fig 1 is a schematic diagram showing the arrangement of two cylinders, definition of symbols and coordinate systems. \( T \) is the center-to-center distance between the cylinders. The position of a point on the surface of a cylinder is defined by the azimuth angle \( \theta \), measured from the direction of free-stream flow. Experiments were performed for \( L/D = 1.1 \sim 4.5 \). Very fine-tuning of \( L/D = 0.1 \) was adopted.

3. RESULTS AND DISCUSSION

3.1. Strouhal Number and Flow Structures

Power spectra of fluctuating lift are shown in Figs 2-5 at \( Re = 9.7 \times 10^4, 1.6 \times 10^5, 3.2 \times 10^5 \) and \( 6.5 \times 10^5 \). At \( Re = 9.7 \times 10^3 \), \( St \) drops suddenly from 0.259 to 0.215 between \( L/D = 1.1 \) and 1.3 (Fig 2). The drop is connected to change in flow structure from an over-shoot flow (Fig 6a) to rear-side reattachment flow (Fig 6b) which corresponds to \( St \) of an isolated cylinder value (= 0.19). Here, front, front-side, rear-side and rear surfaces correspond to \( \theta = 0^\circ \sim 60^\circ, 60^\circ \sim 90^\circ, 90^\circ \sim 120^\circ \), and \( 120^\circ \sim 180^\circ \), respectively. In the reattachment flow, the two shear layers from the upstream cylinder reattach alternately, say in the first half cycle the upper shear layer (Fig 6b1) reattaches and in the next half cycle the lower shear layer reattaches (Fig 6b2). The rear-side reattachment flow takes place between \( L/D = 1.3 \) and 1.7. With increasing \( L/D \), the reattachment point proceeds forward and front-side reattachment therefore occurs for \( L/D = 2.0 \leq L/D \leq 3.5 \) (Fig 6c). The front-side reattachment flow corresponds to a smaller \( St \) than an isolated cylinder flow. \( St \) is again about 0.19 for \( L/D \geq 3.9 \), insinuating a co-shedding flow (Fig 6e). Two peaks are seen at \( L/D = 3.7 \), indicating a bi-stable flow between the front-reattachment and co-shedding flows (Fig 6d). This \( L/D \) is called as the critical spacing.
The power spectrum results at $Re = 1.6 \times 10^4$ (Fig 3) are almost the same as those at $Re = 9.7 \times 10^3$. Here the rear-side reattachment flow regime is shorter, $L/D = 1.3$ - 1.5. The bi-stable flow occurs at a smaller $L/D = 3.6$.

Results at $Re = 3.2 \times 10^4$ and $6.5 \times 10^4$ (Figs 4, 5) are similar, but completely different from those at $Re = 9.7 \times 10^3$ and $1.6 \times 10^4$. Therefore flow structures are presented for $Re = 6.5 \times 10^4$, three $St$ are seen for $L/D \leq 1.5$. The first $St$ is connected to the vortex shedding, while the second and third $St$ are second and third super-harmonics of the first one. The corresponding flow structures (Fig 6f) explains why the second and third super-harmonic $St$ exist for this $L/D$ range. While for the front-side reattachment at $Re = 9.7 \times 10^3$ the shear reattaching on the cylinder splits into two and the high velocity slice of the shear layer goes to the downstream (Fig 6c), that at $Re = 6.5 \times 10^4$ the shear reattaching on the cylinder splits into two and the high velocity slice of the shear layer goes to the upstream and reattach again onto the rear surface of the upstream cylinder (Fig 6f). At $L/D = 1.5 < L/D < 2.2$, where only one peak is seen (Fig 5), the high velocity slice again goes on the same side to the downstream (Fig 6g). The high velocity slice goes on the other side passing over front surface at $2.2 \leq L/D \leq 2.7$, hence the two shear layers provide a strong alternating approaching flow on the downstream cylinder (Fig 6b), hence two peaks prevail in the power spectra (Fig 5). The flow structure generates a large $C_D$ on the downstream cylinder, which will be shown later. With further increasing in $L/D$, the downstream cylinder acts as a stabilizer reducing the curvature of the upstream cylinder, the shear layer again reattaches with the high velocity slice passing on the same side (Fig 6i), hence a single peak is seen in the power spectra at $2.7 < L/D \leq 3.9$ (Fig 5). The bistable flow occurs at $L/D = 4.0$, followed by a co-shedding flow at $L/D \geq 4.2$.

Fig 7 illustrates the trend of $St$ with variation of $L/D$ for the four Re. The dashed line represents the critical spacing. Isolated cylinder Strouhal number ($St_0$) range is also given for the $Re$ range examined. The effect of $Re$ on $St$ is different at different $L/D$. For $L/D < 2$, $St$ decreases with $Re$, but for $L/D = 2$ ~ 3.5, $St$ augments for $Re = 9.7 \times 10^3$ to $1.6\times 10^4$, then wanes for $Re = 1.6\times 10^4$ to $6.5\times 10^4$. Comparing Figs 6 and 7, it is seen that an over-shoot flow corresponds to $St$ higher than $St_0$, a rear-side reattachment flow and co-shedding flow communicate a $St$ equal to $St_0$ and front-side reattachment flow results in a $St$ smaller than $St_0$.

### 3.2. Fluctuating Forces

Fluctuating drag and lift coefficients ($C_Df$, $C_Lf$) are presented in Figs 8 and 9. Both $C_Df$ and $C_Lf$ are very high.
at \( Re = 6.5 \times 10^4 \), decreasing with decreasing \( Re \). They are enhanced around \( L/D = 1.4 \) and 2.4, corresponding to the front-side reattachment flow where the high velocity slice of the shear layer goes to upstream or other side passing over the front surface (Fig 6f, h). Two peaks are formed at \( Re = 3.2 \times 10^4 \) and \( 6.5 \times 10^4 \) but not at \( Re = 9.7 \times 10^3 \) and \( 1.6 \times 10^4 \). It has been discussed in the introduction that a ‘kink’ in \( C_p \) profile with \( L/D \) exists at \( Re < 3.1 \times 10^4 \) and vanishes at \( Re < 3.1 \times 10^4 \). That is, the second peaks in \( C_{DP} \) and \( C_{DF} \) at \( L/D = 2.4 \) (second peak) are 2.8 and 2 times higher, respectively, than those for an isolated cylinder. Alam et al. [5] proved that near the peak (\( L/D = 2.4 \)) the shear layers reattached on the downstream cylinder at a smaller angle of incidence on the front surface (\( \phi = 55^\circ \)), producing a stronger Karman vortex behind the cylinder. Reattachment for other \( L/D \) occurred on the front-side surface. Even \( C_f \) of an isolated cylinder is also a strong function of \( Re \) because of the change in the formation length and transition position in the shear layers. Beyond the critical spacing, the cylinder experiences much higher \( C_{DF} \) and \( C_{FL} \) resulted from the alternating Karman vortices from the upstream cylinder.

3.3. Physical Insight into \( Re \) Effect

\( St \) is dependent on reattachment position, formation length, wake width, etc. A flow structure map displaying how the flow structure vary with \( Re \) and \( L/D \) is given in Fig 10.

\( St \) at \( L/D < 2 \) drops monotonically with increasing \( Re \), which is attributed to a decrease in the vortex formation length. In a single cylinder wake, the formation length reduces rapidly from 2.7D at \( Re = 2.0 \times 10^3 \) to 0.8D at \( Re = 6.5 \times 10^4 \) (Fig 11) and remains unchanged up to \( 2.0 \times 10^3 \) [9]. A shorter length corresponds to a lower \( St \) and vice versa. Since the formation length of the upstream cylinder shear layer retreats with increasing \( Re \), the rollup of the overshooting shear layers (Fig 6a) occurs near and eventually reattaches on the downstream cylinder (Fig 6b), producing a drop in \( St \). Since the downstream cylinder is completely submerged in the recirculation region of the upstream cylinder wake, a larger \( L/D \) implies a longer after-body length of the combined body and hence a reduced \( St \) in 1.3 \( \leq L/D \leq 1.7 \). A wider wake corresponds to a smaller \( St \) and vice versa. A shear layer reattaching on the front-side of the cylinder is diverted away from the centerline, resulting in a wider wake and a lower \( St \).

Transition in the shear layer being another important parameter plays a role in the modification of flow structure and hence of forces and \( St \). For an isolated cylinder, transition length measured from the center of the cylinder decreases rapidly from 1.4D to 0.2D (Fig 11) when \( Re \) increases from \( 2.0 \times 10^3 \) to \( 6.5 \times 10^4 \) [9]. This explains why over-shot flow or rear-side reattachment flow occurs at low \( Re = 9.7 \times 10^3 \) to \( 1.6 \times 10^4 \) but not at higher \( Re = 3.2 \times 10^4 \) to \( 6.5 \times 10^4 \), Fig 6). It has been shown that \( St \) is highly dependent on \( Re \) for \( L/D < 2 \) but not so much for \( L/D > 2 \). This is due to the fact that the upstream shear layer transition happens around the downstream cylinder for \( L/D < 2 \), moving from the downstream to the upstream with \( Re \), hence the reattachment position as well, while that occurs much upstream of the downstream cylinder for \( L/D > 2 \). \( C_{DF} \) and \( C_{FL} \) vary a lot with \( Re \) even for the \( L/D > 2 \). Reattachment position and incidence angle of the shear layer on the cylinder are responsible for this variation. Since transition happens between the cylinders, reattachment position and angle of incidence on the cylinder vary significantly with \( Re \).

4. CONCLUSIONS

The paper leads to the following conclusions.

(i) \( C_{DF} \) and \( C_{FL} \) at smaller \( L/D \) (< 2), reattachment position and incidence angle of the shear layer on the downstream cylinder are responsible for the \( Re \) effect at larger \( L/D \) (> 2). \( St \) diminishes, and \( C_{DF} \) and
C_DA augment with increasing Re for the Re range examined.

(ii) With increasing L/D, flow structure at Re = 9.7×10^3 and 1.6×10^4 changes as over-shoot, rear-side reattachment, front-side reattachment, bi-stable and co-shedding, while that at Re = 3.2×10^4 and 6.5×10^4 changes as reverse-flow reattachment, front reattachment, front-side reattachment, bi-stable, and co-shedding.

(iii) The ‘kink’ in C_D distribution obtained in the literatures or peak in C_Df and C_Lf at Re = 3.2×10^4 and 6.5×10^4 is connected to front-side reattachment flow where the high-velocity slice of the shear layer reattaching on the downstream cylinder goes to the other side of the cylinder.

5. ACKNOWLEDGEMENT
M.M. Alam wishes to acknowledge support given to him by NRF through grant no A0U368, UP RDP through Grant No. AOT366, UP Research and Innovation Support through grant no AOS971. Professor Hiroshi Sakamoto, Kitami Institute of Technology, is also acknowledged for giving opportunity to conduct this experiment.

6. REFERENCES

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