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INSTANTANEOUS KINEMATICS USING DUAL NUMBER ALGEBRA-II: APPLICATION TO HYBRID, PARALLEL MANIPULATORS AND SINGULARITIES

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ABSTRACT

In this paper, we apply the theory developed in the part I [1] to parallel and hybrid manipulators, and analyze their singularities. We present a general method for obtaining the dual velocity Jacobian matrix for parallel and hybrid manipulators. This Jacobian is used directly to obtain the principal screws of ω-basis and $\mathbf{h}-$basis. The motion of the *passive* joint variables are accounted for in arriving at the equivalent screws corresponding to the actuated joints, and the determination of configuration-space singularities is a natural part of the formulation. The gained twists at such a singularity is computed analytically. We also discuss loss type of singularity, and show that the concept of DOF-partitioning helps us to identify loss in the translational DOF or rotational DOF separately. The results are illustrated with the help of the 3-RPS parallel manipulator, and a 6-DOF spatial hybrid manipulator.

Keywords: Instantaneous Kinematics, Dual Number, Screw Theory, Parallel Manipulator

1 INTRODUCTION

Determination of the principal twists of the end-effector of a hybrid or parallel manipulator is significantly more difficult than in the case of serial manipulators. The construction of the velocity Jacobian is difficult due to several reasons. Firstly, the end-effector motion is affected by the passive and the active joint movements, and the elimination of the passive motion in terms of the active variables and their derivatives is difficult in general. Moreover, the nature of singularities are more complicated in such manipulators [2]. Due to these reasons, the use of velocity Jacobians in determining instantaneous kinematics of parallel devices is very limited in literature. However, the statics problem of in-parallel actuated devices is simple, and traditionally, the *wrench-basis* have been used to yield geometric information about the instantaneous kinematics using the concept of reciprocity [3]. While such results are useful, they are not available analytically in general [4], thus limiting their applicability. In this paper, we introduce a general method for the derivation of the dual Jacobian of a parallel or hybrid manipulator. At a non-singular configuration, we determine the passive joint rates, and the *equivalent Jacobian* which incorporates their contribution to the end-effector twists. Using the equivalent dual Jacobian, we find the principal screws of ω-basis and $\mathbf{h}-$basis directly, following the formulation in [1].

At a singular configuration, a parallel device may gain or lose degrees-of-freedom. We derive the conditions for the same and also determine the twists *gained* or *lost* at a singularity. The approach is *direct*, i.e., we do not make use of the concept of reciprocal screws, and we are able to derive all the results analytically.

The paper is organized as follows: in section 2, we describe the derivation of the equivalent dual Jacobian. In section 3, we discuss about singularities leading to gain and loss of DOF. We present an example of a 3-RPS parallel manipular and a 6-DOF spatial hybrid manipulator in section 4. Finally, we conclude in section 5.

2 DERIVATION OF THE DUAL JACOBIAN

In this section, we explain a general method of obtaining the dual velocity Jacobian for constrained mechanisms, such as parallel and hybrid manipulators. The formulation leads naturally to the identification of singularities leading to gain of DOF and determination of the corresponding gained twists[1].

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[1]We use the *right-invariant* or *space-fixed* twists in this paper.

2.1 Determination of the Passive Joint Rates

In parallel manipulators, closed-loop mechanisms, and hybrid manipulators, there are one or more *passive* or unactuated joints in addition to the active joints. For a non-redundant parallel device at a non-singular configuration, it is possible to obtain *m independent holonomic constraints* of the form

$$\eta(\theta,\phi) = \mathbf{0} \tag{1}$$

where m is the number of passive joints, η is a m-vector, θ, and ϕ, are n- and m-vectors denoting the actuated and passive joint variables respectively. The constraints can also be written in the *Pffafian form* as

$$\mathbf{J}_{\eta\theta}\dot{\theta} + \mathbf{J}_{\eta\phi}\dot{\phi} = \mathbf{0} \tag{2}$$

where $\mathbf{J}_{\eta\theta}$ and $\mathbf{J}_{\eta\phi}$ denote the Jacobians $\frac{\partial\eta}{\partial\theta}$ and $\frac{\partial\eta}{\partial\phi}$ respectively. The active joint variables, θ, are functions of time alone, and at a nonsingular configuration, ϕ can be determined *uniquely* in terms of θ from equation (1) [2]. From the implicit function theorem, it may be concluded immediately, that at a non-singular configuration, $\mathbf{J}_{\eta\phi}$ is invertible, and we can obtain the *passive joint rates* as

$$\dot{\phi} = -\mathbf{J}_{\eta\phi}^{-1}\mathbf{J}_{\eta\theta}\dot{\theta} \tag{3}$$

2.2 The Rotational Jacobian ($\mathbf{J}_{\omega}^{eq}$)

In a serial manipulator, we can express the orientation of the end-effector explicitly in terms of the active joint variables, and obtain the Jacobian corresponding to the angular velocity directly from the same [1]. For parallel and hybrid manipulators we compute the angular velocity from the linear velocities of any three *non-colinear* points $\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3$, on the rigid-body. Denoting the *space-fixed* angular velocity as ω, the linear velocities, $\dot{\mathbf{P}}_i$, when expressed in the space-fixed frame, satisfy the relationships

$$\dot{\mathbf{P}}_i - \dot{\mathbf{P}}_j = \omega \times (\mathbf{P}_i - \mathbf{P}_j), i,j = 1,2,3, i \neq j$$

From the three relationships above, we obtain

$$\begin{aligned}\omega = &\frac{1}{\mathbf{P}_1 \times \mathbf{P}_2 \cdot \mathbf{P}_3}((\mathbf{P}_3 \times \mathbf{P}_1) \times (\dot{\mathbf{P}}_2 - \dot{\mathbf{P}}_1) \\ &+ (\dot{\mathbf{P}}_3 - \dot{\mathbf{P}}_1) \cdot \mathbf{P}_1(\mathbf{P}_2 - \mathbf{P}_1))\end{aligned} \tag{5}$$

We can also express the angular velocity as a linear combination of the active and passive joint rates as

$$\omega = J_{\omega\theta}\dot{\theta} + J_{\omega\phi}\dot{\phi} \tag{6}$$

where

$$\begin{aligned}\mathbf{J}_{\omega\theta} = &\frac{1}{\mathbf{P}_1 \times \mathbf{P}_2 \cdot \mathbf{P}_3}((\mathbf{P}_3 \times \mathbf{P}_1) \times (\frac{\partial\mathbf{P}_2}{\partial\theta} - \frac{\partial\mathbf{P}_1}{\partial\theta}) \\ &+ (\frac{\partial\mathbf{P}_3}{\partial\theta} - \frac{\partial\mathbf{P}_1}{\partial\theta}) \cdot \mathbf{P}_1(\mathbf{P}_2 - \mathbf{P}_1))\end{aligned}$$

The matrix $\mathbf{J}_{\omega\phi}$ may be obtained by replacing θ by ϕ in the last equation. Substituting the expression for $\dot{\phi}$ from equation (3) in equation (6), we get

$$\begin{aligned}\omega &= \mathbf{J}_{\omega\theta}\dot{\theta} - \mathbf{J}_{\omega\phi}\mathbf{J}_{\eta\phi}^{-1}\mathbf{J}_{\eta\theta}\dot{\theta} \\ &= \mathbf{J}_{\omega}^{eq}\dot{\theta}\end{aligned}$$

where $\mathbf{J}_{\omega}^{eq}$, the equivalent Jacobian has the expression

$$\mathbf{J}_{\omega}^{eq} = \mathbf{J}_{\omega\theta} - \mathbf{J}_{\omega\phi}\mathbf{J}_{\eta\phi}^{-1}\mathbf{J}_{\eta\theta} \tag{7}$$

2.3 The Translational Jacobian($\mathbf{J}_{v}^{eq}$)

The Jacobian corresponding to the translational velocity is obtained from $\mathbf{v} = (\dot{\mathbf{d}} + \mathbf{d} \times \omega)$, (see equation(5) in [1]) where $\mathbf{d}$ is a chosen reference point on the rigid-body, and $\mathbf{v}$ is the linear velocity of a point on the rigid-body which is instantaneously coincident with the origin of the fixed frame [5]. We express $\mathbf{d}$ in the space-fixed frame in terms of the configuration variables θ and ϕ, and hence $\dot{\mathbf{d}} = \frac{\partial\mathbf{d}}{\partial\theta}\dot{\theta} + \frac{\partial\mathbf{d}}{\partial\phi}\dot{\phi}$. Using the expression of ω from equation (6), we get the ith column of the matrix $\mathbf{J}_{\mathbf{v}\theta}$ as

$$(\mathbf{J}_{\mathbf{v}\theta})_i = (\frac{\partial\mathbf{d}}{\partial\theta})_i + \mathbf{d} \times (\mathbf{J}_{\omega\theta})_i \tag{8}$$

where $(\mathbf{J}_{\omega\theta})_i, (\frac{\partial\mathbf{d}}{\partial\theta})_i$ denote the ith column of $\mathbf{J}_{\omega\theta}, \frac{\partial\mathbf{d}}{\partial\theta}$ respectively. Similarly, we obtain the matrix $\mathbf{J}_{\mathbf{v}\phi}$, by replacing θ by ϕ in equation (8). Finally, using equation (3) to account for the motion of the passive joints, we get the equivalent Jacobian as

$$\mathbf{J}_{\mathbf{v}}^{eq} = \mathbf{J}_{\mathbf{v}\theta} - \mathbf{J}_{\mathbf{v}\phi}\mathbf{J}_{\eta\phi}^{-1}\mathbf{J}_{\eta\theta} \tag{9}$$

2.4 The Dual Jacobian

The dual Jacobian, mapping the active joint rates $\dot{\theta}$ to the end-effector twists can now be written by composing the Jacobians corresponding to the rotational and translational parts:

$$\hat{\mathbf{J}}^{eq} = (\mathbf{J}_{\omega}^{eq} + \varepsilon\mathbf{J}_{\mathbf{v}}^{eq}) \tag{10}$$

The resultant twists can now be expressed in terms $\hat{\mathbf{J}}$, and the columns of $\hat{\mathbf{J}}$ may be interpreted as the equivalent screws as follows[2]:

$$\hat{\mathcal{V}} = \hat{\mathbf{J}}^{eq}\dot{\theta} = \sum_{i=1}^{n} \hat{\$}_i\dot{\theta}_i \tag{11}$$

This method allows us to compute the Jacobian matrices symbolically. Moreover, since we avoid using any particular parameterization of $so(3)$ in obtaining the Jacobian matrices, the only possibility of encountering a singularity in the formulation is when $\det \mathbf{J}_{\eta\phi} = 0$. This case is discussed in detail in the next section.

[2] $\hat{\$}_i$ differs from screws in one respect, that the norm of their real parts need not equal unity. It is more precise to consider $\hat{\$}_i\dot{\theta}_i$ as the input twist corresponding to the ith active joint.

3 ANALYSIS OF SINGULARITIES

In this section, we discuss two types of singularities, leading to *gain* and *loss* of DOF respectively. It is known that serial manipulators can show only loss of DOF, while parallel and hybrid device can show both gain and loss type of singularity [6].

3.1 Gain Type of Singularity

As explained in the previous section, a parallel device gains one or more degrees-of-freedom in the configuration space when one of the constraint Jacobians, $\mathbf{J}_{\eta\phi}$, loses rank. The gain of DOF equals the nullity of $\mathbf{J}_{\eta\phi}$. The *gained* passive motions lie in the nullspace of $\mathbf{J}_{\eta\phi}$, and may be obtained by solving the equation (see, for example, [7])

$$\mathbf{J}_{\eta\phi}\dot{\phi}_i = \mathbf{0},\ \ i = 1,\ldots, nullity(\mathbf{J}_{\eta\phi}) \qquad (12)$$

The effect of this gain is that the manipulator end-effector can now twist about one or more screws even with all the actuators locked. These twists are obtained by setting $\dot{\theta} = \mathbf{0}$ in equation (11):

$$\hat{\mathcal{V}}_i^g = \mathbf{J}_{\omega\phi}\dot{\phi}_i + \varepsilon\mathbf{J}_{\mathbf{v}\phi}\dot{\phi}_i = (\mathbf{J}_{\omega\phi} + \varepsilon\mathbf{J}_{\mathbf{v}\phi})\dot{\phi}_i \qquad (13)$$

We can obtain the *gained screws* $\hat{\$}_i^g$ by normalizing $\hat{\mathcal{V}}_i^g$.

If the nullity of $\mathbf{J}_{\eta\phi}$ is more than one, the gained twists will be a linear combination of the gained screws. Any *gained twist* may be written as

$$\hat{\mathcal{V}}^g = \sum_{i=1}^{nullity(\mathbf{J}_{\eta\phi})} c_i\hat{\$}_i^g,\ c_i \in \Re \qquad (14)$$

Equation (14) is comparable to equation (11), the gained screws replacing the input screws and the arbitrary coefficients c_i taking the place of $\dot{\theta}_i$. Under a normalization constraint, $\sum_{i=1}^{nullity(\mathbf{J}_{\eta\phi})} c_i^2 = 1$ (similar to the unit-speed constraint on $\dot{\theta}$), the principal twists in the subspace of $se(3)$ spanned by the gained screws can be obtained analytically following the formulation mentioned in [1]. This is possible since we need to solve at the most a cubic equation.

3.2 Loss Type of Singularity

The loss kind of singularity is said to occur when the manipulator end-effector fails to twist about certain screws in spite of full actuation. This results in the loss of one or more degrees-of freedom of the end-effector [2]. In our formulation, we treat the rotational degrees-of-freedom as decoupled from purely translational degrees-of-freedom, and hence the loss may occur in either rotational or translational DOF. We first consider loss of rotational DOF.

Loss of Rotational DOF

The manipulator end-effector has $1, 2$ or 3 rotational degrees-of-freedom depending upon the number of non-zero eigenvalues $\hat{\mathbf{g}}$ has at a non-singular configuration. If at a singular configuration, m additional eigenvalues vanish[3], then we say that the manipulator has lost m rotational degrees-of-freedom. It may be noted that the corresponding pitch also vanishes, and hence the corresponding twist can reduce to a pure translation in the nullspace of $\hat{\mathbf{J}}^{eq}$ at that configuration. We look at the possibilities on a case by case basis.

One-degree-of-freedom:

In this case, the principal screw reduces to a null vector, $\mathbf{0} + \varepsilon\mathbf{0}$, unless the original DOF was translational (as in a P-joint), in which case there is no loss of rotational DOF possible.

Two-degrees-of-freedom:

From the set of equations $(19, 20)$ in [1], it can be seen that only one of the $\hat{\lambda}$s ($\hat{\lambda}_2$ in particular) can vanish, under the condition $\sin^2\phi_{12} = 0$. The other eigenvalue can be obtained from equation (19) in [1] as $\hat{\lambda}_1 = 2(1 + \varepsilon(h_1^\omega + h_2^\omega))$. The two principal twists collapse to $\hat{\mathcal{V}}_1^\omega = \frac{1}{\sqrt{2}}(\hat{\$}_1 + \hat{\$}_2)$ which gives the resultant rotational DOF in this case, and $\hat{\mathcal{V}}_2^\omega = \frac{1}{\sqrt{2}}(\hat{\$}_1 - \hat{\$}_2)$, now forms the left nullspace of $\hat{\mathbf{J}}^{eq}$, signifying a translatory DOF in addition to the residual rotational DOF.

Three-degrees-of-freedom:

In this case, there may be loss of one or two angular degrees-of-freedom, the conditions of the same are found from equation(22) in [1] as $c_{12}^2 + c_{23}^2 + c_{31}^2 - 2c_{12}c_{23}c_{31} - 1 = 0$ and $c_{12}^2 + c_{23}^2 + c_{31}^2 - 2c_{12}c_{23}c_{31} - 1 = 0 = (3 - c_{12}^2 - c_{23}^2 - c_{31}^2)$ respectively. As in the case of two-degrees-of-freedom rigid-body motion, the non-zero roots may be computed from the same equation, which reduces to a quadratic and a linear equation in λ in the two cases respectively. The eigenvectors of $\mathbf{g}$ can be computed symbolically, and therefrom the principal twists in the columnspace and null space of $\hat{\mathbf{J}}^{eq}$ can be obtained. It may be noted here that the loss of one or two rotational DOF results in those many principal twists being pushed from the column space into the left nullspace of $\hat{\mathbf{J}}^{eq}$, which has interesting consequences when DOF is greater than 3.

Degrees-of-freedom(n)> 3:

The treatment in this case follows exactly the case of three-degrees-of-freedom. We need to consider equation (25) in [1] instead of equation (22) in [1]. The conditions for loss of one or two rotational DOF are $a_{n-3} = 0$, and $a_{n-3} = 0 = a_{n-2}$ respectively.

Loss of Translational DOF:

The number of pure translational degrees-of-freedom equal the number of linearly independent pure dual vectors in the left null space of $\hat{\mathbf{J}}^{eq}$ and they span the space of pure translational velocities of the rigid body. We write their dual parts as the columns of a $3 \times m$ real matrix, $\mathbf{B}$, and let the rank of $\mathbf{B}$ be $r(r \leq 3)$. At a singularity leading to loss of translational DOF, the rank of $\mathbf{B}$ reduces by 1, 2 or 3. It may be noted

[3] m can be either 1 or 2. All the three eigenvalues can vanish only for a purely Cartesian manipulator, whose analysis can be done much more conveniently by looking at its linear velocity distribution in $\Re^3$.

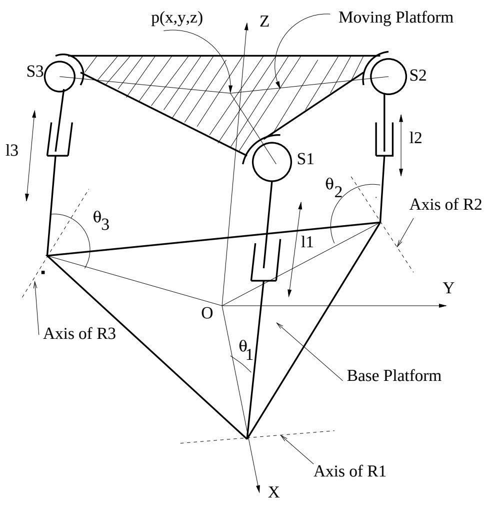

Figure 1. The 3-RPS Parallel Manipulator

that loss of rotational motion also leads to the addition of a column to **B**, but since the rank of **B** is limited to 3, the degeneracy of rotational motion does not lead to an additional translational DOF if rank of **B** is already 3.

4 ILLUSTRATIVE EXAMPLES

In this section, we illustrate above developed theory by an example of a 3-RPS parallel manipulator, and a 6-DOF hybrid gripper.

4.1 3-RPS Parallel Manipulator

Geometry of the 3-RPS

The 3-RPS is a platform-type fully in-parallel actuated device (shown in figure 1). The active and passive variables in this case are given by $\theta = (l_1, l_2, l_3)^T$ and $\phi = (\theta_1, \theta_2, \theta_3)^T$ respectively. The top and bottom platforms are taken to be equilateral triangles with sides a and b respectively, and we assume $a = \sqrt{3}/2,\ b = 2a$. The constraint equations, $\eta_k, k = 1,2,3$, are obtained from the fact that the distance between the spherical joints, $\mathbf{P}_i$, equal the side of the top platform:

$$||\mathbf{P}_i - \mathbf{P}_j|| = a^2,\ i,j = 1,2,3, i \neq j \tag{15}$$

The reference point on the moving platform is chosen as its centroid:

$$\mathbf{d} = (x,y,z)^T = \frac{1}{3}(\mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3)$$

Results of the ω-basis

At a non-singular configuration defined by $l_1 = 1,\ l_2 = 2/3,\ l_3 = 3/4$, and corresponding passive variables $\theta_1 = 0.878516$rad, $\theta_2 = 0.905239$rad and $\theta_3 = 0.120906$rad, the dual eigen values of $\hat{\mathbf{g}}$ are are computed analytically, yielding the numerical values

$$\hat{\lambda}_1 = 3.92612 + \varepsilon(-0.91996)$$
$$\hat{\lambda}_2 = 1.87034 + \varepsilon(0.44710)$$
$$\hat{\lambda}_3 = 0 + \varepsilon(0)$$

The three principal pitches in the ω-basis are given by

$$h_1^\omega = -0.117159,\ h_2^\omega = 0.119524,\ h_3^\omega = \infty$$

respectively. The principal twists, at this configuration, are given by

$$\hat{\mathcal{V}}_1^\omega = (1.61698, -1.11205, 0.27354)^T + \varepsilon(0.59693, 1.14580, -0.55239)^T$$
$$\hat{\mathcal{V}}_2^\omega = (-0.63533, -1.06544, -0.57580)^T + \varepsilon(0.13730, -0.28080, -0.02015)^T$$
$$\hat{\mathcal{V}}_3^\omega = (0,0,0)^T + \varepsilon(0,0,0.90320)^T$$

The DOF decoupling is apparent in the purely translational nature of the third principal twist. Intuitively, the existence of one pure translation mode can be reasoned from the fact that the rotary joint axes in the base are in a plane and the top platform can be made to translate parallel to the **Z** axis, without any angular motion, by changing the leg lengths. The strength of our approach is that we can analytically capture this *partitioning* of DOF.

Results of the h−basis

The principal screws of the **h**−basis are computed as

$$\hat{\mathcal{V}}_1^h = (1.10500, -1.35959, 0)^T + \varepsilon(0.55638, 0.85062, -0.80373)^T$$
$$\hat{\mathcal{V}}_2^h = (9.73597, 7.91281, 6.20082)^T \times 10^{-9} + \varepsilon(0, 5.25180, -0.90320 \times 10^9)^T \times 10^{-9}$$
$$\hat{\mathcal{V}}_3^h = (9.73597, 7.91281, 6.20082)^T \times 10^{-9} + \varepsilon(0, 5.25180, 0.90320 \times 10^9)^T \times 10^{-9}$$

The principal pitches are computed as

$$h_1^h = -0.17648,\ h_2^h = -2.85962 \times 10^7,\ h_3^h = 2.85962 \times 10^7$$

respectively. It may be noted that $h_3^h = -h_2^h \to \infty$, and $||\hat{\mathcal{V}}_2^h|| = ||\hat{\mathcal{V}}_3^h|| \to 0$, even as **g** has rank 2 as seen in equation (17). By observation, the direction of the pure translation can be obtained by deducting $\hat{\mathcal{V}}_2^h$ from $\hat{\mathcal{V}}_3^h$. It may be noted that we get $(0,0,0)^T + \varepsilon(0,0,2 \times 0.9032)$ which is consistent with the translational velocity obtained in ω-basis. The advantage of exact analytical computation, as opposed to numerical computation, is also clearly seen from the values of the principal screws in **h**−basis. One can observe that some entries are $O(1)$ whereas others are $O(10^{-9})$ and most numerical computations will round them off to 0. If they are

Table 1. DH PARAMETERS OF THE jth FINGER

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_j
2	$\frac{\pi}{2}$	l_{j1}	0	ψ_j
3	0	l_{j2}	0	ϕ_j
4	0	l_{j3}	0	0

rounded off to zero, then we will get two translatory modes which is incorrect.

Singular Configuration (Gain)

The 3-RPS gains a DOF when one of its legs lie in the plane of the top platform [8]. We consider the case when the first leg is in such a configuration, and $l_2 = l_3$, $\theta_2 = \theta_3$. The gained passive velocity is obtained analytically as

$$\dot{\phi} = (1,0,0)^T$$

and the corresponding gained twist is obtained from equation (13) as

$$\begin{aligned}\hat{\mathcal{V}}^g = & \left(0, -\frac{2\sqrt{3}l_1}{3a}, 0\right)^T \\ & + \varepsilon\left(\frac{2l_1}{3a}\sqrt{2a^2 - ab - b^2 + 3l_1(l_1 + \sqrt{3}a)}, 0, \frac{l_1}{3}\right)^T\end{aligned}$$

Following [8], it is intuitively clear that the gained passive motion corresponds to a *pure rotation* of the top platform about the axis in the plane of the top platform, perpendicular to the first leg, i.e., the **Y** axis. Correspondingly, we obtain the gained twist as one with zero pitch, indicating a rotation about the **Y** axis.

4.2 6-DOF Spatial Gripper

Geometry of the 6-DOF Gripper

The 6-DOF hybrid spatial manipulator, shown in figure 2, models a three-fingered gripper with the contact points modeled as spherical joints. The *DH parameters* of the jth finger are given in Table1. The first two of the joints in each finger are actuated, and the last link is passive. Hence the active variable is given by $\theta = (\theta_1, \theta_2, \theta_3, \psi_1, \psi_2, \psi_3)^T$, and the passive variable given by $\phi = (\phi_1, \phi_2, \phi_3)^T$. The individual fingers have the same architecture, and their link-lengths are taken such that $l_1 = 2l_2 = 4l_3 = 1$. The other architectural parameters are chosen as follows(see figure 2): $d = 1/2$, $h = \sqrt{3}/2$, $s = \sqrt{3}/2$. The third finger is rotated about the **Y** axis through an angle of $\pi/4$. The constraint equations are formed as in the previous example, i.e., $\eta_k, k = 1,2,3$ has the form:

$$||\mathbf{p}_i - \mathbf{p}_j|| = s^2, \;\; i,j = 1,2,3, i \neq j \qquad (17)$$

Non-Singular Configuration

At a non-singular configuration given by $\theta = (0.2, 0.1, 0.3, -1., -1.2, 1)^T$, and corresponding $\phi = (0.3679, 1.4548, 0.8831)^T$, the dual eigenvalues of $\hat{\mathbf{g}}$ are computed as

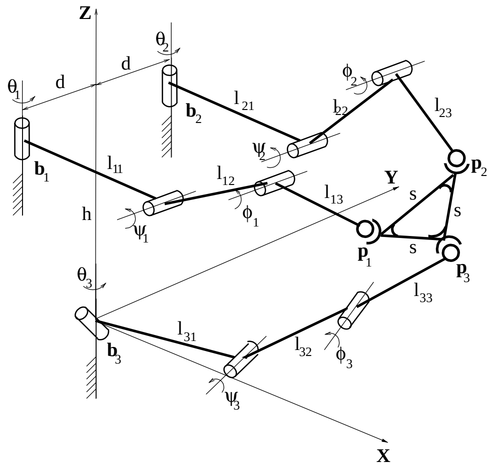

Figure 2. The Spatial 3-fingered Gripper

$$\begin{aligned}\hat{\lambda}_1 &= 0.03 + \varepsilon(0.09) \\ \hat{\lambda}_2 &= 2.10 + \varepsilon(5.45) \\ \hat{\lambda}_3 &= 1496.45 + \varepsilon(1070.41)\end{aligned}$$

The other three eigenvalues are of the form $0 + \varepsilon 0$. The principal pitches are given by

$$h_1^\omega = 1.37,\; h_2^\omega = 1.30,\; h_3^\omega = 0.36,\; h_4^\omega = \; h_5^\omega = \; h_6^\omega = \infty$$

The principal twists in ω-basis, at this configuration, is given by

$$\begin{aligned}\hat{\mathcal{V}}_1^\omega &= (14.56, 35.28, 6.24)^T + \varepsilon(15.16, 7.43, 8.29)^T \\ \hat{\mathcal{V}}_2^\omega &= (-1.34, 0.52, 0.17)^T + \varepsilon(-2.04, -0.03, 0.03)^T \\ \hat{\mathcal{V}}_3^\omega &= (0.01, -0.04, 0.18)^T + \varepsilon(-0.36, -0.19, 0.23)^T \\ \hat{\mathcal{V}}_4^\omega &= (0,0,0)^T + \varepsilon(-0.09, 0.77, 0.11)^T \\ \hat{\mathcal{V}}_5^\omega &= (0,0,0)^T + \varepsilon(-0.03, -0.49, 0.06)^T \\ \hat{\mathcal{V}}_6^\omega &= (0,0,0)^T + \varepsilon(0.19, -0.07, 0.27)^T\end{aligned}$$

Singular Configuration, Loss Type We now consider the singular configuration where all the three fingers are fully stretched [6]. The configuration is defined by $\theta = (0.0500, -0.0500, 0, -1.0998, -1.0998, 1.0026)^T$ and $\phi = (0,0,0))^T$. We expect a loss of three degrees-of-freedom since all the three fingers are in singular configuration, and accordingly we find that the pure dual principal twists vanish identically, signifying the loss of three translational degrees of freedom. The other three principal twists are given as

$$\begin{aligned}\hat{\mathcal{V}}_1^\omega &= (-1.79, -27.73, 0.05)^T + \varepsilon(12.01, -0.01, -8.63)^T \\ \hat{\mathcal{V}}_2^\omega &= (12.25, -0.79, -0.36)^T + \varepsilon(1.75, 0.04, -1.26)^T \\ \hat{\mathcal{V}}_3^\omega &= (0.0001, 0, 0.0050)^T + \varepsilon(-0.00, -0.76, -0.00)^T\end{aligned}$$

Singular Configuration, Gain Type

In this case also, there is a singularity leading to gain of a single DOF when one of the passive links lie in the plane of the moving platform. We obtain such a configuration at $\theta = (0.0462, -0.0462, 0, -0.4732, -0.4732, 1.0472)^T$, $\phi = (-1.0761, -1.0761, 1.0472)^T$. The corresponding gained passive motion in the nullspace of $\mathbf{J}_{\eta\phi}$ is obtained as $(0,0,1)^T$, indicating that ϕ_3 can have an instantaneous variation even with actuators locked. The gained twist is essentially the 3rd column of $\mathbf{J}_{\omega\phi} + \varepsilon \mathbf{J}_{\mathbf{v}\phi}$, whose analytical expression is of the form $(0, \omega_y, 0)^T + \varepsilon(v_x, 0, v_z)^T$. This indicates a pure rotation about the **Y** axis, which corroborates with the intuitive motion of the platform with the instantaneous motion in ϕ_3 alone. In particular, for the chosen architecture and configuration, the gained twist is $\hat{\mathcal{V}}^g = (0, 1/3, 0)^T + \varepsilon(-0.1294, 0, 0.4830)^T$.

It may be noted that as in the case of the PUMA 560 in [1], all the results for the 6-DOF gripper is available only in ω-basis, for reasons explained previously. The principal twists in ω-basis allows us to find all possible finite-pitch, as well as pure translational motions analytically at a non-singular configuration, and also detect singularities, determine lost and gained twists at a singularity. This is a significant improvement over the classical $\mathbf{h}-$basis, where no information can be extracted for 6-DOF rigid-body motion.

5 CONCLUSION

In this paper, we have presented a method of deriving the dual Jacobian of the end-effector of a parallel or hybrid device analytically. The Jacobian is used in determining the instantaneous kinematics of such constrained motion, following the algebraic formulation presented in part I [1]. We also determine in closed form the gained and lost twists at a singularity. The results are illustrated with the help of a 3-RPS parallel manipulator and a 6-DOF hybrid spatial gripper.

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