

GROWTH OF THERMODYNAMIC BOUNDARY LAYER THROUGH POROUS MEDIA

H. N. Mondal

Department of Chemical Engineering
Bangladesh University of Engineering and Technology

ABSTRACT

The governing equations for transient laminar forced convection in the entrance region of a porous concentric annulus are solved numerically. Solutions are obtained through MATLAB programming. The hydrodynamic boundary layer is assumed to be fully developed. Both Darcian and Non-Darcian effects on the hydrodynamic field are taken into consideration. Numerical solutions are obtained for the velocity and temperature fields for several different initial and boundary conditions. The results investigate the effect of the fluid and different solid-matrix parameters. The results show that as the thermal boundary layer becomes thinner, the resistance to heat transfer from the wall to the fluid decreases, and consequently the heat transfer coefficient improves.

Keywords: Porous Media, Entry region, thermal boundary layer

1. INTRODUCTION

Investigation of transient force convective heat transfer in porous media is of considerable practical significance but surprisingly literature on this topic is limited, specially for the entrance section. It has attracted more attention by its intrinsic practical importance. A review of the literature shows that only steady state forced convection problems were considered and mainly two types of geometries, parallel plate channels and circular tubes, have been considered in the research works. In this respect, the main objective of this work was to determine the temperature profile at the very entrance section of an annular section since solutions in this region is not available in the literature.

Vafai and Kim (1989) analyzed the boundary and inertia effects in a steady forced convection over a horizontal plate. They used volume averaging technique for constant porosity media. Kaviany (1985) used a numerical solution of laminar flow in a porous channel bounded by isothermal parallel plates and his work was based on the Darcy model. Poulikakos and Renken (1987) used a variable porosity model and numerically investigated the effects of flow inertia bounded by parallel plates and also for circular tubes. Vafai and Kim (1989) also studied porous forced convection between two parallel plates. Cheng and Hsu (1988) studied the steady forced convection problem in packed sphere beds. In all the above works, steady state flow behavior is assumed for both hydrodynamic and thermal fields.

This work deals with the problem of unsteady state forced convection in the entry region of a porous annulus. Heating starts at the entrance section so that the thermal boundary layer is developing. However, the hydrodynamic boundary layer is assumed to be steady and fully developed. Calculations are performed for the following two conditions: (I) step temperature change at inner wall while the outer wall is kept adiabatic; (II) simultaneous step temperature change at both inner wall and entrance cross section while the outer wall is kept adiabatic.

2. FORMULATION

Fig. 1 and 2 shows the geometry and coordinate systems. The fluid enters the annular passage with uniform velocity distribution and the physical properties of the fluid are assumed to be constant. The equations of motion and energy can be written in the following dimensionless forms:

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial u^*}{\partial r^*} \right) - \left(\frac{u^*}{Da} + \frac{\Lambda}{\sqrt{Da}} u^{*2} \right) = \frac{\partial p}{\partial z^*} \quad (1)$$

$$\frac{\partial T^*}{\partial \tau} + u^* \frac{\partial T^*}{\partial z^*} = \frac{1}{Pe_e} \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T^*}{\partial r^*} \right) \quad (2)$$

τ is the dimensionless time and σ is the heat capacity ratio.

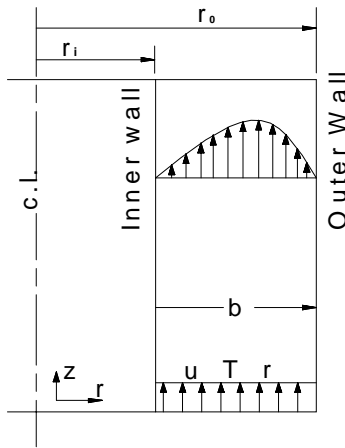


Fig. 1: Two dimensional channel for model of analysis.

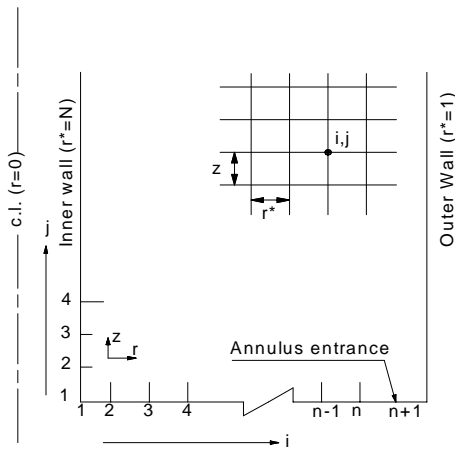


Fig. 2: Finite difference network in r-z plane.

The equation of continuity can be written in the following integral form:

$$\int_N^1 r^* u^* dr^* = \frac{1 - N^2}{2} \quad (3)$$

Since the physical properties of the fluid and the porous medium are assumed to be constant, the momentum equation and the energy equations are solved separately. First the momentum equation is solved with the help of the continuity equation to obtain the velocity and pressure distribution and then the energy equation is solved using these values.

The initial and boundary conditions are as follows:

Initial condition:

For $z^* = 0$, and $N < R < 1$: $u^* = 1$;
 For $z^* > 0$ and $R = N$ or $R = 1$: $u^* = T^* = 0$;

Boundary condition:

At $z = 0$ and $N < R < 1$: $u^* = 1$;
 For $z > 0$ and $R = N$ or $R = 1$: $u^* = 0$;

For $\tau > 0$, the thermal boundary conditions are as follows:

Case (I): step temperature change at inner wall while the outer wall is kept adiabatic;

At $z = 0$ and $N < R < 1$: $T^* = 0$;

For $z > 0$ and $R = N$: $T^* = 1$;

For $z > 0$ and $R = 1$: $\frac{\partial T^*}{\partial r^*} = 0$;

Case (II): simultaneous step temperature change at both inner wall and entrance cross section while the outer wall is kept adiabatic.

At $z = 0$ and $N < R < 1$: $T^* = 1$;

For $z > 0$ and $R = N$: $T^* = 1$;

For $z > 0$ and $R = 1$: $\frac{\partial T^*}{\partial r^*} = 0$;

Very far upstream from the entrance to the heated section, the fluid is at uniform temperature T_0 :

The expression for normalized mixing cup temperature or bulk temperature can be written as follows:

$$T_m^* = \frac{T_m - T_0}{T_w - T_0} = \frac{\int_0^1 T^* u^* r^* dr^*}{\int_0^1 u^* r^* dr^*} \quad (4)$$

The expression for local Nusselt number can be shown to be the following by making an energy balance at any cross-section of the tube:

$$(Nu)_{local} = \frac{-2 \frac{\partial T^*}{\partial r^*} \Big|_w}{T^* \Big|_w - T_m^*} \quad (5)$$

3.METHOD OF SOLUTION

Equations are solved by a self developed code using very popular software MATLAB (www.mathworks.com). The scripting language is syntactically very similar to FORTRAN-77 but it has better visualization capabilities. The hybrid scheme has been used to treat the convection terms and the pressure coupling has been dealt with SIMPLER algorithm (Patankar, 1980) in the solution procedure. The results are not changed significantly if a fully up-winded second order spatial differencing is applied to approximate the convective terms. A typical finite difference grid pattern is shown in Fig. 3. The solution is obtained using marching technique. At each cross section, a set of simultaneous linear equations have to be solved to get temperature distributions. The same procedure is repeated for other values to obtain the temperature field all over the entire annulus length at time $\tau + \Delta\tau$.

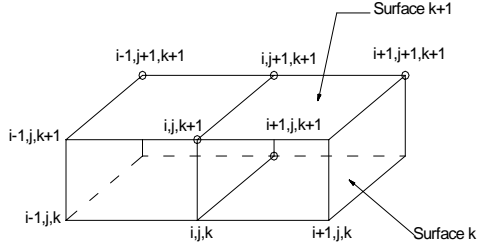


Fig. 3: A typical grid pattern for solution of the energy equation.

4.RESULTS AND DISCUSSION

Numerical calculations are performed for the following values:

$N=0.3, 0.5$ and 0.8

$\sqrt{Da} = 12, 20$ and 30

$\Lambda=10, 50$ and 100

$Pr_e=0.3, 0.6$ and 0.9

Due to space limitation and similar nature of the output of numerical simulation, results are only shown for the following parameters: $N=0.5$, $\sqrt{Da} = 12$, $\Lambda=10$ and 100 and $Pr_e=0.6$ and 0.9 .

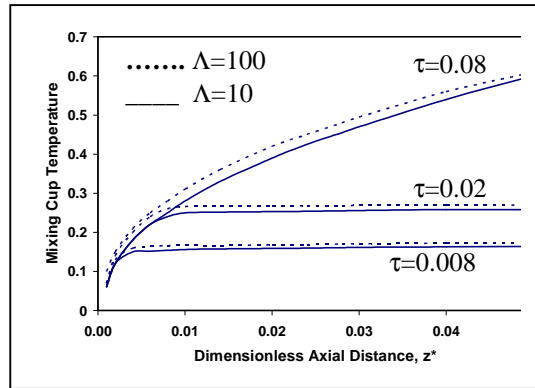


Fig. 4: Mixing cup temperature in the axial direction, case (I), $\sqrt{Da} = 12$ and $N = 0.5$ and $Pr_e = 0.6$.

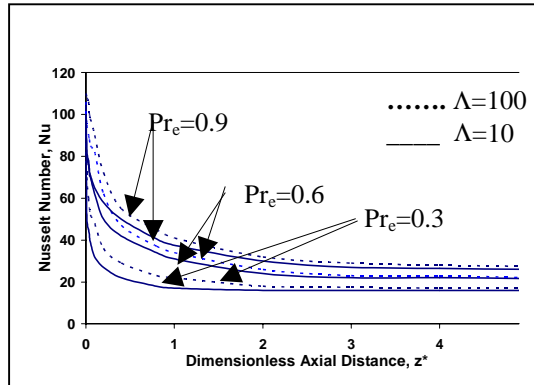


Fig. 5: Axial variation of local Nusselt number, case (I), $\sqrt{Da} = 12$ and $N = 0.5$.

Fig. 4 shows the transient behavior of the mixing cup temperature against the dimensionless axial distance z^* for $\sqrt{Da} = 12$, $N = 0.5$ and $Pr_e = 0.6$. The figure shows that the mixing cup temperature at a given location z^* increases as the inertial effect increases. This is because of the fact that the inertial effect increases the heat transfer by reducing the boundary layer thickness.

Fig. 5 shows the variations of local Nusselt number against the dimensionless axial distance for two values of the effective Prandtl numbers. This figure shows that the Nusselt number increase with an increase of the inertial parameter. An increase in inertial parameter causes a more uniform velocity profile which in turn causes a more uniform temperature distribution thereby resulting a lower value of $T_w^* - T_m^*$. The figure also shows that as the effective Prandtl number increases, the Nusselt number also increases that is the resistance to heat transfer from the wall to fluid decreases and as a result the heat transfer increases.

5.CONCLUSION

The problem of growth of thermodynamic boundary layer in the entry region of a porous annuli is analyzed. Numerical simulations are performed for the velocity and temperature distributions for different initial and boundary conditions. Results show that an increase in either the Prandtl number or the inertial effect or both increase the mixing cup temperature and the Nusselt number.

6.NOMENCLATURE

- c_f = Specific heat of fluid
- c_s = specific heat of solid
- Da = Darcy number, $\frac{K}{R_0^2 \phi}$
- F = Empirical constant
- K = Permeability of the porous medium
- k_f = Thermal conductivity of fluid
- k_s = Thermal conductivity of fluid
- N = Annulus radius ratio
- Pr_e = Effective Prandtl number, $\frac{v_f}{\alpha_e}$
- r^* = r/R_0
- T^* = $\frac{(T-T_0)}{(T_w-T_0)}$
- T_m = Mixing cup temperature over any cross section
- T_w = Heated wall temperature
- T_0 = Fluid temperature at annulus entrance
- u_0 = Axial velocity at the entrance
- z^* = Dimensionless axial coordinate, $\frac{2(1-N)z}{R_0 Re}$
- τ = Dimensionless time, $\frac{v_f t}{R_0^2 \sigma}$
- σ = Thermal capacity ratio, $\frac{\phi \rho_f c_f + (1-\phi) \rho_s c_s}{\rho_f c_f}$
- ρ_f = Fluid density
- ρ_s = Solid density

- α_e = Effective thermal diffusivity of fluid,
 $k_f \phi + (1 - \phi)k_s$
- ϕ = Porosity of the porous medium
- ν_f = Kinematic viscosity of the fluid
- Λ = Inertial parameter, $\phi^{1.5} Fu_0 R_0 / \nu$

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