THERMAL RADIATION EFFECTS ON PERIODIC MHD NATURAL CONVECTION FLOW ALONG A VERTICAL SURFACE

S. Siddiqa ¹ and Md. A. Hossain

¹Department of Mathematics, COMSATS Institute of Information Technology, Pakistan

ABSTRACT

The MHD natural convection periodic boundary layer flow of an electrically conducting and optically dense gray viscous incompressible fluid along a heated vertical plate is analyzed. The aim of this study is to observe the influence of transverse sinusoidal magnetic field which has its applications in induction electromagnetic pumps as well as in heat exchanger devices. Two numerical techniques are adopted to obtain the solution of the problem namely; (i) Direct numerical simulation (DNS) method which is used to integrate the governing equations directly (ii) Keller-box method which is applied after converting the governing equations into a convenient dimensionless form; using stream function formulation (SFF). For the entire range of \( X \), numerical results are demonstrated graphically by showing the influence of important physical parameters, namely, the magnetic field parameter, \( M \), Planck constant, \( R_a \) and the surface temperature parameter, \( \theta_w \), in terms of local skin friction coefficient and local Nusselt number coefficient as well as in terms of streamlines and isotherms; particularly for the case of liquid metals.

Keywords: Thermal Radiation, Magnetohydrodynamics, Natural Convection.

1. INTRODUCTION

Turcotte and Lyons [1] initially considered the periodic form of MHD boundary layer flow between two parallel walls. They also showed that inviscid theory may be used to calculate the overall performance of electromagnetic pumps and generators while the boundary layer theory may be used to obtain the wall shear stress. In order to obtain the solution, they adopted the regular perturbation method up to first order correction for larger values of magnetic interaction parameter \( M' = \sigma \beta B_0 \lambda / \rho U_c \), where \( U_c \) is the reference velocity of the fluid at the outer edge of the momentum boundary layer. Further, to verify the solutions obtained from series solution method, they also given consideration to Karman-Pohlhausen integral method.

Since thermal radiation effects are important in context of space technology and processes involving high temperatures therefore Ozisik [2], Sparrow and Cess [3] and Arpaci [4] initially studied the interaction of thermal radiation and natural convection. Later, considering the Rosseland diffusion approximation, investigations on the natural convection flow as well as on the mixed convection flow of an optically dense gray viscous fluid past or along heated bodies of different geometries have been accomplished by Hossain et al. [5]-[6], Hossain and Munir [7], Hossain and Rees [8], Molla and Hossain [9], Siddiqa et al. [10].

In this work, natural convection flow along a semi-infinite vertical plate of electrically conducting and optically dense gray fluid is considered in the presence of sinusoidal transverse magnetic field (see Turcotte and Lyons [1]). The aim of this study is to look at the effects of transverse sinusoidal magnetic field which has its applications in induction electromagnetic pumps; since magnetohydrodynamic power generators also use induction designs. The boundary layer equations are solved numerically with two different methods. On one hand direct numerical simulation method is applied to discover the flow pattern; while on the other hand governing equations are reduced to convenient form by the introduction of the stream function formulation (SFF). The non-similar equations obtained from the SFF are solved by Keller-box method for the whole range of \( X \) that measure the axial distance from the leading edge. Numerical results thus obtained are expressed graphically in terms of local skin friction and local Nusselt number coefficients with effect of physical parameters, such as the magnetic field parameter, \( M \), thermal radiation parameter, \( R_a \) and surface temperature parameter, \( \theta_w \), for the fluids having Prandtl number.
Pr=0.05. Effect of sinusoidal magnetic field on streamlines and isotherm lines have also been discussed and shown graphically.

2. MATHEMATICAL FORMULATION

Consider the steady 2D natural convection flow of a viscous, electrically conducting and optically dense gray fluid along a semi-infinite vertical heated surface in the presence of magnetic field prescribed in the streamwise direction, $B_\nu(\bar{x})$, following Turcotte and Lyons [1], is given as

$$B_0 = 0, \quad B_\nu = B_s \sin \left( \frac{\pi \bar{x}}{\lambda} \right). \quad (1)$$

where $B_0$ and $\lambda$ are the constants related to transverse magnetic field and wavelength of the applied magnetic field, respectively. From the relation given in (1) it is depicted that induced magnetic field is not considered and induced fields can be neglected if the magnetic Reynolds number ($R_m = \mu_0 \sigma \nu \lambda$, where $\mu_0$ is the magnetic permeability of the free space) is much less than unity. The devices which are functional due to liquid metals usually have magnetic Reynolds number between 0.1 to 0.01. If the induced magnetic field effects are negligible then the effects of applied field are obtained by solving Laplace's equation in the boundary layer region. Its worthy to mention here that Turcotte and Lyons [1] were the first to consider the magnetic field of periodic form as given in (1), which has special applications in the induced electromagnetic pumps and generators. It is assumed that the surface temperature, $T_s$, of the flat plate is higher than the ambient fluid temperature, $T_a$. Further, all the thermo-physical fluid properties are considered to be constant with negligible viscous dissipation effects.

Thus the fundamental equations under the usual Boussinesq approximation for steady magnetohydrodynamic flow with Ohm's law and Maxwell's equations associated with thermal radiation may now be written as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (2)$$

$$\bar{\nabla} \cdot \bar{\nabla} \pi = \nabla \cdot \frac{\sigma}{\rho} \frac{\partial \bar{B}^2}{\partial \bar{y}} = -\frac{\sigma}{\rho} \frac{\partial \bar{B}^2}{\partial \bar{y}} \sin \left( \frac{\pi \bar{x}}{\lambda} \right) \bar{\nabla} \pi + \beta \left( T - T_s \right) \quad (3)$$

$$\frac{\partial T}{\partial \bar{x}} + \nabla \bar{v} \cdot \nabla T = \alpha \left( \frac{\partial T}{\partial \bar{y}} \right)^2 - \frac{1}{\kappa} \frac{\partial T}{\partial \bar{y}} \quad (4)$$

where $\bar{u}$ and $\bar{v}$ are the $\bar{x}$ and $\bar{y}$ -components of the velocity field, respectively, in the momentum boundary layer, $T$ is the temperature of the fluid in the thermal boundary layer, $\rho$ the density of the fluid, $\nu$ the kinematic coefficient of viscosity, $-g$ identifies the gravitational vector, $\beta$ the coefficient of thermal expansion, $\kappa$ the thermal conductivity and $\alpha$ the thermal diffusivity.

The term $q_r$ in energy equation represents radiative heat flux in the $\bar{y}$-direction. In order to reduce the complexity of the problem and to provide a means of comparison with further studies that might employ a more detailed representation for the radiative heat flux, here the optically thick radiation limit, known as Rosseland diffusion approximation (see Ozisik [2]), is considered. Due to this assumption, the radiative heat flux $q_r$ is as given in (5).

$$q_r = -\frac{4 \sigma_o}{3(\alpha + \sigma_v)} \frac{\partial T}{\partial \bar{y}} \quad (5)$$

In Eq. (5) ‘$\alpha$’ is the Rosseland mean absorption coefficient, $\sigma_v$ is the scattering coefficient and $\sigma_o$ is the Stefan-Boltzmann constant. Diffusion approximation is valid in the interior of a medium but not employed near the boundaries and is good only for intensive absorption, that is, for an optically thick boundary layer. The coordinate system and the flow configuration of the problem are shown in Fig. 1.

The boundary conditions to be satisfied are

$$\bar{y} = 0: \pi = \bar{v} = 0, \quad T = T_a \quad (6)$$

$$\bar{y} = \infty: \pi = 0, \quad T = T_a \quad (7)$$

In order to obtain the governing equations in dimensionless dependent and independent variables, we introduce the following parameters.

$$\bar{\pi} = \frac{\nu}{\lambda} \frac{1}{Gr_r} \bar{u}, \quad \bar{v} = \frac{1}{\lambda} \frac{v}{Gr_r}, \quad x = \bar{x},$$

$$y = \frac{\bar{y}}{\lambda} \frac{1}{Gr_r}, \quad \theta = \frac{T - T_s}{T_a - T_s}, \quad Gr_r = g \beta \left( T_s - T_a \right) \lambda^3 \quad (8)$$

where $Gr_r$ is the Grashof number which measures the ratio of buoyancy force to the viscous force. Substituting (7) in (2)-(6), we get the following set of governing equations:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (8)$$

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -M^2 \sin^2 \left( \pi x \right) \frac{\partial \bar{u}}{\partial \bar{y}} + \theta$$

$$\frac{\partial \bar{\theta}}{\partial \bar{x}} + \frac{\partial \bar{\theta}}{\partial \bar{y}} = \frac{1}{Pr} \left( 1 + \frac{4}{3} R_m (1 + \Delta \theta) \right) \frac{\partial \bar{\theta}}{\partial \bar{y}}$$

The corresponding boundary conditions are
\( y = 0 : \quad u = v = 0, \quad \theta = 1 \) \hspace{1cm} (11)
\( y \to \infty : \quad u \to 0, \quad \theta \to 0 \)

Where
\[
M' = \frac{\sigma B^2}{\nu \gamma^{2/3}}, \quad R_d' = \frac{4 \sigma T^*}{\kappa (a + \sigma)}, \quad \Delta = \theta^{*} - 1 \quad (12)
\]

In Eq. (12) \( M' \) is the magnetic interaction parameter (or simply, the magnetic parameter), that controls the strength of the sinusoidal magnetic field, \( R_d' \) is the Planck constant or thermal radiation parameter which acts as a heat source due to the thermal radiation, \( \theta^{*} \) denotes the surface temperature parameter that measures the ratio of the surface temperature to the ambient fluid temperature and \( Pr \) symbolizes the Prandtl number which gives the ratio of momentum diffusivity to the thermal diffusivity.

Now we draw our attention in finding the solution of equations (8)-(11) with the direct numerical simulation (DNS) method. For this purpose Eqs. (8)-(11) are discretized with the help of difference quotients. The equations (8)-(11) are replaced with central difference quotients in \( x \) direction.

Conclusively, we get the discretized equations with unknown’s \( f \) and \( v \). These coupled equations are solved independently with the aid of tri-diagonal solver. The computation procedure, the accuracy of \( 10^{-6} \) is noted that in the present investigation the value of \( Pr = 0.05 \) and compared quantitatively with those obtained from stream function formulation (SFF). It is inferred from these numerical values that local skin-friction coefficient and local Nusselt number coefficient enhances considerably, owing to increase in the thermal radiation parameter, \( R_d \). Due to the intense thermal radiation inside the boundary layer, rate of energy

where \( \psi \) is the stream function which satisfies the equation of continuity given in (8). Introducing (14) into the boundary layer equations (8)-(11), we acquire the following non-similar equations

\[
f'' + \frac{3 + 2X}{4(1 + X)} \theta'' = \frac{1}{2(1 + X)} f'' - M' X^{3/4}(1 + X)^{3/4} \sin(\pi X) f' \\
+ (1 + X) \theta = X \left( f' \frac{\partial f}{\partial X} + f \frac{\partial f}{\partial X} \right) \quad (15)
\]

The boundary conditions to be satisfied are
\[
f(\xi, 0) = f' (\xi, 0) = 0, \quad \Theta (\xi, 0) = 1
\]
\[
f' (\xi, \infty) = 0, \quad \Theta (\xi, \infty) = 0 \quad (17)
\]

In the light of the previous investigations, we noted that in the absence of thermal radiation \( (R_d = 0) \) Hunt and Wilks [11] discussed the above posed problem (15)-(17) with constant transverse magnetic field \( i.e. \) by taking \( B_x = B_0 \). In this investigation, full numerical solutions via algorithm based upon Keller box technique had been obtained for the whole range of local Hartmann number, \( \zeta \). However, Hunt and Wilks [11] also obtained the asymptotic solutions for the regions where \( \zeta \) is treated to be sufficiently large and small.

The formulation obtained in (15)-(17) has been integrated numerically via Keller-box scheme, introduced by Keller and Cebeci [12]. Now, one can obtain the local skin friction coefficient, \( C_{fl} G_{Rd}^{-1/4} \), and the local Nusselt number, \( Nu_{G_{Rd}}^{1/4} \), from the following relations:

\[
C_{fl} G_{Rd}^{-1/4} = X^{1/4} \left( 1 + X \right)^{-3/4} f' (0, X), \quad (18)
\]
\[
Nu_{G_{Rd}}^{1/4} = \left( 1 + \frac{4}{3} R_d \xi \right) X^{1/4} \left( 1 + X \right)^{-3/4} \Theta (0, X) \]

### 4. RESULTS AND DISCUSSION

Present analysis deals with the free convection flow of electrically conducting and optically dense gray fluid over a vertical plate in the presence of sinusoidal magnetic field. Solutions of the governing equations are obtained by two numerical techniques for the entire range of \( X \), namely, (i) direct numerical simulation (DNS) method and (ii) Keller-box scheme (which is applied after reducing boundary layer equations into non-similar equations with the help of SFF). It should be noted that in the present investigation the value of Prandtl number, \( Pr \), is taken to be 0.05 (that is appropriate for lithium). Numerical values of local skin-friction coefficient, \( C_{fl} G_{Rd}^{-1/4} \), and local Nusselt number, \( Nu_{G_{Rd}}^{1/4} \), obtained by DNS are entered, respectively, in Table 1 against \( X [0.01,10.0] \) for \( R_d = 0.0 \) and 2.0 while values of other parameters are taken as \( \Theta = 2.1, M = 0.5 \) and \( Pr = 0.05 \).
transport of the fluid is increased sharply which in turn gives rise to the temperature of the fluid in the surrounding of the plate surface. Although the comparison shows good agreement between the two methods, but, in view of computational time and accuracy, SFF sounds to be more economical as compared to DNS method.

Table 1: Numerical values of coefficient of skin-friction with \( R_d = 0.0, 2.0 \) and \( \theta_w = 2.1, \) Pr = 0.05 and \( M = 0.5 \)

<table>
<thead>
<tr>
<th>( X )</th>
<th>( C_f \lambda^{1/4} )</th>
<th>( R_d = 0.0 )</th>
<th>( R_d = 2.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DNS</td>
<td>SFF</td>
<td>DNS</td>
</tr>
<tr>
<td>0.01</td>
<td>0.3125</td>
<td>0.4055</td>
<td>0.3610</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8216</td>
<td>0.8383</td>
<td>0.9359</td>
</tr>
<tr>
<td>0.5</td>
<td>0.9392</td>
<td>0.9522</td>
<td>1.0618</td>
</tr>
<tr>
<td>1.0</td>
<td>1.1983</td>
<td>1.2023</td>
<td>1.3431</td>
</tr>
<tr>
<td>2.0</td>
<td>1.3648</td>
<td>1.3680</td>
<td>1.5240</td>
</tr>
<tr>
<td>3.0</td>
<td>1.4605</td>
<td>1.4645</td>
<td>1.6270</td>
</tr>
<tr>
<td>4.0</td>
<td>1.5264</td>
<td>1.5316</td>
<td>1.6967</td>
</tr>
</tbody>
</table>

Table 2: Numerical values of coefficient of Nusselt number with \( R_d = 0.0, 2.0 \) and \( \theta_w = 2.1, \) Pr = 0.05 and \( M = 0.5 \)

<table>
<thead>
<tr>
<th>( X )</th>
<th>( \theta_w \lambda^{1/4} )</th>
<th>( R_d = 0.0 )</th>
<th>( R_d = 2.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DNS</td>
<td>SFF</td>
<td>DNS</td>
</tr>
<tr>
<td>0.01</td>
<td>0.6181</td>
<td>0.3808</td>
<td>2.7330</td>
</tr>
<tr>
<td>0.2</td>
<td>0.1798</td>
<td>0.1772</td>
<td>0.7770</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1313</td>
<td>0.1306</td>
<td>0.5782</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1164</td>
<td>0.1159</td>
<td>0.5151</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0937</td>
<td>0.0924</td>
<td>0.4144</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0838</td>
<td>0.0819</td>
<td>0.3661</td>
</tr>
<tr>
<td>4.0</td>
<td>0.0773</td>
<td>0.0749</td>
<td>0.3360</td>
</tr>
</tbody>
</table>

Further, we discuss the results in terms of coefficient of local skin friction, \( C_f \lambda^{1/4} \) and coefficient of local Nusselt number, \( \theta_w \lambda^{1/4} \) for different values of the physical parameters, \( \lambda, \) magnetic field parameter (or Hartmann number), \( M, \) thermal radiation parameter (or Planck constant), \( R_d \) and surface temperature parameter, \( \theta_w, \) against the dimensionless axial distance, \( X. \)

4.1 EFFECT OF PHYSICAL PARAMETERS \( M \) AND \( \theta_w \) ON \( C_f \lambda^{1/4} \) AND \( \theta_w \lambda^{1/4} \)

Firstly, the influence of magnetic field parameter, \( M = 0.0, 0.5, 1.0 \) and 2.0 is illustrated in Fig. 2 for \( R_d = 2.0, \) \( \theta_w = 2.1 \) and Pr = 0.05 on coefficient of local skin friction, \( C_f \lambda^{1/4}, \) and coefficient of local Nusselt number, \( \theta_w \lambda^{1/4}. \) In can be seen from these figures that both coefficients of local skin friction and local Nusselt number decreases considerably due to the increase in magnetic field parameter which acts as retarding force. Intense amount of magnetic field inside the boundary layer literally increases the Lorentz force which significantly opposes the flow in the reverse direction. Thus, coefficients of local skin friction and local Nusselt number diminish. In these figures one can see that amplitude of the flow pattern increases considerably as \( M \) is intensified, which is expected since the waves build each other due to the constructive interferences. Further, in Fig. 2(b) it is observed that the sinusoidal waves decay down smoothly as the fluid moves away from the leading edge of the plate, which signifies that gradually the sinusoidal flow patterns settle down to their asymptotic values.

The variation of local skin friction coefficient \( C_f \lambda^{1/4} \) and local Nusselt number coefficient \( \theta_w \lambda^{1/4} \) is inspected for \( \theta_w = 1.1, 2.1 \) and 3.1 while \( R_d = 2.0, \) \( M = 0.5 \) and Pr = 0.05 in Fig. 3. Here, notable enhancement is recorded in the coefficient of local skin friction and the local Nusselt number coefficients as we intensify the surface temperature parameter \( \theta_w. \) Such a behavior is possible because increment in surface temperature parameter, \( \theta_w, \) leads to an increase in the motion of the fluid particles. This rapid increase in the motion of the fluid accelerates the flow rate near the plate. Thus, surface temperature parameter is responsible for the raise in the temperature of the fluid and ultimately wall shear stress and heat transfer rate increases. Further, impact of surface temperature parameter tends to decrease the momentum boundary layer thickness whereas thermal boundary layer thickness increases a little.
Fig 3. (a) Variation of coefficient of local skin friction and (b) coefficient of local Nusselt number with $X$ for $\theta_w = 1.1, 2.1, 3.1$ while $R_d = 2.0$, $M = 0.5$ and $Pr = 0.05$.

4.2. EFFECT ON $M$ ON STREAMLINES AND ISOTHERMLINES

We now focus our attention to observe the effects of magnetic field parameter, $M$ on streamlines and isotherm lines for the case of liquid metals.

The influence of magnetic field on streamlines and isotherms with effect of thermal radiation parameter (for $R_d = 2.0$) is depicted in Figs. 4 and 5. In these figures magnetic field parameter is chosen to be 2.0 and 4.0 whilst Prandtl number, $Pr$, is 0.05 and surface temperature parameter, $\theta_w = 2.1$. From the Figs. 4(a) and 4(b) it is retrieved that $\psi_{max}$ diminishes steadily from 21.3 to 15.5 for the increasing values of magnetic field parameter. Again, this indicates that magnetic field opposes the flow, as it is noticed earlier. Basically Figs. 4-5 are drawn to show how the velocity and temperature of the flow becomes stronger because of the response gained due to the presence of thermal radiation. For $R_d > 0$, fluid energy increases rapidly as the fluid molecules are heated up sharply which conclusively give rise to the temperature of the fluid in the neighborhood of the surface of the plate that leads to increase in the temperature gradient. Thus due to the increment of energy at the molecular level, the fluid moves more frequently and in turn increase the momentum as well as thermal boundary layer thicknesses.

Fig 4. Streamlines for (a) $M = 2.0$ (b) $M = 4.0$ while $\theta_w = 2.1$, $R_d = 2.0$ and $Pr = 0.05$.

Fig 5. Isotherm lines for (a) $M = 2.0$ (b) $M = 4.0$ while $\theta_w = 2.1$, $R_d = 2.0$ and $Pr = 0.05$.

5. CONCLUSIONS

In this paper, the interaction of magnetohydrodynamic and thermal radiation on the two dimensional free convection flow of an electrically
conducting and optically dense gray viscous fluid over a semi-infinite vertical flat surface has been studied. For entire range of locally varying parameter $X$, the governing equations are reduced to parabolic partial differential equations using SFF which are then integrated numerically by employing the Keller-box method. Further, DNS technique is also applied to obtain the solution of the boundary layer problem and results obtained are compared quantitatively with the former method. The numerical values thus obtained in terms of local skin friction coefficient $C_{f} \sqrt{\text{Gr}}^{-3/4}$ and local Nusselt number, $\text{Nu} \sqrt{\text{Gr}}^{-1/4}$ for various values of the physical parameters for low Prandtl number. From the above investigation it may conclude that (i) Coefficient of local skin friction and coefficient of local Nusselt number diminishes owing to the increase in the magnetic field parameter $M$, (iii) increase of surface temperature parameter is responsible for the upraise in the temperature of the fluid and ultimately it leads to increase wall shear stress and heat transfer rate and (iv) it is noted that coefficient of local skin friction and coefficient of local Nusselt number diminishes due to the retarding effects of magnetic field on the fluid flow.

6. REFERENCES


7. MAILING ADDRESS

S. Siddiqa
Department of Mathematics,
COMSATS Institute of Information Technology,
Park Road, Chak Shahzad, Islamabad,
Pakistan