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OPTIMUM BLOCKAGE RATIO FOR STACKS: STEADY, CONJUGATE PROBLEM

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ABSTRACT

We have performed an EGM (Entropy Generation Minimization) based analysis to optimize the blockage ratio for multi-plate stacks of a thermoacoustic device. We mainly focus on the steady flow inside the gaps made of consecutive stacks in this paper. Analyses have been performed for fully developed forced convection and entropy generation in a fluid-saturated porous medium channel bounded by two parallel plates. The channel walls are assumed to be finite in thickness. Conduction heat transfer inside the channel walls is also accounted and the full problem is treated as a conjugate heat transfer problem. The flow in the porous material is described by the Darcy-Brinkman momentum equation. The outer surfaces of the solid walls are treated as isothermal. A temperature dependent volumetric heat generation is considered inside the solid wall.

Keywords: Blockage ratio, Darcy number, EGM.

1. INTRODUCTION

Axial heat conduction along the walls bounding the fluid is usually overlooked in the design and analysis of heat transfer devices as well as in the interpretation of experimental data. Nevertheless, in actual practice, the temperature and heat flux distributions on the boundary depend only on the thermal properties and the flow characteristics of the fluid, but also on the properties of the wall. This is especially true in regions of large temperature gradients, such as the neighborhood of the leading edge in the flow over a heated body or the thermal entrance region in a duct. Such situations where heat conduction in the solid interacts with convective heat transfer are referred to as conjugate problems.

Convection process in porous media is a well-developed field of investigation because of its importance to a variety of situations. For example, thermal insulation, geothermal system, solid matrix heat exchangers, nuclear waste disposal, microelectronic heat transfer equipment, etc. Another potential application of the convection processes in porous media is found in thermoacoustic prime movers and heat pumps (Swift [1]). Despite recent developments in thermoacoustic engines (see Swift [1]), there are many areas requiring further investigation in order to better predict their performance and guide future designs for thermoacoustic engines. Any thermoacoustic device (system) can be divided into four basic components (resonant tube, speaker, heat exchanger, and stack); among them the stack serves as the heart of a thermoacoustic device. Starting from the single plate, stacks are available in different sizes and shapes. Multi-plate arrays, honeycombs, spiral roles, and pin arrays' are some example of stacks commonly used in thermoacoustic engines and refrigerators [1]. To improve the thermal contact and heat transfer area, a porous media (a fine wire mesh made of a material with moderate or good thermal conductivity) of relatively moderate permeability may be embedded inside the fluid gap between two consecutive stacks. In developing a thermoacoustic theory that incorporates porous media, a step by step development is required; starting from the simplest possible case (steady-incompressible-conjugate problem in porous media), proceeding to the complicated (unsteady-compressible-conjugate problem in porous media). In this paper, we only focus on the steady-state, incompressible, porous media, conjugate problem. The unsteady, compressible, version of this problem is left for future work.

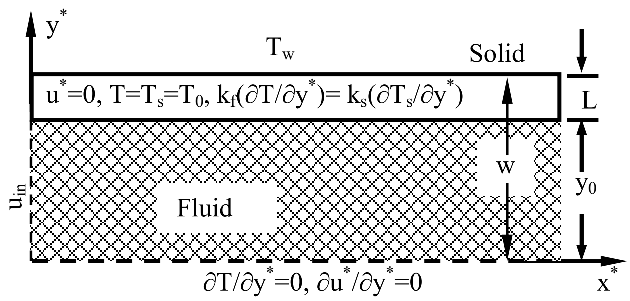

Fig. 1 Schematic diagram of the problem.

2. PROBLEM FORMULATION

The analysis is carried out for a steady-state, incompressible, and one-dimensional flow in a parallel-plate channel (as illustrated in Fig. 1) filled with a homogeneous and isotropic porous medium and heated with uniform wall temperatures at the walls. Corresponding equations for axial momentum, energy in fluid, and energy in solid in dimensionless forms are

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{1}{\mathrm{Re}\,Da}v \qquad (1)$$

$$u\frac{\partial \Theta}{\partial x}+v\frac{\partial \Theta}{\partial y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \Theta}{\partial x^2}+\frac{\partial^2 \Theta}{\partial y^2}\right)+\frac{Ec}{\mathrm{Re}\,Da}\left(u^2+v^2\right) \qquad (2)$$

$$\left(\frac{\partial^2 \Theta_s}{\partial x^2}+\frac{\partial^2 \Theta_s}{\partial y^2}\right)+G^2\Theta_s=0 \qquad (3)$$

where different parameters can be defined as

$$\begin{aligned}
&x=\frac{x^*}{w},y=\frac{y^*}{w},u=\frac{u^*}{u^*_{av}},v=\frac{v^*}{u^*_{av}},\\
&p=\frac{p^*}{\rho(u^*_{av})^2},\Theta=\frac{T-T_w}{T_m-T_w},\Theta_s=\frac{T_s-T_w}{T_m-T_w},\\
&\mathrm{Re}=\frac{\rho w u^*_{av}}{\mu},Da=\frac{K}{w^2},Ec=\frac{(u^*_{av})^2}{C_p\Delta T},\mathrm{Pr}=\frac{\mu C_p}{k_f},\\
&G=w\sqrt{\frac{Q_0}{k_s}},\alpha=\frac{y_0}{w},\beta=\frac{L}{w}
\end{aligned} \qquad (4)$$

In Eq. (4) any symbol with a superscript '*' represents a dimensional quantity. α and β are the clearance ratio and blockage ratio for the channel. Sum of α and β is 1.

3. FLOW AND THERMAL FIELDS

Assuming a fully developed and unidirectional flow, the momentum equation (Eq. (1)) reduces to a second order non-homogeneous equation of velocity u. With the help of boundary conditions: (a) no slip, i.e., $u(y=\alpha)=0$ and (b) symmetry, i.e., $(\partial u/\partial y)_{y=0}=0$, the final expression for velocity becomes

$$u=\left[\frac{\alpha\cosh\left(\alpha/\sqrt{Da}\right)\;-\alpha\cosh\left(y/\sqrt{Da}\right)}{\alpha\cosh\left(\alpha/\sqrt{Da}\right)\;-\sqrt{Da}\;\sinh\left(\alpha/\sqrt{Da}\right)}\right] \qquad (5)$$

The solution to the energy equation inside the fluid is given by

$$\begin{aligned}\Theta=\Phi_0\left[2\cosh\left(y/\sqrt{Da}\right)-\frac{\cosh\left(\alpha/\sqrt{Da}\right)}{2Da}y^2\right]\times\\
\cosh\left(\alpha/\sqrt{Da}\right)-\frac{\Phi_0}{4}\cosh^2\left(y/\sqrt{Da}\right)+\Psi\end{aligned} \qquad (6)$$

The solution to the energy equation inside the solid wall is given by

$$\Theta_s=\Theta_0\frac{\sin(G\,y)-\tan(G)\cos(Gy)}{\sin(G\alpha)-\tan(G)\cos(G\alpha)} \qquad (7)$$

where Θ_0 is the solid-fluid interface temperature and Φ_0 and Ψ are two constants and is given by the following equations:

$$\Theta_0=\Phi_0\left[\frac{\alpha^2}{2Da}+\frac{7}{4}\right]\cosh^2\left(\alpha/\sqrt{Da}\right)+\Psi \qquad (8)$$

$$\Phi_0=\frac{Ec\,\mathrm{Pr}\,\alpha^2}{\left[\alpha\cosh\left(\alpha/\sqrt{Da}\right)-\sqrt{Da}\sinh\left(\alpha/\sqrt{Da}\right)\right]^2} \qquad (9)$$

$$\begin{aligned}\Psi=&\frac{\Phi_0\left[3Da\sinh\left(2\alpha/\sqrt{Da}\right)-4\alpha\cosh^2\left(\alpha/\sqrt{Da}\right)\right]}{4Da\varepsilon G\Gamma}\\
&-\Phi_0\left[\frac{\alpha^2}{2Da}+\frac{7}{4}\right]\cosh^2\left(\alpha/\sqrt{Da}\right)\end{aligned} \qquad (10)$$

The parameter, Γ, used in Eq. (10) can be expressed as

$$\Gamma=\frac{\cos(G\alpha)-\tan(G)\sin(G\alpha)}{\sin(G\alpha)-\tan(G)\cos(G\alpha)} \qquad (11)$$

In Eqs. (1)–(11), the following dimensionless numbers are used: Reynolds number (Re), Prandtl number (Pr), Darcy number (Da), Eckert number (Ec), and heat generation number (G).

4. ENTROPY GENERATION

The convection process in a channel is inherently irreversible. Nonequilibrium conditions arise due to the exchange of energy and momentum, within the fluid and at the solid boundaries. This causes continuous entropy generation. One part of this entropy production is due to the heat transfer in the direction of finite temperature gradients, which is common in all types of thermal engineering applications. Another part of the entropy production arises due to the fluid friction and is generally termed as fluid friction irreversibility. For the present problem, the simplified form of the dimensionless entropy generation is given by

$$Ns=\left[\left(\frac{\partial\Theta}{\partial y}\right)^2+\frac{Ec\,\mathrm{Pr}}{\Lambda}\left(\frac{\partial u}{\partial y}\right)^2\right]+\varepsilon\left[\frac{\partial\Theta_s}{\partial y}\right]^2 \qquad (12)$$

where Ns, Λ, and ε represent entropy generation number, dimensionless temperature difference, and solid-fluid conductivity ratio ($=k_s/k_f$), respectively. In Eq. (12), terms inside the first square bracket represent the total entropy generation rate inside the fluid region (Ns_{fluid}) and valid only for $0\le y\le\alpha$. The second square bracketed term is the total entropy generation inside the solid wall (Ns_{solid}) and valid only for $\alpha\le y\le 1$. Average entropy generation rate can be calculated from the following equation:

$$Ns_{av}=\frac{1}{\forall}\int_{\forall}Ns\,d\forall=\frac{1}{\alpha}\int_0^{\alpha}Ns_{\mathrm{fluid}}dy+\frac{1}{1-\alpha}\int_{\alpha}^{1}Ns_{\mathrm{solid}}dy \qquad (13)$$

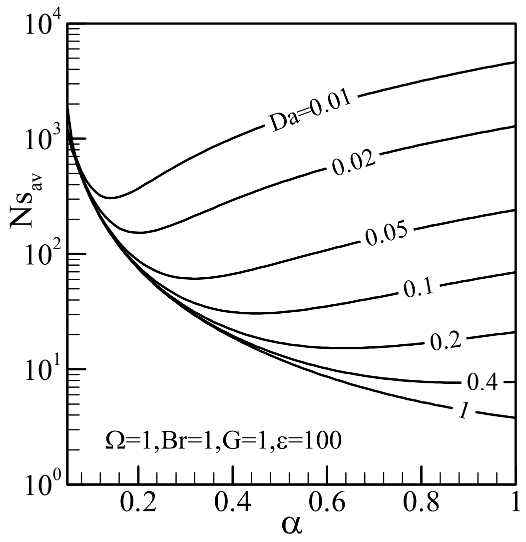

Fig. 2 Average entropy generation as a function of clearance ratio (α) at different Darcy numbers.

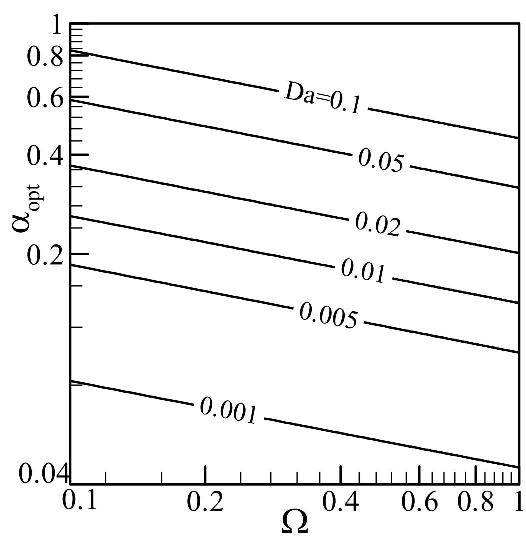

Fig. 3 Optimum clearance ratio as a function of group parameter at different Darcy numbers.

5. RESULTS AND DISCUSSION

Local entropy generation rate inside the solid and the fluid region is calculated using Eq. (12) after substituting u, Θ, and Θ_s from Eqs. (5), (6), and (7). Once the expression for local entropy generation rate is available, the average entropy generation rate is calculated from Eq. (13). Most of the calculation is performed by using the software MAPLE (version 7) developed by Advanced Mathematics Group in University of Waterloo.

Fig. 2 shows the distribution of average entropy generation number as a function of clearance ratio (α) at different Darcy number (Da). We fix the other parameters, i.e., Brinkman number ($=Ec\times\Pr$), heat generation parameter (G), conductivity ratio (ε), and group parameter ($Ec\times\Pr/\Lambda=\Omega$). The influence of these parameters on average entropy generation rate inside the channel is insignificant which is checked, but not reported here. A high clearance ratio means a low blockage ratio ($\beta=1-\alpha$). Entropy generation rate is higher in magnitude for smaller α. An increases in α decreases the value of Ns_{av}. The $Ns_{av}-\alpha$ profile shows a minimum ($\alpha=\alpha_{opt}$). The location of minimum average entropy generation rate strongly depends on Darcy number. For the parameters considered in Fig. 2, minimum Ns_{av} occurs at $\alpha=1.0$ for $Da=0.4$. For $Da>0.4$, no α_{opt} exists.

Fig. 3 shows the variation of α_{opt} as a function of group parameter at different Darcy numbers. An increases in the Darcy number increases the value of α_{opt} at a particular Ω. In the logarithmic plot, the $\alpha_{opt}-\Omega$ profiles show linear variations which enables us to propose a correlation in the following form

$$\alpha_{opt}=1.4182352\times Da^{0.5}\times\Omega^{-0.265}$$
$$\beta_{opt}=1-1.4182352\times Da^{0.5}\times\Omega^{-0.265} \quad (14)$$

The above equations are function of Darcy number and group parameter only.

6. CONCLUSION

We calculate the optimum blockage ratio (or clearance ratio) for multi-plate stacks of a thermoacoustic system at steady state. Entropy Generation Minimization is used as a working tool for optimization. For $Da>0.4$, there is no such optimum blockage ratio for the present model. A simplified correlation, based on the power curve fit, is proposed for calculating blockage ratio or clearance ratio.

7. REFERENCE

1. Swift, G. W., 2001, *Thermoacoustics: A Unifying Perspective for Some Engines and Refrigerators*, ASA Publication, New York.