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PREDICTION OF FATIGUE CRACK INITIATION AND PROPAGATION LIFE FOR A PIPE USING FRACTURE MECHANICS

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ABSTRACT
Engineering components and structures are frequently subjected to repetitive or cyclic loads which cause the failure of those components at a stress much lower than that required to cause fracture on a single application of load. Failure occurring under such conditions of loading is called fatigue failure and process of damage and failure due to cyclic loading is called fatigue. Therefore it is necessary to know the behavior of the growing crack. In this paper, fatigue analysis has been carried out for different size of pipes to validate the analytical procedure for evaluation of fatigue crack initiation and propagation life. This will also help in predicting the useful life of the piping component. Effect of notch tip radius and characteristic distance d on fatigue crack initiation is also discussed. Analytical results have been compared with experimental results available for the same conditions.

Keywords: Fatigue crack initiation and propagation, strain life approach, fracture Mechanics approach

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1. INTRODUCTION
The fatigue failure of components has been studied extensively because of immense industrial interest as about 90% of failure are by fatigue. The general topic has been divided into a number of inter-related fields. These divisions include high and low cycle fatigue; fatigue of notched members, fatigue crack initiation and fatigue crack propagation among others. Hence understanding of fatigue failure is very important to many industrial applications. Another common engineering problem is the prediction of fatigue life reduction due to effect of local stress raisers. These stress raisers may be surface irregularities or fabrication flaws.

At present there are various approaches to analyse and design against fatigue failure. The traditional approach is based on the analysis on nominal stress that can be resisted under cyclic loading. This is determined by considering mean stresses and by making adjustments for the effects of stress raisers such as grooves, fillets and keyways. This is known as stress based approach. Another approach is the strain-based approach, which involves more detailed analysis of the localised yielding that may occur at stress raisers under cyclic loading. The third approach is the fracture mechanics approach, which specifically treats growing cracks using the methods of fracture mechanics.

2. LITERATURE REVIEW
2.1 Cyclic Stress-Strain Curve
The response of material subjected to cyclic inelastic loading is in the form of hysteresis loop as shown in Fig.1. The width of the loop is total strain range $\Delta\varepsilon$ and total height of the loop is $\Delta\sigma$, total stress range [1].

Total stress amplitude (σ_a) and strain amplitude (ε_a) are given by the equation,

$$\varepsilon_a = \Delta\varepsilon/2, \qquad \sigma_a = \Delta\sigma/2$$

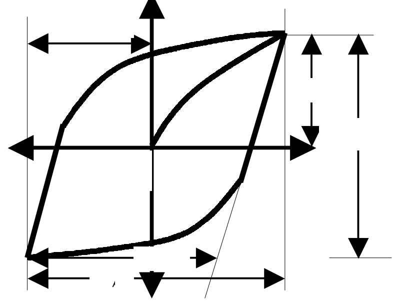

Fig.1 Hysteresis loop

Further the strain range $\Delta\varepsilon$ is,

$$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p$$

In terms of amplitudes, we can write,

$$\Delta\varepsilon/2 = \Delta\varepsilon_e/2 + \Delta\varepsilon_p/2$$

$$\Delta\varepsilon/2 = \Delta\sigma/2E + \Delta\varepsilon_p/2 \quad \ldots\ (i)$$

Cyclic stress-strain curve may be described by power curve equation,

$$\Delta\sigma/2 = K'\ (\Delta\varepsilon_p/2)^{n'} \quad \ldots(ii)$$

$$\Delta\varepsilon_p/2 = (\Delta\sigma/2K')^{1/n'} \quad \ldots(iii)$$

Putting equation (ii) and (iii) in equation (i)

$$\Delta\varepsilon/2 = (\Delta\sigma/2E) + (\Delta\sigma/2K')^{1/n'} \quad \ldots(iv)$$

Equation (iv) is the Cyclic Stress-Strain equation.

2.2 Low Cycle Fatigue

The usual way of presenting low cycle fatigue tests results is to plot plastic strain range $\Delta\varepsilon_p/2$ against $2N_i$. Fig. 2 shows that a straight line is obtained when plotted on log-log coordinates.
This behavior is best described by Coffin-Manson relation [1],

$$\Delta\varepsilon_p/2 = \varepsilon_f'\ (2N_i)^c \quad \ldots(i)$$

Also, c can be expressed according to Morrow as,

$$c = -1\ /\ (1 + 5n')$$

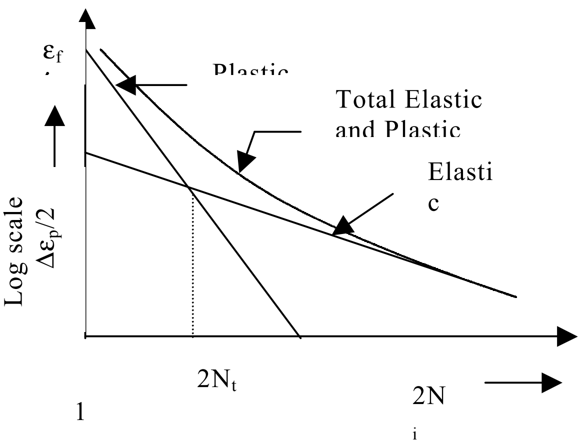

Fig. 2 Strain-Life Curve

Further using stress amplitude, the stress-life plot can be linearized as

$$\Delta\sigma/2=(\sigma_f/2)(2N_i)^b$$

The total strain is sum of the elastic and plastic strains. So in terms of amplitude, we can write,

$$\Delta\varepsilon/2=\Delta\varepsilon_e/2\ + \Delta\varepsilon_p/2 \quad \ldots(iii)$$

$$(\Delta\varepsilon/2) = (\Delta\sigma/2E)\ + (\Delta\varepsilon_p/2) \quad \ldots(iv)$$

From equations (i), (ii), (iii) and (iv)

$$\Delta\varepsilon/2 = \sigma_f'/E\ (2N)^b + \varepsilon_f'\ (2N)^c \quad \ldots\ (v)$$

Equation (v) is the strain life equation

2.3 Neuber's Rule

Neuber's rule [2] states that the theoretical stress concentrations the geometric mean of the stress and strain concentration. Neuber analyzed a specific notch geometry and derived the following relationship:

$$K_t = (K_\sigma K_\varepsilon)^{1/2}$$

By substituting the values of $K_\sigma = \sigma/S$ and $K_\varepsilon = \varepsilon/\ e$

$$K_t^2\ S\ e = \sigma\ \varepsilon$$

For nominally elastic behavior, remote strain, e, can be related to the remote stress, S, using Hook's law. The notch response in terms of applied load can be described as

$$K_t^2\ S\ (S/\ E) = \sigma\ \varepsilon$$

$$\underbrace{\sigma\ \varepsilon}_{\text{Notch response}} = \underbrace{(K_t\ S)^2\ /\ E}_{\text{applied load}}$$

The values on the right hand side are known while σ and ε are to be determined. Thus for loading S, the value on the right hand side is constant. Thus,

$$\sigma\ \varepsilon = \text{Constant} \quad \ldots\ (i)$$

This equation (i) shows the Neuber's relationship, which is the Equation of hyperbola. Since the notch stress – strain response must lie on stress - strain curve of the material. The intersection of the two curves (the cyclic stress strain curve and Neuber's hyperbola) provides the correct values of σ and ε for the initial loading, as shown in Fig. 3.

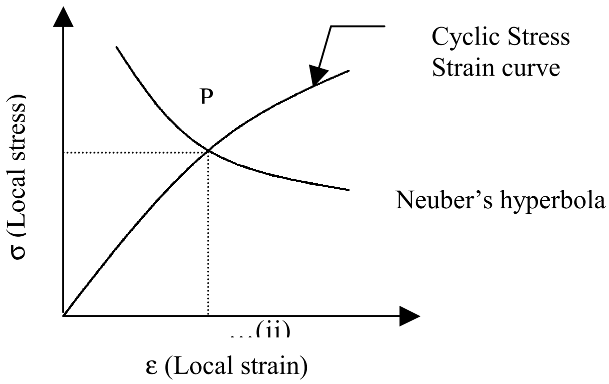

Fig. 3. Intersection of cyclic stress strain curve and Neuber's hyperbola**.**

Analytically, the stress and strain coordinates of this point, P may be found using the equation of the cyclic stress-strain curve.

$$\varepsilon = (\sigma / E) + (\sigma/K')^{1/n}$$

Substituting values from equation of Neuber's hyperbola.

$$[(\sigma/E) + (\sigma/K')^{1/n}]\ \sigma = (K_t S)^2 / E$$

3. FRACTURE MECHANICS APPROACH:

The fracture approach specifically treats growing cracks using method of fracture mechanics. The Paris relation [2] can be used, which was proposed in the early 1960s. In this equation

$$da/dN = C(\Delta K)^m \qquad \ldots \text{(i)}$$

where C and m are material constants and ΔK is the stress intensity factor range ($K_{max} - K_{min}$).

The material constants, C and m can be found in the literature and in data books. Values of the exponent, m, are usually between 3 and 4.

The crack growth life, in terms of cycles to failure, may be calculated using above equation (i). Thus, cycles to failure, N_p, may be calculated as Using the Paris formulations,

$$N_p = \int_{a_i}^{a_f} da/C(\Delta K)^m$$

where a_i is the initial crack length and a_f is the final (critical) crack length.

Because ΔK is a function of the crack length and a correction factor that is dependent on crack length, the integration above must often be solved numerically. As a first approximation, the correction factor, f(g), can be calculated at the initial crack length and equation can be evaluated in closed form [2].

The stress intensity factor range is

$$\Delta K = f(g)\ \Delta\sigma \sqrt{\pi a}$$

Substituting into the Paris equation yields

$$da/dN = C\ (f(g)\ \Delta\sigma \sqrt{\pi a}\ )^m$$

Separating variables and integrating (for $m \neq 2$) gives

$$N_p = \int_{a_i}^{a_f} da/C(\ f(g)\Delta\sigma \sqrt{\pi a}\ )^m$$

$$= \frac{2}{(m-2)\ C(\ f(g)\Delta\sigma \sqrt{\pi}\ )^m} \left(\frac{1}{a_i^{(m-2)/2}} - \frac{1}{a_f^{(m-2)/2}} \right)$$

3. MATERIAL PROPERTIES

Analysis has been carried out on piping components of two materials. The materials employed are low carbon steel and stainless steel. The material properties are as follows:

1) Low Carbon Steel [3]

a) Tensile stress strain curve

Young's modulus,	E = 203 GPa
Poisson's Ratio,	$\mu = 0.3$
Yield strength,	Syt = 302 Mpa
Ultimate strength,	Sut = 450 Mpa

b) Cyclic stress strain curve constants

Cyclic strength coefficient, K' = 629 MPa

Cyclic strain hardening coefficient, n' = 0.168

c) Strain life equation constants

Fatigue strength coefficient, $\sigma'_f = 586$ MPa

Fatigue ductility coefficient, $\varepsilon'_f = 0.2406$

Fatigue strength exponent, b = - 0.0752

Fatigue ductility exponent, c = - 0.4814

d) Paris law constants

$C = 3.807 \times 10^{-12}$ m/cycle, m = 3.03445

2) Stainless Steel 304LN [4]

a) Tensile stress strain curve

Young's modulus,	E = 195 GPa
Poisson's Ratio,	$\mu = 0.3$
Yield strength,	Syt = 318 MPa
Ultimate strength,	Sut = 617 MPa

b) Cyclic stress strain curve constants [6]

Cyclic strength coefficient, K' = 454 MPa

Cyclic strain hardening coefficient, n' = 0.351

c) Strain life equation constants [6]

Fatigue strength coefficient, $\sigma'_f = 211.52$ MPa

Fatigue ductility coefficient, $\varepsilon'_f = 0.1135$

Fatigue strength exponent, b = - 0.12741

Fatigue ductility exponent, c = - 0.362

d) Paris law constants

$C = 2.33 \times 10^{-12}$ m/cycle, m= 3.00

4. EXPERIMENTAL CONDITIONS:

The experimental basis proposed for the benchmark is the case of a pipe subjected to a four point bending at room temperature. In this pipe, the geometric singularity is an axi-symmetrical notch, machined in the midsection of pipe.

4.1 Specimen Geometry

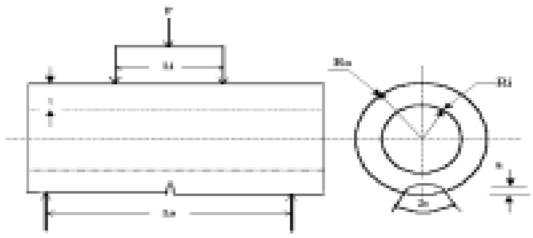

Fig.4 Specimen Geometry And Loading Conditions

The description of test geometry is given in Fig.4. In the cross-section, an axi-symmetric notch machined. The precise descriptions of the pipe and notch geometry for six different cases are tabulated below.

Table 1. Test geometry for low carbon steel

Case	Do (mm)	t (mm)	A (mm)	2C (mm)	Lo (mm)	Li (mm)
I	324	21.5	5	100	5000	1480

Table 2. Test geometry for stainless steel

Case	Do (mm)	t (mm)	a (mm)	2C (mm)	Lo (mm)	Li (mm)
II	170	14.4	3.42	14	1700	680

Table 3. Loading conditions for low carbon steel

Case	Maximum load, kN	Minimum load ,kN	Load ratio R
I	308	50.31	0.16

Table 4 Loading conditions for stainless steel

Case	Maximum load, kN	Minimum load, kN	Load ratio R
II	258	25.8	0.1

5. ANALYTICAL PROCEDURE

5.1 Prediction Of Fatigue Crack Initiation Life

Strain-life approach is used for calculation of crack initiation life, as it is reliable under number of circumstances. The general methodology consists of calculation of notch tip strain and stress for obtaining Neuber's hyperbola. The intersection of Neuber's hyperbola and cyclic stress-strain curve gives a strain value, which can be used in strain-life equation for calculating crack initiation life.

1. Calculation of Stress Intensity Factor Range

a) Calculate the reaction at the supports due to applied load 'p' and calculate maximum and minimum bending moments i.e. $(M_b)_{max}$ and $(M_b)_{min}$.

b) Calculate bending stress 'σ_b' due to bending moment by using formula.
$\Delta\sigma_b = \Delta M_b\ Y/I$
where, $\Delta M_b = M_{b\ max} - M_{b\ min}$

c) By knowing crack depth a, crack length L, and thickness of pipe t, determine a/t, R_i/t and l/a ratios and then find out the geometric factor f(g) from the data available in the handbooks.

d) Stress intensity factor range is given by
$\Delta K = \Delta\sigma\ f(g)\ \sqrt{(\pi a)}$,

2. Calculation of Crack Initiation Life:

a) The local stress $(\Delta\sigma)_{tip}$ at the tip if the notch is given by following equation,

$$(\Delta\sigma)_{tip} = (\Delta K / \sqrt{2\pi r}\ [\cos\theta/2\ (1+ \sin\theta/2\ \sin 3\theta/2)\ ]\ [1+ \rho/2r]$$

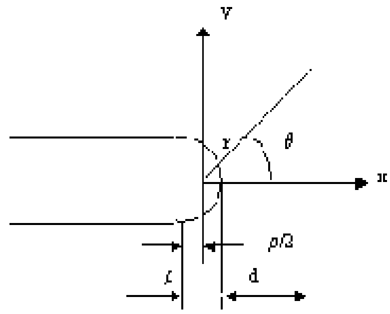

Fig.5 Reference coordinates and stresses in the near tip region of a notch in a plate.

Where, $r = d + \rho/2$ as shown in Fig.5

b) Strain range at the tip is given by the equation,

$$(\Delta\varepsilon)_{tip} = (\Delta\sigma)_{tip}/E\ (2\ (1+\mu)\ /\ 3\ )$$

c) Equation for Neuber's hyperbola is given by

$$(\Delta\sigma)_{tip}\ (\Delta\varepsilon)_{tip} = \text{Constant.}$$

d) From cyclic stress-strain curve strain amplitude is given by

$$(\Delta\varepsilon/2) = (\Delta\sigma/2E) + (\Delta\sigma/2K')^{1/n'}$$

e) Solving equations from (c) and (d) simultaneously for obtaining point P as shown in Fig.3. The corresponding value of strain is used for calculating initiation life in step (f).

f) Then by using strain life curve the crack initiation life can be calculated by the equation.

$$(\Delta\varepsilon/2) = (\sigma'_f /E)\ (2N_i)^{b} + \varepsilon'_f\ (2N_i)^{c}$$

5.2 Prediction Of Fatigue Crack Propagation Life

In fracture mechanics approach it is assumed that crack is present in the component. We can calculate No. of cycles (N_p) required to propogate the initial crack to its critical length.

From the Paris law relationship, we have derived the equation,

$$N_p = \int_{a_i}^{a_f} da / C(f(g)\Delta\sigma \sqrt{\pi a}\ )^m$$

From this equation N_p can be calculated

6. RESULTS AND DISCUSSION

6.1 Fatigue Crack Initiation

Analytical studies have been carried out for fatigue crack initiation from machined notch. In this analysis, crack driving force i.e. Stress Intensity Factor (K) at the notch tip has been evaluated for given loading condition of pipes. The SIF has been used for evaluating the stress at the notch tip. The notch tip radius'ρ' and characteristic distance 'd' has been considered for calculating stress at the notch tip. Actual stress and strain at the notch tip has been evaluated considering the Neuber's rule and cyclic stress-strain curve for the pipe material under study. Number of cycles for crack initiation has been obtained from the low cycle fatigue curve of the material and strain range, calculated above. Effect of notch tip radius 'ρ' for given characteristic distance 'd' is shown in Fig.6. for case I . It has been found that in general with increase in notch tip radius, number of cycles for fatigue crack initiation increases. This effect becomes significant for d >70μ. Finally it can be said that, the notch tip radius has significant role to play in number of cycles required for initiation of crack from machined notch. Therefore for analytical calculation, for number of cycles for fatigue crack initiation, notch tip radius should be known accurately.Effect of characteristic distance 'd' for given notch tip radius sre shown in Fig.7 for case I . Here also number of cycles required for fatigue crack initiation increase with increase in 'd' in general for given notch tip radius. This effect becomes more significant at ρ > 0.4mm. Characteristic distance 'd' depends on the material characteristics. For materials under study 'd' varies from 40-70 μ. Since there is significant effect of 'd' on number of cycles for fatigue crack initiation, this parameter also should be known accurately.

6.2 Fatigue Crack Propagation

Fatigue crack propagation analysis has been carried out on the basis of Paris law. In this study, analysis has been carried out for the crack growth up to a/t = 0.8 as the geometry factor available are up to a/t = 0.8. The crack depth Versus number of cycle curves for case I is shown in Fig.8

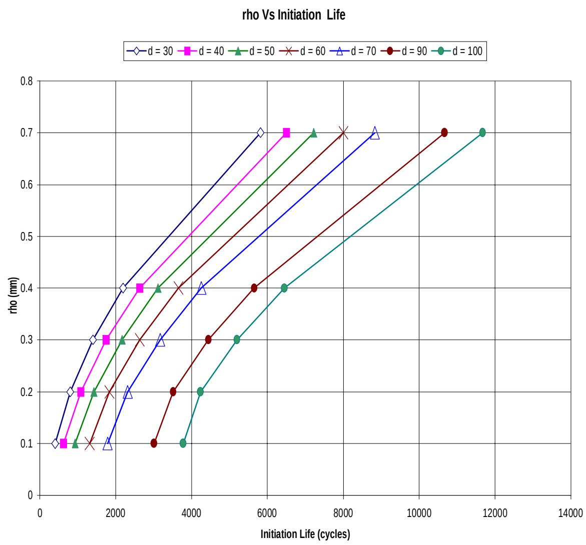

Fig. 6. Variation of 'ρ' versus N_i for CASE I

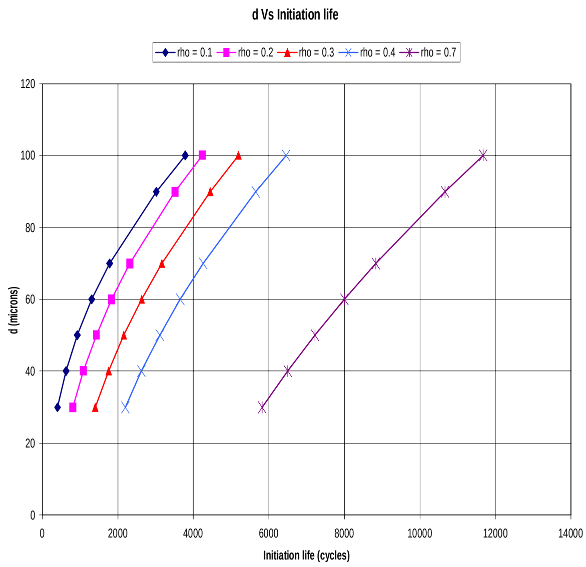

Fig. 7. Variation of 'd' versus N_i for CASE I

The figures show that the crack growth curve for carbon steel is in good agreement with the experimental results. This is because the material properties have been used in the analysis, which has been determined from the same pipe material. The Fig. 9 shows that crack growth curve for stainless steel differs from experimental curve to some extent. Out of the reasons for this variation may be due to the assumed material properties taken from the literature for similar steel. The graphs show the relationship between changing crack depth and corresponding elapsed stress cycles. The graph shows that most of the life of the component is spent while the crack depth is relatively small. While, considerably least of the life of a component is spent while depth is high. This is because initially material offers resistance to cracking and more stress is required to cause growth. After certain growth of crack, resistance offered reduces drastically and crack growth occurs faster, thus taking fewer cycles. Also, it is observed that for low cycle fatigue, majority of life is consumed in propagation of crack.

7. CONCLUSIONS:

Following conclusions can be drawn from the analytical studies carried out for the pipe having machined notch.

1. Number of cycles required for crack initiation can be predicted by evaluating fracture mechanics mere accurately. Notch tip radius ‘ρ’ and characteristic distance ‘d’ has significant effect on the number of cycles for crack initiation.
2. Fatigue crack growth life can be predicted well by Paris law. Paris law constants for the material shall be known using standard specimen for more accurate prediction.
3. The crack growth curve for carbon steel is in good agreement with the experimental results while the crack growth curve for stainless steel differs from experimental curve to some extent.

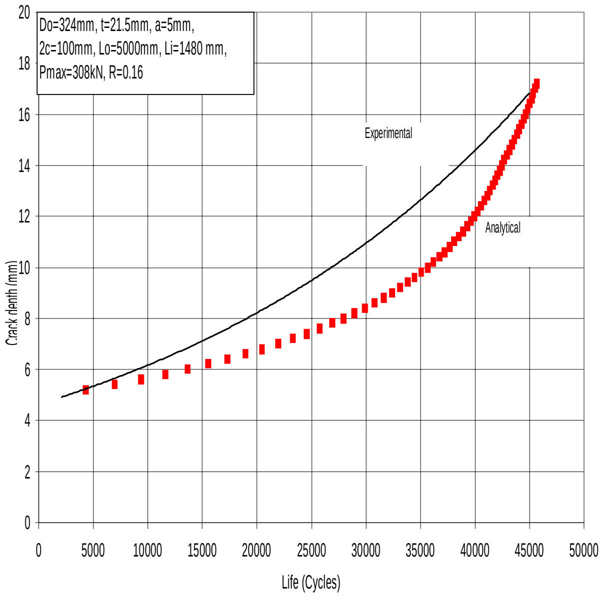

Fig. 8 Variation of ‘a’ versus N_p for CASE I

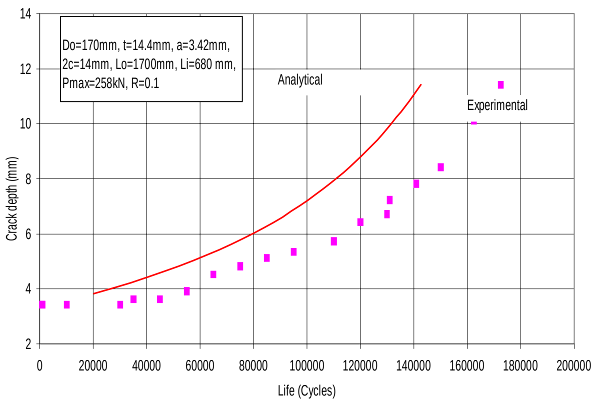

Fig.9. Variation of ‘a’ versus N_p for CASE II

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9. NOMENCLATURE

Symbol	**Meaning**	**Unit**
ρ	Notch root radius	mm
Δσ	Stress range	MPa
$(\Delta\sigma)_{tip}$	Stress range at crack tip	MPa
$(\Delta\varepsilon)_{tip}$	Strain range at crack tip	----
$\Delta\varepsilon_e$	Elastic strain range	----
$\Delta\varepsilon_p$	Plastic strain range	----
ΔK	Stress intensity factor range	MPa√m
2c	Crack length of elliptical crack	mm
$2N_i$	Reversals to crack initiation	cycles
a	Instantaneous crack depth	mm
C	Coefficient of Paris law	m/cycle
d	Characteristic distance	microns
da/dN	Crack growth rate	mm/cycle
D_i	Inner diameter	mm
D_o	Outer diameter	mm
E	Young’s modulus	GPa
f(g)	Geometric factor	----
K	Stress intensity factor	MPa√m
L_i	Distance between two loading points (Inner span)	mm
L_o	Distance between supports (outer span)	mm
N_p	Propagation life	Cycles
P	Applied load	kN
P_{max}	Maximum applied load	kN
P_{min}	Minimum applied load	kN
R	Stress ratio	----
S	Remote stress	MPa