

NON-PROPORTIONAL BENDING AND TORSION LOADING OF SOLID SQUARE BAR WITHIN THE ELASTIC-PLASTIC REGION

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ABSTRACT

A series of tests has been carried out to observe the inelastic behavior of solid square bar under combined bending and torsion loading. The tests were conducted using a newly designed test rig, which is capable of applying bending and torsional loads both separately and simultaneously. Two different loading paths were considered to examine the mechanical properties of the bar both in the elastic and plastic regions. In the first type of non-proportional loading, different levels of torque were initially applied and keeping the initial torques constant, bending moment was gradually applied. Similarly in the second type of non-proportional loading, different levels of bending moments were initially applied and keeping the initial moments constant, torque was gradually applied. It is observed that in the elastic region, stiffness of the material of the bar is independent of the initially applied constant bending moment and torque, whereas in the inelastic region the magnitudes of the stiffness and strain hardening are comparatively higher for the lower values of the initially applied constant bending moment and torque.

Keywords: Non-proportional loading, bending, torsion, solid square bar.

1. INTRODUCTION

In case of machine tools, robots and machineries, solid square bars are frequently subjected to the combined bending and torsional load. Variation of the deformation as well as stiffness of the solid square bar under such type of loading is different from that of the shaft. When a shaft is subjected to above mentioned type of loading, maximum stresses occur at the outer circumference and the entire outermost surface first goes to inelastic state from the elastic state, and with the increase of the load, this inelastic region increases approaching towards the center. But in case of the solid square bar, maximum stresses concentrate at some localized points at the outermost surface and these points first go to inelastic state from elastic state, and with the increase of the load, these inelastic regions increase in volume approaching towards the center.

In most cases, thin walled specimens have been used to investigate combined bending and torsion loading [1-5]. Bathe and Wieser [6] have conducted theoretical investigations regarding the biaxial bending and torsion of channel section whereas Pi and Trahair [7] have conducted theoretical investigations regarding the biaxial bending and torsion of I-beam. Pi and Trahair have conducted their research to develop a theory considering material inelasticity based on incremental theory of plasticity using the von-Mises yield criteria. Zhao and Hancock [8] have researched with square and rectangular hollow section bar to investigate the effect of the bearing

length on the failure loads in case of combined loading. However, experimental works regarding the material behavior of solid bars under combined loadings are very few. Maruyama and Nakagawa [9], Newnham, et al [10], Chapman, et al [11], Hagiwara, et al [12] and Hariri [13] have carried out experimental investigations on the behavior of the bolted joints (i.e., solid bar) in elastic-plastic region. Recently, Ali [14] has carried out experimental investigation regarding the behavior of the solid rod of BS 2874-C120 copper under combined loading within the elastic-plastic region. Here he considered torsion and tension loading of the circular rod. In the current investigation, material behavior of the solid square bar under combined bending and torsion loading within the inelastic region is presented. For that purpose a series of tests has been carried out using a newly designed test rig, where bending and torsional loadings are applied non-proportionally.

2. EXPERIMENTAL PROGRAMME

2.1 Experimental Setup

A test rig along with the auxiliary components/parts was designed and fabricated to perform the current experimental investigation, which is shown in Figure 1. This apparatus with its different features extended the range of experiments to cover virtually all requirements necessary to apply pure bending, pure torsion and combined bending and torsion loadings. Its basic units provide facilities for supporting horizontal bars on fixed

supports, to apply loads, and also to measure beam deflections and twisting angles.

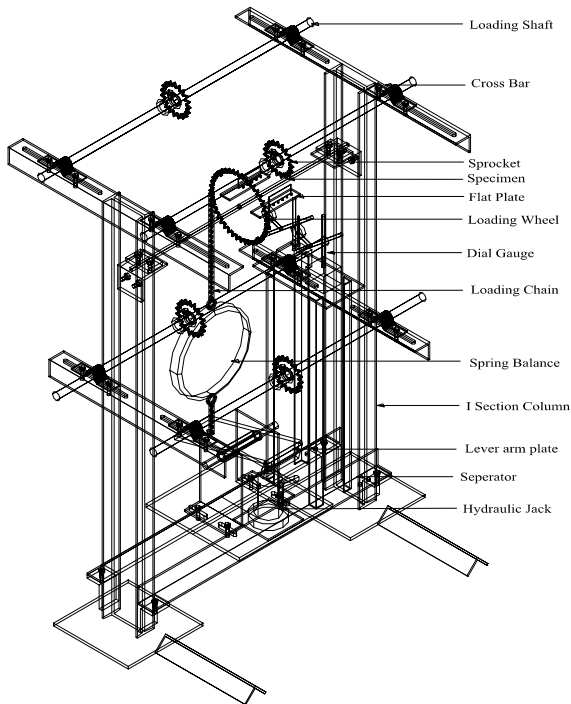


Figure 1: Isometric view of the test rig

2.2 Experimental Procedure and Specimen Selection

At first the specimen was inserted in the loading wheel so that the wheel was firmly placed at the midspan of the specimen. The test specimen with the loading wheel was then placed on base plates, and the two ends of the specimen were firmly clamped with the help of clamping plates and bolts. Then loads were applied to the specimen with the help of a hydraulic jack and dead weights. Figures 2 (a)- (d) show loading arrangements of the loading wheel for different types of combined loading. In the first type of non-proportional loading, the initial torques were maintained at 25%, 50% and 75% of the yield torque at pure torsion. In the second type of non-proportional loading, the initial moments were 25%, 50% and 75% of yield bending moment at pure bending.

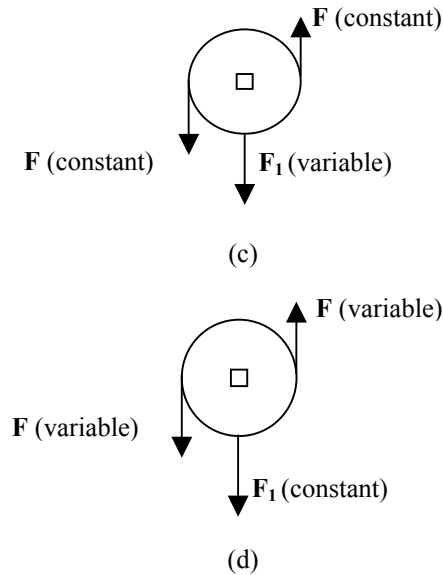
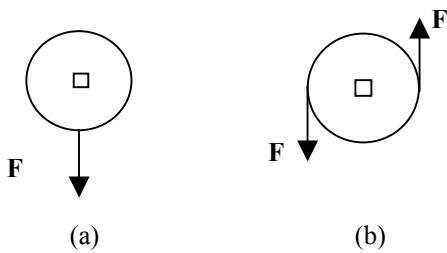


Figure 2. (a) Loading pattern of Pure Bending. (b) Loading pattern of Pure Torsion. (c) Loading pattern of combined loading with initial constant torque and different level of bending moment. (d) Loading pattern of combined loading with initial constant bending moment and different levels of torque.

8mm × 8mm bar of length 700 mm was used as the test specimen. The material of the specimen was mild steel with 0.15 % carbon. All the specimens were made as received material.

3 RESULTS AND DISCUSSION

Figure 3 shows the variation of the bending loads with the midpoint deflections of the beam. From this figure it is seen that whatever is the level of the initial constant torque, the initial slopes of the bending load versus deflection curves are nearly same and the yield bending loads decrease with the increase of initial torques. Figure 4 depicts the variation of the tangent modulus of the material with the midpoint deflection the bar investigated. From figure 4 it is seen that up to the yield points the corresponding slopes are constant and their values are nearly same, but there is a drastic change just after the yield point. It is further observed that at $1.50\delta_y$, $2.00\delta_y$ and $2.50\delta_y$ (δ_y is the yield deflection), the values of the corresponding tangent modulus of elasticity are higher for the lower values of the initial torques, which is shown in figure 5. This means, strain hardening is more noticeable in case of the lower value of the initially applied torque. Similarly, variation of the torsional load with the angle of twist of the beam is shown in Figure 6. From this figure it is found that whatever is the level of initially applied constant bending moment, the initial slopes of the torsional load versus angle of twist curves are nearly same and yield torsional loads decrease with the increase of initial bending moment. From figure 7 it is seen that, up to the yield points the slopes are constant and their values are nearly same, but there is a drastic change just after the yield point. It is further observed that at $1.25\theta_y$, $1.50\theta_y$ and $2.00\theta_y$ (θ_y is the yield angle of

twist), the values of the corresponding tangent modulus of rigidity are higher for the lower values of the initial bending moment, which is shown in Figure 8. This means, strain hardening was more noticeable in case of lower values of the initially applied constant bending loads.

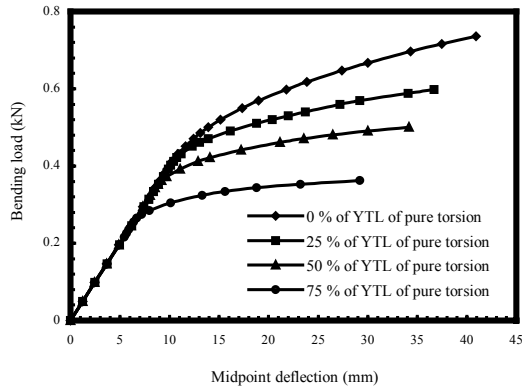


Figure 3: Bending load versus midpoint deflection curves for different levels of initially applied constant torsional loads

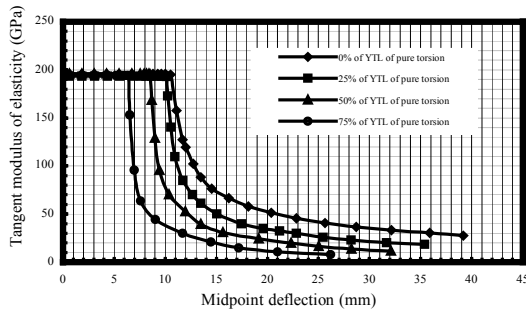


Figure 4: Variation of the tangent modulus of elasticity with respect to the midpoint deflection for different levels of initially applied constant torsional loads (Non-proportional loading)

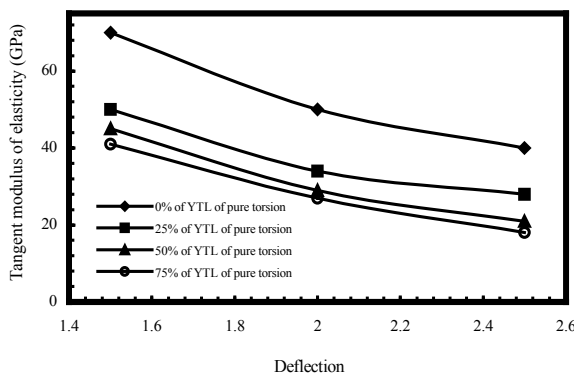


Figure 5: Variation of the tangent modulus of elasticity at 1.5, 2, and 2.5 times of the corresponding yield deflection for different levels of initially applied constant torsional loads

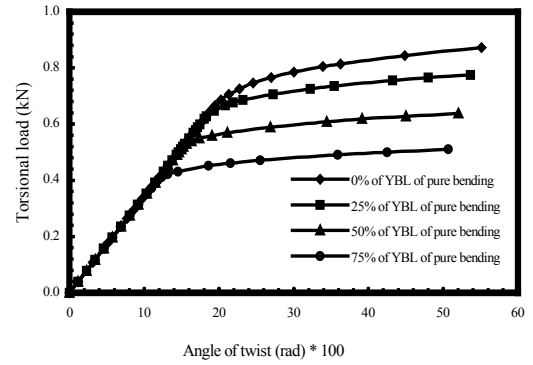


Figure 6: Torsional load versus angle of twist curves for different levels of initially applied constant bending loads

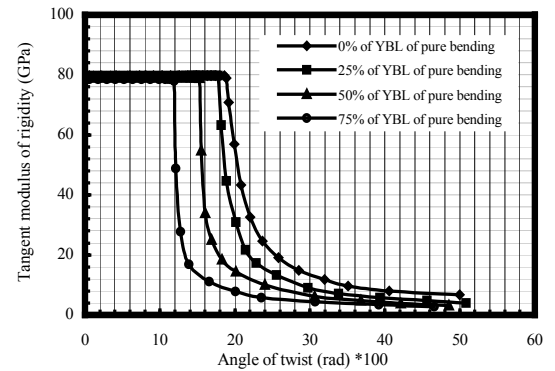


Figure 7: Variation of the tangent modulus of rigidity with respect to the angle of twist for different levels of initially applied constant bending loads

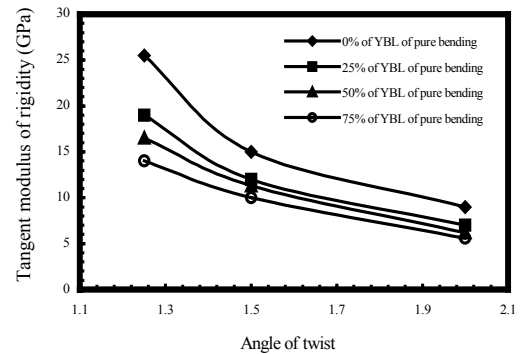


Figure 8: Variation of the tangent modulus of rigidity at 1.25, 1.5, and 2 times of the corresponding yield angle of twist for different levels of initially applied constant bending loads

4. CONCLUSIONS

The following conclusions can be drawn from the present investigation:

1. Whatever be the level of initial torque or bending moment, the initial slopes of the bending load versus midpoint deflection curves as well as torsional load versus angle of twist curves are similar to those of pure bending and pure torsion curves respectively, i.e., modulus

of elasticity and modulus of rigidity are nearly constant for all levels of initially applied constant bending moment and torque respectively. Hence, stiffness of the material remains unaffected in the elastic region for the all levels of initially applied constant bending moment or torque.

- 2) Whatever be the level of initially applied constant torque, tangent modulus of elasticity beyond the yield point steadily decreases with the increase of load, and at the same corresponding deflection its value is higher for the lower level of initial torque. Similarly, whatever be the level of initially applied constant bending moment, tangent modulus of rigidity beyond the yield point steadily decreases with the increase of load, and at the same corresponding angle of twist, its value is higher for the lower level of initial bending moment. The above concludes that in the inelastic region the magnitudes of the stiffness and strain hardening are comparatively higher for the lower values of the initially applied constant bending moment and torque

5. REFERENCES

- 1) Siebel, M. P. L. "The combined bending and twisting of thin cylinders in the plastic range" Jour. Mech. and Phys. Solids, Vol. I, 1953, pp. 189-206.
- 2) Hill, R and Siebel, M. P. L., "On the combined bending and twisting of thin tubes in the plastic range", Philosophical Mag., Vol. 42(7), 1951, pp. 722-33.
- 3) Imegwu, E. O., "Plastic flexure and torsion of thin tubes", Jour. Mech. and Phys. Solids, Vol. 3, 1954, pp 156-66.
- 4) Banerjee, J. K., "Plastic instability in tubes of finite length", *ibid*, vol. – 17, 1975, pp. 659
- 5) Menkin, C. M. and Veldpavs, F. E., "The non linear flexural-torsional behaviour of straight slender elastic member with arbitrary cross sections", Thin walled Structure, Vol – 6 (5), 1988, pp. 385-404.
- 6) Bathe, K. J. and Wieser. P. M., "On elastic-plastic analysis of I beam in bending and torsion", J. of Struc. Engineering, Vol- 17, 1983, pp. 711-718.
- 7) Pi, Y. L. and Trahair, N. S., "Inelastic bending and torsion of steel I beams", Journal of Structural Engineering, Vol-120 (12), 1994, pp. 3397-3417.
- 8) Zhao, X. L. and Hancock, G. J., "Square and Rectangular Hollow Sections Subjected to Combined Actions", Journal of Structural Engineering, Vol-118, No. 3, March 1992, pp. 648-668.
- 9) Maruyama, K. and Nakagowa, M., " Proposal of New Torque Control Method and New Design System in Bolted Joint", Belletin of Research Laboratory, Precision of Machine Electron, Tokyo Institute of Technology, vol-55, 1985.
- 10) Newnham, J., Curley, L. and Boos, H. P., " The Response to External Loads of Bolted Joint Tightened to Yield", *VDI BERICHTE NR*, vol-766, 1989.
- 11) Chapman, I. Newnham, J. and Wallace, " The Tightening of Bolts to Yield and Their Performance under load", Journal of Vibration, Acoustics, Stress and Reliability in Design, vol-108, 1986.
- 12) Hagiwara, M., Ohashi, N. and Yoshimoto, I., " Characteristics of Bolted Joints in Plastic Region Tightening", Bulletin of Japan Society of Precision Engineering, vol-20, no 4, 1986.
- 13) Hariri, B. H., M E Thesis, Dublin City University, Ireland, 1990.
- 14) Ali, A. R. M., "Alternatively Applied Torque Tension Loadings to Copper Rod beyond Combined Yield Point", Journal of the Institute of Engineers, India, vol-82, March 2002.