

# INTEGRATED CONTROL CHART SYSTEM FOR MULTI-STAGE AND MULTI-STREAM MANUFACTURING PROCESSES

M. Shamsuzzaman<sup>1</sup> and Zhang Wu<sup>2</sup>

<sup>1</sup>PhD Research Student, MPE, Nanyang Technological University, Singapore 639798

<sup>2</sup>Associated Professor, MPE, Nanyang Technological University, Singapore 639798

## ABSTRACT

This article proposes algorithm for the optimization design of an integrated control chart system monitoring a multi-stage and multi-stream manufacturing system. The optimization design is carried out within the available resources (inspectors equipped with measuring instruments) of the manufacturing system. The effectiveness of the chart system is improved and the product quality is enhanced without additional cost and effort. Furthermore, the shop operators can utilize and understand the optimal control chart system as easily as for the traditional system. The proposed algorithm is illustrated through an example.

**Keywords:** Statistical process control, Integrated control chart system, Optimization design.

## 1. INTRODUCTION

During the fabrication of a product, it usually passes through different manufacturing process stages. The integration of all these stages results in a *manufacturing system*. In the manufacturing of a mechanical part, each stage corresponds to the machining of a dimension. As an example, several process stages are required in order to fabricate the dimensions of the part shown in Fig 1(a). Some of the dimensions are critical to the overall quality of the product and control charts are used to monitor the corresponding process stages. In this article, a *control chart system* is defined as an integrated system consisting of all of the charts used to monitor the critical stages of a manufacturing system. Some of the stages may have more than one parallel streams or machines (because of lower production rate and other factors). Typically, the quality characteristics in parallel streams in a single stage have the same mean, standard deviation and target [1]. Therefore, in this article, a group of identical control charts are used to monitor the quality characteristics of the parallel streams in a stage.

For the part shown in Fig 1(a), dimensions to be machined in stages one, two and three are functional (critical) ones, which directly affect the function of the product. Therefore, three control charts as a system are required to carry out on-line monitoring of the manufacturing processes. Furthermore, two identical machines are employed to turn dimension 1 (the outer surface). Thus, control chart one for the first stage has two duplicates; each monitors one of the two parallel machines. A block diagram of this control chart system is displayed in Fig 1(b). It is assumed that the quality requirements on the non-functional dimensions can be easily met and on-line monitoring is unnecessary.

In this article, a process means one single-stream stage or one stream in a multi-stream stage. The total number  $M$  of the processes that are monitored by the control chart system is equal to

$$M = \sum_{i=1}^s g_i \quad (1)$$

where,  $s$  is the number of the functional stages and  $g_i$  is the number of streams in the  $i$ th stage ( $g_i \geq 1$ ). Since  $g_i$  streams in the  $i$ th stage are monitored by identical control charts (with same sample size, sampling interval and control limits), there are only  $s$  different charts in the control chart system.

Traditionally, the control charts monitoring different process stages of a manufacturing system are designed in isolation. For example, the  $\bar{X}$  chart is a charting tool for monitoring the process mean shifts and is used most widely in Statistical Process Control (SPC). Designing a control chart means selecting the sample size  $n$ , sampling interval  $h$ , lower control limit  $LCL$  and upper control limit  $UCL$ . In traditional system,  $n$  is usually set around 5, control limit co-efficient  $k$  is made identical (typically taken as 3) for each chart and  $h$  is mainly decided by the working shifts (rational sub-grouping) [2].

However, if all of the charts in a system are designed in an integrated and optimal manner, the overall effectiveness of the chart system may be improved significantly. The optimization design of the control chart system optimizes sample size  $n$  and sampling interval  $h$  of the chart at each stage of the manufacturing system in such a manner that the overall performance of the chart system is improved without additional cost.

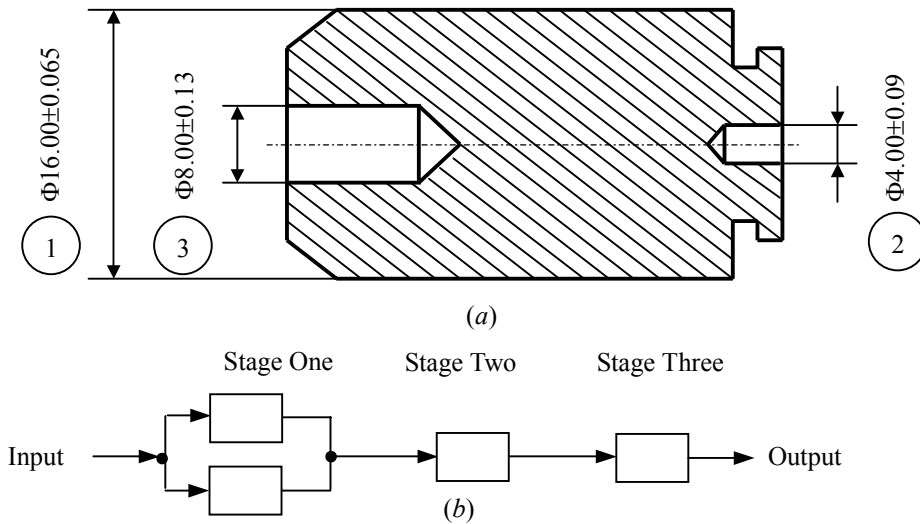


Fig 1. (a) The Dimensions of the Part and (b) The Process Stages

The optimization design of the control chart system may result in different  $n$  and  $h$  values or allocate different power to different charts in a system based on the values of certain parameters (e.g., process capability, magnitudes of process shifts) that would affect the performance of the chart system.

The idea of applying multiple univariate control charts to individual variables has been discussed in Montgomery's textbook on SPC [2]. However, little literature is found in the optimization design of an integrated control chart system for a manufacturing system. Woodall and Ncube [3] investigated the multivariate CUSUM procedures and defined the out-of-control condition for a system comprising several charts. Mortell and Runger [4] proposed and developed the group control chart for monitoring the output from multiple streams of a single stage. Runger *et al.* [1] used principal component analysis to develop the control charts that are able to detect both common and assignable causes in multiple process streams. Nonetheless, in these studies, the charting parameters (e.g., sample size and sampling interval) were not designed in an integrated and optimal manner.

It is convenient to evaluate the performance of a control chart in terms of out-of-control Average Time to Signal (ATS) [2]. In this article, ATS is defined as the average time that one of the control charts in the chart system gives an out-of-control signal subsequent to any process in a manufacturing system going out of control [3]. When any process is out of control, the control chart system should signal quickly (i.e., minimum out-of-control ATS), but when the whole manufacturing system is in control, the chart system should produce large in-control  $ATS_0$  (i.e., minimum false alarms).

A few assumptions and conventions are adopted in this article.

- (1) All of the processes are independent of each other.
- (2) Only one process is out of control at any moment in a manufacturing system.

- (3) Only the  $\bar{X}$  chart is studied in details in this study.
- (4) The probability distribution of the quality characteristic  $x$  (e.g., the dimension of a mechanical part) is assumed normal with a constant standard deviation.
- (5) The  $g_i$  parallel streams in the  $i$ th stage have the same mean, standard deviation, and target and use the identical control charts.
- (6) If several operators are engaged in the inspection activities in a manufacturing system, they can share their inspection time in all the stages.
- (7) The time required to inspect a unit in a stage is substantially smaller than the sampling interval of that stage.

## 2. OPTIMIZATION DESIGN

### 2.1 Specifications

For the optimization design of the integrated control chart system, the following specifications are needed:

- |             |                                                                                                 |
|-------------|-------------------------------------------------------------------------------------------------|
| $s$         | number of stages in the control chart system                                                    |
| $g_i$       | number of streams in the $i$ th stage                                                           |
| $\mu_{0,i}$ | in-control process mean in the $i$ th stage                                                     |
| $\sigma_i$  | constant standard deviation of the process in the $i$ th stage                                  |
| $t_i$       | time required to inspect a unit in the $i$ th stage                                             |
| $\delta_i$  | maximum allowable mean shift in the $i$ th stage                                                |
| $p_i$       | probability of out-of-control occurrence in the $i$ th stage                                    |
| $\tau$      | minimum allowable in-control $ATS_0$                                                            |
| $R$         | available number of operators (equipped with measuring instruments) in the manufacturing system |

Most of the above parameters are used in designing a traditional control chart and can be determined easily. The numbers  $s$  and  $g_i$  can be determined from the block diagram (e.g., Fig 1(b)). The process parameters  $\mu_{0,i}$  and  $\sigma_i$  are usually estimated from the data observed in the pilot run. The value of  $t_i$  can be easily estimated from historical data or the results of a field test.

The maximum allowable mean shift  $\delta_i$  indicates the rejectable quality level (the barely tolerated mean shift) and can be determined either directly on the basis of the required level of quality, manufacturing cost and customers' requirements, or from the specified tolerance and the allowable process capability ratio  $C_{pk}$ . To maintain the quality of a product,  $C_{pk}$  for any process must be greater than a minimum allowable value,  $c_{pk,\min}$ . The product quality will degrade significantly if  $C_{pk}$  is lower than  $c_{pk,\min}$ . Usually,  $c_{pk,\min}$  is decided in accordance with the considerations, for example, whether the process is an existing one or a new one, or whether the product is critical to safety [2]. A default value of one may be used for  $c_{pk,\min}$ . For the  $i$ th process stage,

$$\delta_i = USL_i - \mu_{0,i} - 3\sigma_i c_{pk,\min,i} \quad (2)$$

Where, the upper specification limit  $USL_i$  in a manufacturing system can be found from the engineering drawings.

The probability  $p_i$  can be estimated from the historical data of the out-of-control cases or it may be reasonably estimated by,

$$p_i = g_i / M = g_i / \sum_{j=1}^s g_j \quad (3)$$

This means that the estimated probability that the out-of-control case happens in the  $i$ th stage is proportional to the number of parallel streams in this stage.

The variable  $\tau$  is specified based on the trade-off between the false alarm rate and the detection power. If the cost of handling the false alarms is high, a larger  $\tau$  should be used in order to reduce the false alarm frequency. However, a large  $\tau$  may impair the effectiveness of the control chart at the same time. The actual in-control  $ATS_0$  must be greater than or equal to  $\tau$ .

The value of  $R$  depends on the total time that the operators are engaged with the inspections. For example, if each of three operators spends one fifth of his/her time in the inspection activities, then,  $R$  is equal to 3/5.

## 2.2 Optimization Model

Based on the above specifications, the optimization design of the integrated control chart system can be conducted by using the following nonlinear optimization model:

$$\text{Objective function: } ATS = \text{minimum} \quad (4)$$

$$\text{Constraint functions: } ATS_0 \geq \tau \quad (5)$$

$$r \leq R \quad (6)$$

$$\text{Design variables: } n_i, h_i (i = 1, 2, \dots, s)$$

where,  $ATS$  is the average time to signal of the control chart system when any process in the manufacturing system is out of control (or more specifically, when any process mean has shifted by an amount of  $\delta_i$ ).  $ATS_0$  is the average time to signal when the whole system is in control (or all process means remain in  $\mu_{0,i}$ ).  $r$  is the

required number of operators equipped with measuring instruments.

## 2.3 Optimization Process

The first step in the optimization process is to decide the initial values of  $n_i$  and  $h_i$ . Then, the step sizes of  $n_i$  and  $h_i$  are determined. Finally, a simple gradient based search algorithm is employed to approach the optimal set of  $n_i$  and  $h_i$  step by step.

### Determination of the initial values of $n_i$ and $h_i$

It is a common practice to take sample size around 5 for the  $\bar{X}$  chart, so the initial value of the sample size at all the stages can be set as,

$$n_i = 5 \quad (7)$$

In general, small samples are taken at short intervals or large samples at long intervals. That means, the sampling intervals are generally proportional to the sample sizes. Therefore, it can be approximately written as,

$$h_i \propto n_i, \text{ or, } h_i = an_i \quad (8)$$

Where,  $a$  ( $a > 0$ ) is a proportionality constant, and is given by,

$$a = \frac{1}{R} \sum_{i=1}^s g_i t_i \quad (9)$$

### Determination of step sizes of $n_i$ and $h_i$

The minimum incremental value of sample size is one. Therefore, the step size ( $\Delta n_i$ ) of  $n_i$  for all stages is taken as one. It is rational to make the step size  $\Delta h_i$  of  $h_i$  proportional to  $h_i$  itself, that is

$$\Delta h_i \propto h_i \text{ or, } \Delta h_i = bh_i \quad (10)$$

where,  $b$  ( $b > 0$ ) is a proportionality constant, and can be calculated by,

$$b = \frac{\sum_{i=1}^s g_i t_i}{\sum_{i=1}^s h_i} \quad (11)$$

### A simple gradient based search algorithm

During optimization,  $n_i$  and  $h_i$  is increased or decreased step by step until the out-of-control  $ATS$  cannot be reduced any more. For each step, the search algorithm is described below.

It is well known that, increasing  $n_i$  by  $\Delta n_i$  results in the decrease of  $ATS$  (or gaining in detection power) and decreasing  $n_i$  results in the increase of  $ATS$  (losing in detection power). Conversely, decreasing  $h_i$  by  $\Delta h_i$  means moving the sampling interval in the gaining direction (gaining in detection power) and increasing  $h_i$

by  $\Delta h_i$  means moving the sampling interval in the losing direction (losing in detection power).

If  $n_i$  and  $h_i$  are handled jointly by a general variable  $X_j$  ( $j = 1, 2, \dots, 2s$ ). The gaining factor  $G_j$  pertaining to  $X_j$  per unit increase of  $r$  is given by,

$$G_j = \frac{|\Delta ATS_j|}{\Delta r_j} = \frac{-\Delta ATS_j}{\frac{\partial r}{\partial X_j} \Delta X_j} \quad (12)$$

Where,  $\Delta ATS_j$  is the change of the out-of-control  $ATS$  when one  $X_j$  moves one step in the gaining direction (i.e., 1 for  $n_i$ , or  $-\Delta h_i$  for  $h_i$ ) and all other  $X_j$ s are kept unchanged.  $\Delta ATS_j$  is always smaller than zero. The out-of-control  $ATS$  of the chart system is calculated by using equation (A6) in the Appendix.

The steps of the search algorithm for the optimization process are summarized below:

- (1) Specify  $s, g_i, \mu_{0,i}, \sigma_i, \delta_i, p_i, \tau, t_i$  and  $R$ .
- (2) Set  $\Delta n_i = 1$ , for  $i = 1, 2, \dots, s$ .
- (3) Decide the starting point (i.e., the initial values of  $n_i$  and  $h_i$ ) by Eqs. (7) and (8).
- (4) Determine  $ATS(I)$ , that is the out-of-control  $ATS$  value at the starting point by Eq. (A6).
- (5) Calculate  $\Delta h_i$  by Eqs. (10) and (11) at the current design point.
- (6) Calculate the gaining factor  $G_j$  by Eq. (12) for each general variable  $X_j$  ( $n_i$  and  $h_i$ ) at the current design point.
- (7) Rank  $X_j$  according to the descending order of  $G_j$ .
- (8) Find the delimiting point  $K$ , then move the first  $K$  general variables  $X_j$  one step in the gaining directions and move other general variables one step in the losing directions. It results in a new design point.
- (9) Determine  $ATS(II)$ , that is the  $ATS$  values in the new design point by Eq. (A6). The in-control  $ATS_0$  (calculated by Eq. (A2) in the Appendix) and the control limits (calculated by Eqs. (A7) and (A8) in the Appendix) are determined in the meantime.
- (10) If  $(ATS(I) - ATS(II))$  is substantial with reference to some predetermined criterion, the current design point is replaced by the new point and  $ATS(I)$  is replaced by  $ATS(II)$ . Then, go back to step (5) for a new search.
- (11) Otherwise (i.e., if  $(ATS(I) - ATS(II))$  is negligible), the optimization is terminated.

In this optimization, a set of optimal  $n_i$  and  $h_i$  is obtained, which improves the out-of-control  $ATS$  of the chart system and ensures ( $r \leq R$ ) and ( $ATS_0 \geq \tau$ ).

A computer program **QualityExpert** in C language has been developed to carry out the optimization design. Usually, an optimal solution is obtained in a few CPU seconds using a personal computer.

### 3. EXAMPLE

A mechanical part as in Fig 1(a) is to be manufactured. Among all process stages, only three of them are

functional and have to be closely monitored by the  $\bar{X}$  charts. Stage one turns the outer surface ( $\phi 16.00 \pm 0.065mm$ ); stage two drills the small hole ( $\phi 4.00 \pm 0.09mm$ ); and stage three bores the large hole ( $\phi 8.00 \pm 0.13mm$ ). The block diagram of the control chart system is shown in Fig 1(b). The specifications of the system are listed below:

- Number of stages in the system:  $s = 3$
- Number of streams:  $g_1 = 2, g_2 = g_3 = 1$
- In-control process means (mm):  $\mu_{0,1} = 16.00, \mu_{0,2} = 4.00, \mu_{0,3} = 8.00$
- Process standard deviations (mm):  $\sigma_1 = 0.012, \sigma_2 = 0.023, \sigma_3 = 0.038$

Originally, the system uses  $3-\sigma \bar{X}$  charts for all the processes, i.e., all charts have the same type I error probability of 0.0027. The values of  $n_i, h_i, LCL_i$  and  $UCL_i$  of the system are as follows:

- Stage one:  $n = 5, h = 39$  min,  $LCL = 15.984mm, UCL = 16.016mm, (k = 3)$
- Stage two:  $n = 5, h = 39$  min,  $LCL = 3.969mm, UCL = 4.031mm, (k = 3)$
- Stage three:  $n = 5, h = 39$  min,  $LCL = 7.949mm, UCL = 8.051mm, (k = 3)$

To improve the effectiveness of the chart system, optimization design is carried out by using the proposed algorithm. For this purpose, some more specifications are required. From the historical data, the values of  $t_i$  are obtained:

$$t_1 = 0.25 \text{ min}, t_2 = 0.40 \text{ min and } t_3 = 0.50 \text{ min.}$$

For this set of  $t_i$  values, the in-control  $ATS_0$  of the traditional  $3-\sigma$  chart system is equal to 3602 min.

Since each of the three operators assigned to the manufacturing system spends (on average) 6% of his/her time (per shift per day) for the inspection activities, the value of  $R$  is equal to 0.18.

The  $\tau$  and  $R$  value are specified as 3602 min and 0.18, respectively, with the aim to maintain the false alarm rate and required manpower at the same level as in the traditional chart system.

The values of the maximum allowable mean shift  $\delta_i$  are calculated from  $USL$  (shown in Fig 1(a)) and  $c_{pk,\min}$ . In this example,  $c_{pk,\min}$  is specified as one. The maximum allowable mean shifts are calculated by Eq. (2):

$$\delta_1 = 0.029mm, \delta_2 = 0.021mm \text{ and } \delta_3 = 0.016mm.$$

For this set of mean shifts, the out-of-control  $ATS$  of the traditional  $3-\sigma$  chart system is equal to 436 min. The probabilities  $p_i$  of out-of-control occurrences can be estimated from historical records, which show that the numbers of out-of-control cases occurring in each of the three stages are 8, 6 and 5, respectively. Therefore,

$$p_1 = 0.421, p_2 = 0.316, \text{ and } p_3 = 0.263.$$

Based on the above specifications, the computer program **QualityExpert** designs the optimal chart system at almost no time in an Intel Pentium IV 2.4GHz PC. The results are listed below.

Stage one:  $n = 2, h = 25 \text{ min}, LCL = 15.975\text{mm},$   
 $UCL = 16.025\text{mm}, (k = 3)$

Stage two:  $n = 4, h = 38 \text{ min}, LCL = 3.965\text{mm},$   
 $UCL = 4.035\text{mm}, (k = 3)$

Stage three:  $n = 15, h = 77 \text{ min}, LCL = 7.971\text{mm},$   
 $UCL = 8.029\text{mm}, (k = 3)$   
 $ATS_0 = 3602 \text{ min}, r = 0.18, ATS = 281 \text{ min}$

It is observed that both the optimal chart system and the traditional 3- $\sigma$  chart system have the same  $ATS_0 (= 3602)$  and  $r (= 0.18)$  values. However, the optimization design reduces  $ATS$  from 436 to 281. It translates to an improvement of 64.4% in detection effectiveness.

#### 4. CONCLUSION

This article proposes algorithm for the optimization design of an integrated control chart system for multi-stage and multi-stream processes. It is found that, by properly allocating the power among the individual charts based on the values of the influential parameters, the effectiveness of the system as a whole can be significantly improved, and therefore, the product quality is further guaranteed. The optimization design also ensures that no extra resources to maintain the quality control activities are required and the false alarm rate is not increased.

The design algorithms of the optimal control chart system can be easily computerized. It is believed that the general algorithm discussed in this paper can be modified and then applied to the designs of other types of control charts in addition to the  $\bar{X}$  chart.

#### 5. APPENDIX

##### Calculation of in-control $ATS_0$ of the chart system

The probability  $p_0$  that the whole control chart system generates a false alarm in a time unit is,

$$p_0 = 1 - \prod_{i=1}^s (1 - \alpha_i / h_i)^{g_i} \quad (A1)$$

Where,  $\alpha_i$  is the type I error probabilities. Finally,

$$ATS_0 = 1 / p_0 \quad (A2)$$

##### Calculation of out-of-control $ATS$ of the chart system

Given that one process in the  $i$ th stage goes out of control, then the value of the out-of-control  $ATS$  in the  $i$ th stage is given by,

$$ats_i = \left( \frac{1}{q_i} - 1 \right) \times h_i + 0.5 \times h_i \quad (A3)$$

This is the steady-state formula, which provides a more accurate and realistic evaluation of the out-of-control

$ATS$  than the zero-state counterpart [5]. In Eq. (A3),  $1/q_i$  is actually the Average Run Length ( $ARL$ ).

$$q_i = 1 - (1 - \alpha_i)^{g_i-1} \beta_i \prod_{j=1, j \neq i}^s \left( 1 - \frac{h_i \alpha_j}{h_j} \right)^{g_j} \quad (A4)$$

In Eq. (A4),  $\beta_i$  is the type II error probability.

$$\beta_i = \Phi \left( \frac{UCL_i - (\mu_{0,i} + \delta_i)}{\sigma_i / \sqrt{n_i}} \right) - \Phi \left( \frac{LCL_i - (\mu_{0,i} + \delta_i)}{\sigma_i / \sqrt{n_i}} \right) \quad (A5)$$

Since an out-of-control case may occur in any of the  $s$  stages, the final  $ATS$  for the manufacturing system is,

$$ATS = \sum_{i=1}^s (ats_i \cdot p_i) \quad (A6)$$

Where,  $p_i$  is the probability that the out-of-control case takes place in the  $i$ th stage.

##### Calculation of $LCL_i$ and $UCL_i$ of the chart system

The lower control limit  $LCL_i$  and upper control limit  $UCL_i$  of each chart in the chart system can be calculated by,

$$LCL_i = \mu_{0,i} - k_i \frac{\sigma_i}{\sqrt{n_i}} \quad (A7)$$

$$UCL_i = \mu_{0,i} + k_i \frac{\sigma_i}{\sqrt{n_i}}$$

$$k_i = \Phi^{-1}(1 - 0.5\alpha_i) \quad (A8)$$

Where,  $\Phi^{-1}()$  is the inverse function of the cumulative probability function of the standard normal distribution and  $k_i$  is the control limit co-efficient.

#### 6. REFERENCES

1. Runger, G. C., Alt, F. B. and Montgomery, D. C., 1996, "Controlling Multiple Stream Processes with Principal Components", International Journal of Production Research, 34 (11):2991-2999.
2. Montgomery, D. C., 2001, *Introduction to Statistical Quality Control*. John Wiley & Sons, NY.
3. Woodall, W. H. and Ncube, M. M., 1985, "Multivariate CUSUM Quality Control Procedures", Technometrics, 27(3):285-292.
4. Mortell, R. R. and Runger, G. C., 1995, "Statistical Process Control of Multiple Stream Processes", Journal of Quality Technology, 27:1-12.
5. Wu, Z., Zhang X. L. and Yeo, S. H., 2001, "Design of the Sum-of-Conforming-Run-Length Control Charts", European Journal of Operational Research, 132(1):187-196.