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# DEVELOPMENT OF A SOFTWARE FOR AUTOMATED DESIGN OF MACHINING FIXTURES USING KINEMATIC ANALYSIS AND SYNTHESIS

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## ABSTRACT

This paper presents the methodology for automatic configuration of machining fixture for prismatic and non prismatic parts based on screw theory and kinematic analysis. The necessary and sufficient conditions for deterministic positions of the workpiece in 3-2-1 location is first identified and then used to synthesize the locator scheme. The methodology takes into account forces and moments during all time and ensure the clamping stability and positive reaction forces on locators. The mixed integer linear programming techniques are used to formulate the deterministic positioning. The number and position synthesis of the clamps is then made by mathematical formulation of the fixture reasoning forces. With input as 3-D drawing of the workpiece, the output is position of locators and clamps with respect to work origin.

**Keywords:** Twist,Wrench, 3-2-1 Location.

## 1. INTRODUCTION

The use of fixture in metal working industry is increasing in the current scenario of computer-integrated manufacturing. To cope with flexibility of modern computer controlled systems, the design lead-time should be reduced. Hence the fixture design process needs to be automated.

When the workpiece is subject to machining forces, the dynamic machining condition occurs. A viable fixture designed for a workpiece experiencing dynamic machining conditions must ensure,workpiece is restrained for all times, clamping forces are optimum, deterministic positioning, accessibility, stability of workpiece in fixture while under no external forces and positive clamping sequence.

Although 3-2-1 principle provides the basis for the location system, do not suggest any criteria for positioning of locators on the particular surface selected for location. For the force equilibrium of the workpiece during machining not only the correct position of location points but also the position of claming point and magnitude of clamping forces are very important.

Number of attempts have been made to find optimum position of location and clamping points using linear programming techniques, expert systems, screw theory etc. by various researchers. In this work an attempt is made to optimize positions of location and clamping points using motion analysis based on screw theory.

## 2. LITERATURE REVIEW

The progress in automating the design of fixture has been mainly in the form of computer aided fixture design[1] . Usually a number of standard and modular fixture elements are maintained in the database. An interactive graphics interface provides the designer with easy access to fixture element of workpiece information. However the selection of fixture elements and the configuration of fixture are made by the designer and thus very little of the design process is automated.

Darvishi and Gill[2] provides optimum solution for the fixture design problem by rule based method. The expert system is divided in four modules. The rules presented in the four selection modules have been written for a milling fixture, and the expert rules have been developed for more common types, that is, plate, angle plate, vice jaw indexing, duplex and the electromagnetic chuck. The rules need the physical and geometric attributes associated with the component specified. The manufacturing operations also require other attributes, typically, production rates, machining cycle, and fixture complexity, to be specified. Each of these can be assigned values. Each module is intended to achieve a particular goal, like fixture selection, reference plane selection, generic element selection and selection of standard element. The output of this can be used as preprocessor for the proposed methodology.

Lee and Haynes[3] used finite element analysis to minimize fixturing force and workpiece deflection. They were able to determine the deflection of the part for a specific fixture assembly. Menassa and DeVries[4] further extended this work using specific fixturing elements considering the complete fixture as elastic body. Finite element analysis provide an excellent tool to analyze the configuration of fixture before actual assembly thus reducing the set up time and the labor cost. The assumptions made in this work can be applied to proposed methodology with considerable accuracy. DeMeter et.al[5] presented a linear program which takes as input, specification of the fixturing scheme, the coefficient of friction at the contact and the expected machining forces. The solution to the linear program is the minimum clamping forces required to achieve the total restraint. They also presented the technique based on non linear programming for the computation of the position and the force magnitude applied at the clamps, subject to friction with work surface so that total mechanical leverage on the part is maximized. However it does not provide prescriptive analysis for the fixture element placement.

Rodrigo A Marin et.al[6] addresses the problem of generating well conditioned near optimal deterministic 3-2-1 location scheme. Given is the workpiece with identified planer datum faces, each with an associated convex region of admissible locator position. The necessary and sufficient conditions for deterministic positioning of workpiece in 3-2-1 locator scheme and the scheme is synthesized. However, it discusses only locator scheme for the prismatic parts.

The work presented in this paper deals with fixture configuration complete in respect of deterministic location, clamping stability and total restraint of the workpiece.

## 3. METHODOLOGY

From the mechanistic point of view, fixture must satisfy four functional requirements 1. locating stability 2.Deterministic location 3. Clamping stability 4. Total restraint.

### 3.1 Screw Theory

As screw theory based on concepts of twists and wrench is very suitable for freedom and constraint analysis for the object in 3-D. Hence, it is used for the work discussed here. Basic screw theory is founded on two essential principles. The first is that any displacement of a rigid body can be described as rotation 'θ'about the line in space and translation 'd' along the same axis. This is represented by entity twist 't'. Mathematically, twist is represented as,

$$t = [\theta, r * \theta + d]$$

Second principle is that any system of forces and couple acting on a body can be reduced to force 'f' along a line in space and couple 'c' about it. This is represented by entity called as wrench 'w'. Mathematically wrench is represented as,

$$w = [f, r * f + c]$$

where, 'r' is the vector for any point on the axis. The entity that allows unified representation of twist and wrench is called as 'screw'.Thus a twist is defined as screw with certain amplitude and wrench is defined as screw with certain intensity. Based on the notion of virtual work, virtual coefficient between 'w' and 't' can be expressed as,

$$\sigma(w, t) = (r * \theta + d) . f + w . (r * f + c)$$

The sign of $\sigma(w, t)$ distinguishes the following cases,

1. $\sigma(w, t) = 0$, 'w' and 't' are said to be reciprocal and two bodies remains in contacts for the infinitesimal displacement 't'.
2. $\sigma(w, t) < 0$, 'w' and 't' are said to be contrary i.e. 'w' opposes 't' and two bodies tend to penetrate each other.
3. $\sigma(w, t) > 0$, , 'w' and 't' are said to be repelling and the contact breaks.

When the workpiece is placed in a fixture, a system of contact wrenches is formed between the part and the locator. The axes of these wrenches are passing through the contact point and are normal to the surface of the part. Further, each wrench has a pitch of zero. The location scheme is thus deterministic if and only if for any twist 't' undergone by the part relative to the fixture, there is at least one contact wrench in the location that is either contrary or repelling.

### 3.2 Deterministic Location

Let the six locators of 3-2-1 scheme be represented by wrenches $w_1, w_2, w_3, w_4, w_5, w_6$, where $w_1, w_2$ and $w_3$ are placed on primary datum ($R_1$), $w_4$ and $w_5$ are placed on secondary datum ($R_2$) and $w_6$ is placed on tertiary datum ($R_3$). The arrangement of these wrenches is shown in figure 1.

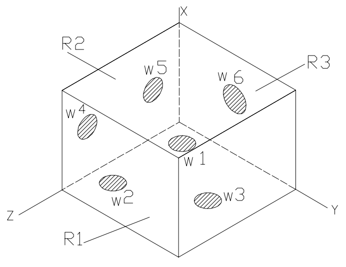

Fig.1: 3-2-1 location scheme

The forces and moments arrested by these wrenches are based on their positions from origin and are given in the table 1.

Table1: Fixture reasoning forces.

| | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ |
|---|---|---|---|---|---|---|
| $n_z$ | 0 | 0 | 0 | 0 | 0 | 1 |
| $n_y$ | 0 | 0 | 0 | 1 | 1 | 0 |
| $m_x$ | 0 | 0 | 0 | 2 | 4 | -2 |
| $n_x$ | 1 | 1 | 1 | 0 | 0 | 0 |
| $m_z$ | 1 | 1 | 3 | -2 | -2 | 0 |
| $m_y$ | -1 | -5 | -3 | 0 | 0 | 2 |

This will completely define the matrix W as:

$$W = \begin{pmatrix} f_{1x} & 0 & 0 & 0 & m_{1y} & m_{1z} \\ f_{1x} & 0 & 0 & 0 & m_{2y} & m_{2z} \\ f_{1x} & 0 & 0 & 0 & m_{3y} & m_{3z} \\ f_{4x} & f_{4y} & 0 & 0 & 0 & 0 \\ f_{4x} & f_{4y} & 0 & m_{5x} & m_{5y} & m_{5z} \\ f_{6x} & f_{6y} & f_{6z} & m_{6x} & m_{6y} & m_{6z} \end{pmatrix}$$

$$\mathit{det}(W) = f_{1x} f_{4y} f_{6z} m_{5x}((m_{2y} m_{1z} - m_{1y} m_{2z}) + (m_{3y} m_{2z} - m_{2y} m_{3z}) + (m_{1y} m_{3z} - m_{3y} m_{1z}))$$

To have a deterministic location scheme 'W' should be of full rank. This is possible if :

1.$f_{1x}$ must not be zero hence there is well defined

direction for $w_1$, $w_2$ and $w_3$.
2.$f_{4y}$ should not be zero i.e. $R_2$ cannot be parallel to $R_1$.
3.$f_{6z}$ not to be zero i.e. $R_3$ cannot be parallel to $R_1$ and $R_2$ .
4. $m_{5x}$ must not be zero, hence axis of $w_5$ can not be located on xy plane.
5. $(m_{2y}\, m_{1z} - m_{1y}\, m_{2z}) + (m_{3y}\, m_{2z} - m_{2y}\, m_{3z}) + (m_{1y}\, m_{3z} - m_{3y}\, m_{1z})$ should not be zero means axes of $w_1$,$w_2$ and $w_3$ are non co-planer, but are the vertices of the non degenerate triangle.

### 3.3 Behavior of Locators with respect to Twist

As discussed earlier two bodies will remain in contact, if $\sigma$ (w, t ) is zero and 'w' and 't' will be reciprocal. By setting $\sigma$ (w, t ) =0, a plane $\pi_o$ can be obtained which contains all locator wrenches reciprocal to 't'. The intersection of $\pi_o$ with given locator plane 'R' is the line $L_0$ containing all possible points on 'R' at which locator reciprocal to 't' can be placed. As shown in figure 2, the locator plane 'R' is subjected to twist $t_1$,$t_2$ and $t_3$. The reciprocal lines $L_{01}$,$L_{02}$ and $L_{03}$ of the three twists respectively divides 'R' in two sub regions '$R^+$' and '$R^-$'.

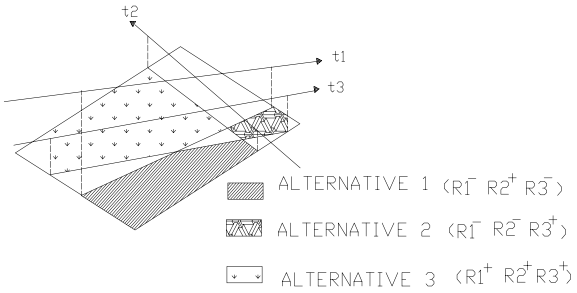

Fig.2: Behavior of a locator in different regions.

As shown in figure 2, three alternative regions are analyzed. For alternative 1, the locators in this region are on right side of twists '$t_1$' and '$t_3$', whereas on left side of '$t_2$'.Thus, the locators in this region are repelling to $t_2$ and contrary to'$t_1$' and '$t_3$'. For alternative 2, the locators are contrary to $t_1$ and $t_2$ and repelling to $t_3$. For alternative 3,any locator placed in this region is repelling to all the twists. Note that, if all the locators are placed in the region of alternative 3, then no locator is contrary to any twist '$t_j$' and thus the location is not possible.

### 3.4 Rules for optimization

The optimization the location scheme is based on the rules discussed below.
1.In the set of locators, there should be at least one locator wrench $w_i$ such that $\sigma (w_i, t_j) < 0$ i.e. should be contrary to $t_j$ .
2. Let 'n' is the number of locators and 'm' is the number of twists.Then i = {1-----n} and j = {1------m}.The selection of optimal position of locators from the feasible region is where, sum of all 'Z' is maximum. Where,
$Z = \Sigma\, \delta_{ij}$ subject to, $\delta_{ij} = 1$ if $\sigma (w_i, t_j) < 0$, else $\delta_{ij} = 0$.
3.We can thus position $w_6$ on $R_3$ so that it offers maximum opposition to the largest possible subset of twists '$T_3$' in 'T'; position $w_4$ and $w_5$ on $R_2$ so that they offer maximum opposition to the largest possible subset of twist '$T_2$' in 'T-$T_3$', and position $w_1$, $w_2$, $w_3$ on $R_1$ so that they offer maximum opposition and with respect to the twist 'T-$T_3$-T2'.

### 3.5 Positioning Locators on $R_1$, $R_2$ and $R_3$

The set of locators $w_1$, $w_2$ and $w_3$ has largest influence followed by $w_4$ and $w_5$ and wrench $w_6$ has least influence. Therefore, it is desirable to as much flexibility as possible in selecting positions of $w_1$, $w_2$ and $w_3$. This flexibility is enhanced if the selection of the locators is limited by coverage requirement involving as few twists in 'T' as possible. The same is true for w4 and w5. Based on the criteria discussed previously, the positioning of the locators on $R_1$, $R_2$ and $R_3$ is suggested as below:
Positioning $w_6$ on $R_3$: As discussed $w_6$ should cover the largest possible subset of twist in 'T' and makes maximum opposition to them. This can be accomplished in two stages, a: Maximization of coverage is achieved by selecting that sub region which maximizes 'Z' and
b: Maximization of opposition is the position within $R_3$ such that total opposition by $w_6$ will be maximized.
2. Positioning $w_4$ and $w_5$ on $R_2$: The following algorithm is used to position $w_4$ and $w_5$ on $R_2$.

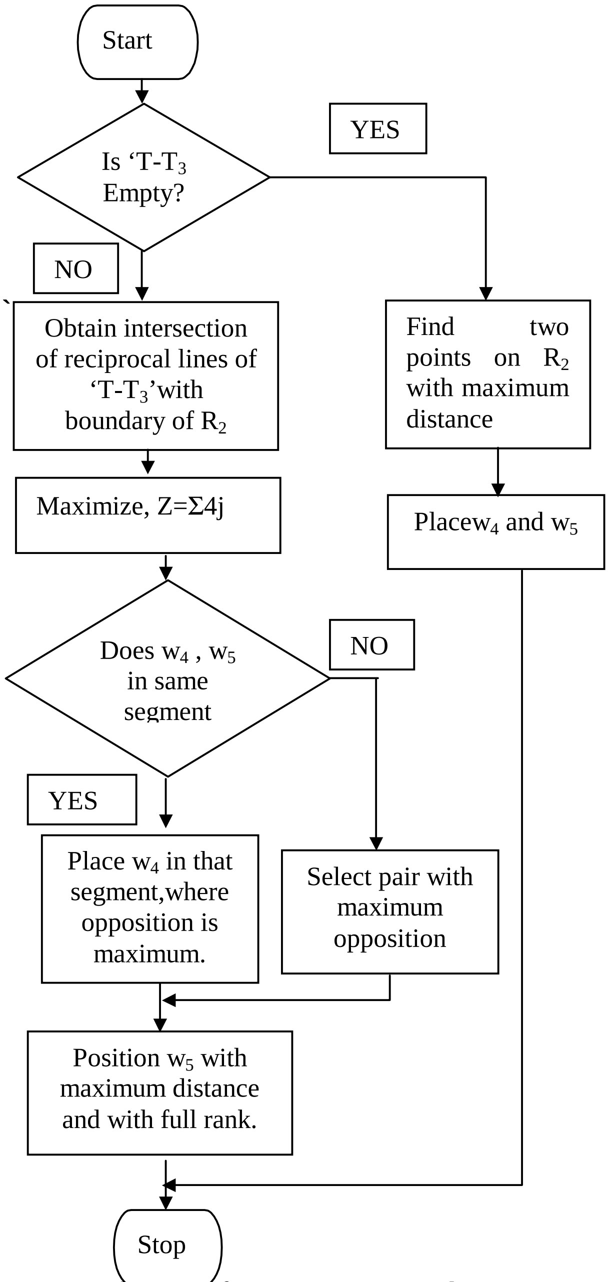

Figure 3: Flowchart for positioning $w_4$ and $w_5$ on $R_2$.

3.Positioning $w_1$,$w_2$ and $w_3$ on $R_1$: The algorithm shown in figure 3, can be used to position $w_1$,$w_2$ and $w_3$ on $R_1$with provision of best conditioned configuration for these locators, which is achieved by placing them at the vertices of the largest isosceles right angle triangle inscribed in $R_1$.

### 3.6 Clamping stability

Let '$w_p$' be the matrix of passive wrenches. With reference to table 1, '$w_p$' is given as,

$$w_p = \begin{pmatrix} 0 & A \\ B & T \end{pmatrix} \text{ and } \quad w_p^{-1} = \begin{pmatrix} 0 & A^{-1} \\ B^{-1} & -B^{-1}TA^{-1} \end{pmatrix}$$

Based on particular position and their distances from the origin, values of '$A^{-1}$', '$B^{-1}$' and '$-B^{-1}TA^{-1}$' are known and thus '$w_p^{-1}$' can be obtained.

The vector 'V' which is the coefficient of clamping force can be calculated as, [ $-w_p^{-1}$, $w_i$], where, '$w_i$' is the clamping wrench whose effect is to be studied.The clamping force is divided in two parts,

1.Horizontal clamping for which, $V_{hi} = -[A^{-1}, w]$.

2.Vertical clamping for which, $Vvi = -[-B^{-1}TA^{-1}, B^{-1}] w$.

The method for selection of horizontal clamping is discussed through following illustration.

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 4 & -2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1 & 2 & -0.5 \\ 1 & 1 & 0.5 \\ 1 & 0 & 0 \end{pmatrix}$$

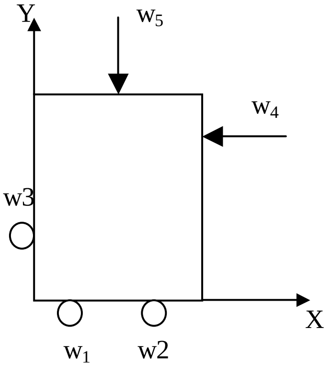

Fig.4 : Illustration for horizontal clamping.

As indicated in figure 4, $w_4 = [-1,0,6]$ then,

$V_{h1} = [f_1,f_2,f_3] = -[A^{-1}, w_4]$ = **[ 2, -2, 1 ]**.

Note that, '$V_{h1}$' has $f_2$ as a negative component, which implies that negative reaction force at a specific locator is needed to maintain the workpiece equilibrium, if it is used as a first clamping wrench.As it is not possible, it should not be used as the first clamp. Now consider another clamping wrench, $w_5 = [0,-1,-3]$. Then,

$V_{h2} = [f_1,f_2,f_3] = -[A^{-1}, w_5]$ = **[ 0.5, 2.5, 0 ].**

Since '$V_{h2}$' has either positive or zero components, it can be selected as the first clamp. However, as '$f_3$'=0, workpiece is still free to move away from the locator 3. As '$w_4$' has '$f_3$' positive it could be selected as the second clamp. Thus, two horizontal clamps are required such that '$w_5$' is applied first and then '$w_4$' is applied.

To ensure the total restraining of the workpiece, consider the external wrench w=[ -4, -1.5, 2.8, -3, -0.6, -0.4 ]. With reference to figure1,

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 5 \\ -1 & -5 & -3 \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} 1.75 & -0.5 & 0.25 \\ -0.25 & 0 & -0.25 \\ -0.5 & 0.5 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 4 & -2 \end{pmatrix}, \quad A^{-1} = \begin{pmatrix} -1 & 2 & -0.5 \\ 1 & 1 & 0.5 \\ 1 & 1 & 0 \end{pmatrix}$$

The total restraint is achieved by,

$$V = -[-B^{-1}TA^{-1}, B^{-1}]\, w$$

Where,

$$T = \begin{pmatrix} 0 & 0 & 0 \\ -2 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$-B^{-1}TA^{-1} = \begin{pmatrix} -0.5 & -1 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$V_i = -\left\{ \begin{pmatrix} -0.5 & -1 & 0 & 1.75 & -0.5 & 0.25 \\ 0.5 & 0 & 0 & -0.25 & 0 & -0.25 \\ 0 & 1 & 0 & -0.5 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} -4 \\ -1.5 \\ 2.8 \\ -3 \\ -0.6 \\ -4 \end{pmatrix} \right\}$$

$$= \begin{pmatrix} f1 \\ f2 \\ f3 \end{pmatrix} = \begin{pmatrix} \mathbf{2.45} \\ \mathbf{0.25} \\ \mathbf{0.30} \end{pmatrix}$$

Since all the components of 'V' (f1,f2 and f3) are positive, we can say that the workpiece is stable under the effect of wrench 'w'.

Initially, the weight of the workpiece is a downward wrench, which has the null 'V' vector. Therefore the initial state of the fixture is "zzz" (all zero), At any intermediate states, a clamp is feasible if its force instantiation does not make any component of 'f' negative. The goal fixture state is "ppp" (all positive).Each feasible clamp has a vector which represents its effect on clamping. Clamping wrenches are thus classified into twenty seven classes (viz. pnp,ppz,zzz,pzz etc.) The feasible clamps in class "ppp" is singly able to create a force closure. In contrast clamps in classes containing all zero, or all negative or combination of zero and negative (viz. "zzz", "zzn"etc.), should be categorily ruled out from consideration, as they do not contribute to constrain the workpiece. This elimination leaves only nineteen feasible classes. As the state proceeds from "zzz" to "ppp" the focus of the search shifts accordingly. When the combination of the clamps is selected, to ensure clamping stability,

$$\sum_{i=1}^{n} f_i v_i \geq 0; \quad \text{for all} \quad n \leq N_c$$

Where, $N_c$ = Number of clamps.

The program thus starts with searching that position of clamps, which has either all positive (PPP) or combination of positive and zero components. The next position of the clamp is the one which has that positive component which was zero in the previous wrench . The search is completed when the goal state ‘PPP’ (i.e. all positive) is achieved.

The methodology discussed above can be applied to locate the circular part from the internal surface (i.e. Internal cylinder) or external surface. In practice, the common elements used for this purpose are cylindrical plug locator or vee - block locator respectively.

The internal cylinder in the component is identified by feature recognition techniques [7] . The effect of using a single plug locator is equivalent to the effect of locator wrenches $w_{4,}$ $w_5$ and $w_6$ used in the prismatic part shown in figure 1, for the purpose of analysis. However, the problem of jamming may occur with the plug locator. This problem is eliminated by providing a) circular relief on the diameter of the plug locator. b) Triangular relief c) Chamfer d) Shape of the locator so as to contain in the sphere.

Similarly, the effect of vee block location to locate from the external surface of the part can be analyzed. In this case the vee block is considered to replace the locating wrenches $w_1$, $w_3$, and $w_6$ shown in figure 1. Though $90^0$ angle of vee is commonly used to centralize the workpiece, based on the tolerance analysis, the optimum angle of vee is calculated as,

$$\alpha = 2 \sin^{-1} ( \delta_w / (\tau - \delta_w ).$$

## 4.RESULTS

To check the validity of the software extensive runs were carried out for a variety of components. The results obtained for a typical component is discussed below. The input is 3-D drawing of the component.

Figure 5(a) is the component for which optimum fixture configuration is desired. Figure 5 (b) shows feasible regions for optimum position of the locators and subsequently, the final positions of the locators in these regions. Figure 5 (c) shows the placement of locators on the component. Figure 5(d) shows candidate-clamping wrenches. Figure 5 (e) gives the optimum positions of the clamping wrenches along with optimum locator position

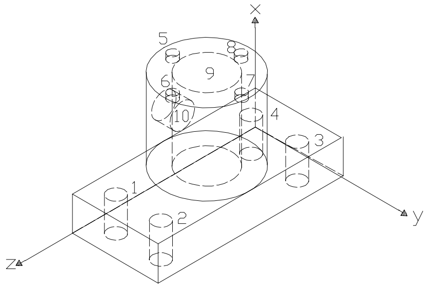

Figure 5(a)

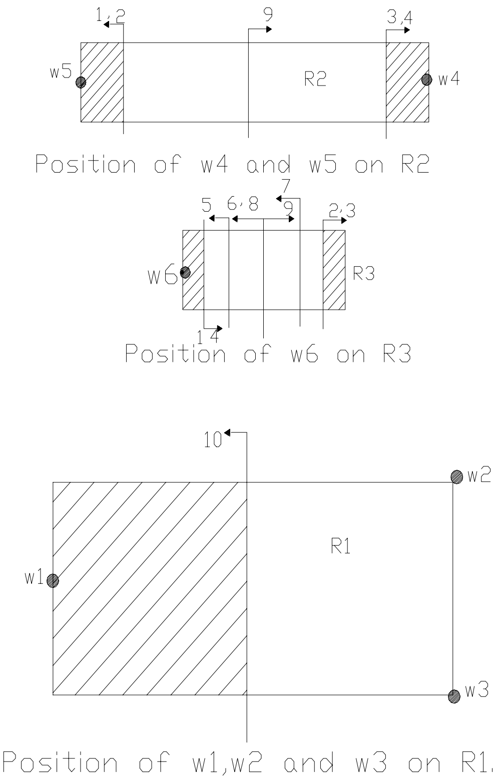

Fig.5 (b )

The total set of twists, T = { $t_1$, $t_2$----------$t_{10}$}.After positioning $w_6$ on $R_{3,}$ $T_3$ = { $t_5$, $t_6$, $t_7$, $t_8$ }. Then after positioning $w_4$ and $w_5$ on $R_2$ , $T_2$ = { $t_1$, $t_2$, $t_3$, $t_4$, $t_9$}. Thus, finally twist to be opposed by $w_1$, $w_2$ and $w_3$ is $T_1$= { $t_{10}$}

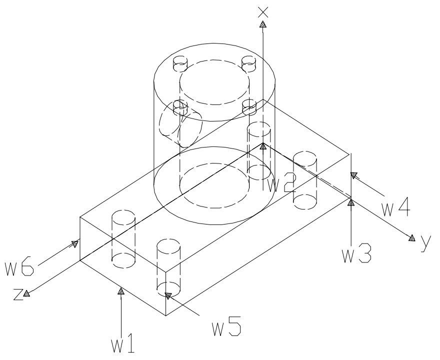

Fig. 5 (c )

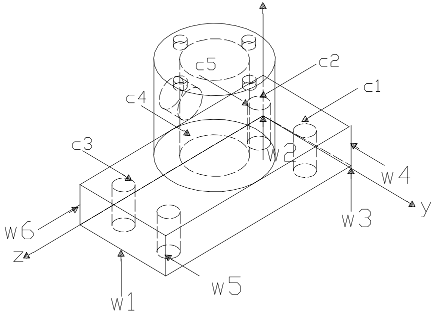

Fig. 5(d)

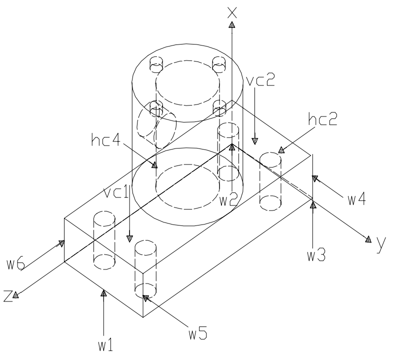

Fig. 5 (e)

## 5.CONCLUSIONS

The major conclusions of this work are as discussed below:

1.Deterministic location is an essential property of a locator scheme and conditions discussed here allows simple and quick evaluation.

2. As compared to other techniques, the technique discussed here provides faster solution as most of the operations are comparisons and the search is drastically reduced due to focus shift.

3. As the software can be used for the non prismatic parts also, has rendered it as a general purpose software.

4. The software has its own input module and eliminates the necessity of any other drafting software for this purpose.

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## 7. NOMENCLATURE

| **Symbol** | **Meaning** | **Unit** |
|---|---|---|
| $\theta$ | Angle of rotation | Degrees |
| d | Displacement | mm |
| f | force | N |
| c | Couple | N-mm |
| $\alpha$ | Angle of Vee | Degrees |
| $\delta_w$ | Tolerance | mm |
| $\tau$ | Machining Allowance | mm |