ICME05-AM-01

MODELING PROGRESSIVE SLIDING/CRACKING IN GEOMECHANICS

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ABSTRACT

This paper presents a discontinuous approach for describing progressive sliding/cracking in geomechanics, based on the partition of unity method or the extended finite element method (PUM/XFEM). In contrast to the classical interface approach, the approach of PUM/XFEM uses enhanced interface elements without employing extra nodes. Those enhanced interfaces can be inserted into the bulk elements in a mesh-structure independent way. For a general non-homogeneous situation, the present paper uses the onset/propagation criterion based on a threshold value for an averaged equivalent plastic shear-strain. Further, the orientation of the interfaces is approximated based on an averaged gradient of incremental displacements at the tip of the progressing shear band. Gravity effects and the initial lateral stresses have been taken into account by means of K0-procedure. The constitutive behavior at discontinuity is described by means of a traction-jump relation. The capability of the recently developed PUM/XFEM-interface is illustrated by considering some standard cases in geomechanics.

Keywords: Geomechanics, Shear-band, Localization, Discontinuity.

1. INTRODUCTION

Within the framework of finite-element analysis, the classical interface approach has been an important tool for modeling contact behavior between interconnected bodies. In the classical approach the modeling of relative jump is described using a double set of nodes on a predefined "weak plane". This weak plane can only be defined at the element boundaries (i.e. not across the elements). For e.g. laminated composites, the classical interface approach is adequate [1-3]. However, the path of separation is limited by the mesh structure and suffering from a mesh bias [4, 5].

Regarding the plasticity-based localization models, the incorporation of discontinuous approach has been tackled by different authors (e.g. [6-10]). Recently, the incorporation of discontinuous approach in (frictional and dilatational) soil plasticity models has been developed and implemented into the Plaxis Finite Element Code for Soil and Rock Analyses. The implemented approach makes use of the PUM/XFEM framework of Wells and Sluys [8]. To account for the friction and dilatation effects, the present approach treats discontinuity as a special-purposed interface element. The PUM/XFEM-based interfaces do not have to be aligned with element boundaries and they can cross soil elements at any place with arbitrary orientation. The onset/ propagation of these interfaces have been based on a threshold value of an average equivalent plastic strain at the tip – rather than on the singular acoustic tensor criterion. The idea of an averaged localization criterion has been utilized earlier by e.g. Ortiz *et al.* [11] in which the local onset/propagation is related to the onset of plasticity at the element centre. (Note that despite the bifurcation condition is not satisfied at the center point and the utilized direction is no longer localization directions, this averaged localization criterion is shown to be adequate). Moreover, the present approach uses the averaged gradient of incremental displacements to determine the orientations of discontinuity.

The mainframe of the present discontinuous approach is shortly by-passed in Section 2. Then, a short description of the onset/propagation criterion and orientation are presented in Section 3. Section 4 illustrates the capability of the present approach by considering biaxial tests and soil retaining wall problems.

2. FRAMEWORK OF THE PROPOSED MODEL

The point of departure is the PUM/XFEM approach, as described in e.g. [8, 12], which employs an enhanced displacement field to describe strong localization or discontinuity phenomenon.

2.1 Kinematics of Discontinuous Field

Due to a discontinuity Γ_d, a body Ω bounded by Γ is divided into two sub-domains Ω^+ and Ω^-, which lie on either side of the discontinuity. **n** is the unit normal to the discontinuity, which points to Ω^+ (Figure 1).

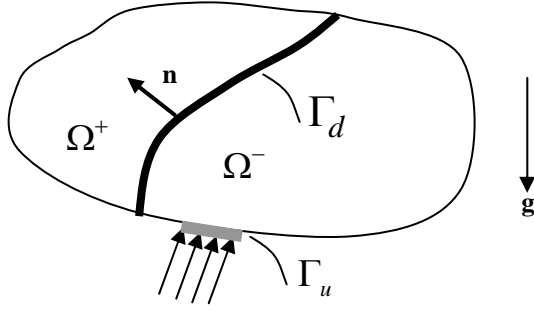Fig 1. Body Ω crossed by a discontinuity Γ_d.

The discretised displacement field $\mathbf{u}$ and strain field $\boldsymbol{\varepsilon}$ can be rewritten as

$$\mathbf{u} = \mathbf{Na} + \mathcal{H}_{\overline{\Gamma}d}(\mathbf{x})\mathbf{Nb} \qquad (1)$$

$$\boldsymbol{\varepsilon} = \mathbf{Ba} + \mathcal{H}_{\overline{\Gamma}d}(\mathbf{x})\mathbf{Bb} + \delta_{\overline{\Gamma}d}(\mathbf{x})\,(\mathbf{RN})\mathbf{b} \qquad (2)$$

where $\mathbf{a}$ and $\mathbf{b}$ represent the regular nodal displacements and the enhanced displacement jumps, $\mathbf{B}$ contains the derivatives of the shape function matrix $\mathbf{N}$, $\mathcal{H}_{\Gamma d}$ is the Heaviside function (1 for $\mathbf{x} \in \Omega^+$ and 0 for $\mathbf{x} \in \Omega^-$), and $\delta_{\Gamma d}$ is the Dirac-delta function (1 at Γ_d and 0 elsewhere). $\mathbf{R}$ is a orientation matrix containing the components of $\mathbf{n}$. Consider an orthogonal system $\mathbf{nst}$ with $\mathbf{s}$ and $\mathbf{r}$ two mutually unit orthogonal vectors perpendicular to $\mathbf{n}$. The components of the displacement-jump (u_n, u_s, u_r) at Γ_d can be found as (u_n, u_s, u_r) = $\boldsymbol{\Lambda}.(\mathbf{n}, \mathbf{s}, \mathbf{r})$, where $\boldsymbol{\Lambda}$ is a so-called displacement-jump vector.

2.2 Finite Element Formulation

The finite element formulation is derived by employing two separate variational equations [12]. Using discrete formulations of displacements and strains, the corresponding discrete formulation of weak equations can be expressed as

$$\int_{\Omega} \mathbf{B}^{\mathrm{T}}\boldsymbol{\sigma}\,d\Omega = \int_{\Gamma_u} \mathbf{N}^{\mathrm{T}}\mathbf{t}_u\,d\Gamma \qquad (3a)$$

$$\int_{\Omega^+} \mathbf{B}^{\mathrm{T}}\boldsymbol{\sigma}\,d\Omega + \int_{\Gamma_d} \mathbf{N}^{\mathrm{T}}\mathbf{t}\,d\Gamma = \int_{\Gamma_u} \mathcal{H}_{\overline{\Gamma}d}\,\mathbf{N}^{\mathrm{T}}\mathbf{t}_u\,d\Gamma \qquad (3b)$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor in a vector notation, $\mathbf{t}_u$ is external traction forces along surface Γ_u and $\mathbf{t}$ is the tractions at discontinuity Γ_d. (Note: the first expression governs the global equilibrium in body Ω, while the second expression can be considered as the governing equilibrium equation in sub-region Ω^+ including tractions at the discontinuity). Next, assuming the following relations for the stress rate in the continuum $\dot{\boldsymbol{\sigma}}$ and the traction rate at the discontinuity $\dot{\mathbf{t}}$

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}^{\mathrm{ep}}\dot{\boldsymbol{\varepsilon}} = \mathbf{D}^{\mathrm{ep}}(\mathbf{B}\dot{\mathbf{a}} + \mathcal{H}_{\overline{\Gamma}d}\mathbf{B}\dot{\mathbf{b}}) \qquad (4)$$

$$\dot{\mathbf{t}} = \mathbf{T}^{\mathrm{ep}}\dot{\mathbf{b}}_\Gamma = \mathbf{T}^{\mathrm{ep}}(\mathbf{RN})\dot{\mathbf{b}} \qquad (5)$$

where: $\mathbf{D}^{\mathrm{ep}}$ is the elastoplastic tangent stiffness matrix of the continuum and $\mathbf{T}^{\mathrm{ep}}$ represents a (pseudo) elastoplastic tangent stiffness matrix of the discontinuity, taking into account for the orientation of discontinuity in $\mathbf{b}_\Gamma$. After substitutions of the rate equations (4) and (5) into the discretised weak equations (3a) and (3b) and linearization, one arrives at

$$\mathbf{K} \begin{Bmatrix} \Delta\mathbf{a} \\ \Delta\mathbf{b} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_a^{ext} \\ \mathbf{f}_b^{ext} \end{Bmatrix} - \begin{Bmatrix} \mathbf{f}_a^{int} \\ \mathbf{f}_b^{int} \end{Bmatrix} \qquad (6)$$

where the stiffness matrix $\mathbf{K}$ contains the contribution of the regular and the enhanced parts. $\mathbf{f}^{ext}$ and $\mathbf{f}^{int}$ are the external and the internal forces of the regular and respectively, the enhanced part. Note that the initial stresses from gravity loading and K0-procedure is taken into account in the internal forces $\mathbf{f}_a^{int}$ and $\mathbf{f}_b^{int}$.

2.3 Elasto-Plastic Model for Intact Soil

Without referring to any particular plasticity model, the governing equation for the intact soil continuum can be formulated as

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}^{\mathrm{e}}(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p), \quad \dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\frac{\partial g}{\partial \boldsymbol{\sigma}} \qquad (7a)$$

$$F = F(\boldsymbol{\sigma},q) \le 0, \quad \dot{\lambda} \ge 0, \quad \dot{\lambda}\,F = 0, \quad \dot{\lambda}\,\dot{F} = 0 \qquad (7b)$$

where: $\dot{\boldsymbol{\sigma}}$ and $\dot{\boldsymbol{\varepsilon}}$ are the stress rate and the strain rate, $\dot{\boldsymbol{\varepsilon}}^p$ is the plastic part of the strain rate, λ is the plastic multiplier rate, F and G are the yield function and the plastic potential function, q is an internal variable to accommodate e.g. hardening behavior, $\mathbf{D}^{\mathrm{e}}$ is the elastic stiffness matrix.

2.4 PUM/XFEM Interface (Discontinuity)

Again, without referring to any particular discrete plasticity model the governing equations at discontinuity can be formulated in terms of the traction vector $\mathbf{t}$ and the jump vector $\mathbf{b}$ as follows

$$\dot{\mathbf{t}} = \mathbf{T}^{\mathrm{e}}(\dot{\mathbf{b}}_\Gamma - \dot{\mathbf{b}}_\Gamma^p), \quad \dot{\mathbf{b}}_\Gamma^p = \dot{\lambda}_\Gamma\frac{\partial \mu}{\partial \mathbf{t}} \qquad (8a)$$

$$\phi = \phi(\mathbf{t}, \kappa) \le 0, \quad \dot{\lambda}_\Gamma \ge 0, \quad \dot{\lambda}_\Gamma\,\phi = 0, \quad \dot{\lambda}_\Gamma\,\dot{\phi} = 0 \qquad (8b)$$

where $\mathbf{T}^{\mathrm{e}}$ represents the pseudo stiffness matrix of the interface, $\mathbf{b}^p$ represents the "plastic part" of the jump, $\mathbf{b}_\Gamma$ represents the total jump, $\dot{\lambda}_\Gamma$ is the plastic multiplier rate at discontinuity, ϕ and μ are the discrete yield function and the plastic potential function of discontinuity, κ is an internal variable related to the state of sliding/cracking at discontinuity.

3. ONSET/PROPAGATION AND ORIENTATION

At the onset of discontinuity, a bifurcation model assumes plastic yielding inside the shear band and elastic

unloading outside the band. For an elastic perfectly-plastic model, discontinuity along the slip-line is incepted when the determinant of the acoustic tensor **A** is zero (e.g. [11, 13]). The onset of discontinuity results in a bifurcated mechanical response: the plastic deformation being concentrated at discontinuity surface while the remaining part of the structure unloads elastically, e.g. as described in [7].

3.1 Global Criterion

When using a non-local criterion, the local bifurcation condition is not necessarily satisfied at the tip/triggering point [7]. The resulting direction of discontinuity **n** is no longer the localization direction, but can be thought of as an approximation of the general trend of shear-band orientation towards localization/failure. The present approach uses a threshold value for the equivalent plastic strain γ^p_{eq} as the onset/propagation criterion. γ^p_{eq} has been averaged using a Gaussian weight function [8]

$$w = \frac{1}{(2\pi)^{3/2} l^3} \exp\left(-\frac{r^2}{2l^2}\right) \qquad (9)$$

where w is the weight function, r is the distance from the tip, and l determines how fast the weight function decays from the tip. When the threshold value of $\tilde{\gamma}^p_{eq}$ at the tip is violated, discontinuity is inserted across the element touched by the tip.

3.2 Orientation of Discontinuity

For a general non-homogeneous case the orientation of discontinuity is according to the gradient of displacement-increment at the tip. For this purpose, a incremental displacement function $\Delta V \equiv \Delta V(\mathbf{x})$ is determined based on the nodal values $\{\Delta V_i\}$. ΔV at a Gaussian point (ξ, η, ζ) is interpolated using the element shape function matrix **N** and the orientation follows from the gradient of ΔV at the tip, i.e.

$$\frac{\partial(\Delta V)}{\partial \mathbf{x}} = [\frac{\partial \mathbf{N}}{\partial \mathbf{x}}] \{\Delta V_i\} \qquad (10)$$

in which the derivatives of the shape functions are computed based on their local derivatives and the Jacobean matrix J. To obtain more reliable information of the sliding/cracking direction, a non-local averaging has been applied using a Gaussian weight function. Note that the proposed displacement gradient approach is mainly for handling non-homogeneous cases, for homogeneous cases one simply uses the singular acoustic tensor criterion.

4. NUMERICAL EXAMPLE

For illustration purposes, the implemented approach has been applied to analyze mode-II failure in 2D case by employing the elastic perfectly-plastic Mohr-Coulomb soil (see also [14, 15]). The analysis is performed using 6-noded triangular elements (Note: in a similar retaining wall problem Borja and Lai [16] used constant strain triangular CST-element). For a plane-strain situation, the yield function f and the plastic potential function g can be formulated as

$$F = \tfrac{1}{2}(\sigma_3-\sigma_1) + \tfrac{1}{2}(\sigma_3+\sigma_1)\sin\varphi - c.\cos\varphi \qquad (11)$$

$$G = \tfrac{1}{2}(\sigma_3-\sigma_1) + \tfrac{1}{2}(\sigma_3+\sigma_1)\sin\psi \qquad (12)$$

where: σ_1 is the major compressive principal stress, σ_3 is the minor compressive (or the major tensile) principal stress, c is the cohesion, φ and ψ are the frictional angle and the dilatational angle. For the PUM/XFEM-interface element at discontinuity we assume the following yield function ϕ and plastic potential function μ

$$\phi = |t_s| + t_n \tan\varphi_i - c_i(\kappa) \quad \mu = |t_s| + t_n \tan\psi_i \qquad (13)$$

where t_s and t_n are the tangential and normal components of the tractions, φ_i and ψ_i are the frictional and dilatational angle of the interface. Considering $\psi_i=0$ (pure sliding at discontinuity) and a simple linear softening model one obtains

$$\begin{Bmatrix} \dot{t}_n \\ \dot{t}_s \end{Bmatrix} = \begin{bmatrix} k_n & 0 \\ -k_n \tan\varphi_i & -h \end{bmatrix} \begin{Bmatrix} \dot{b}_n \\ \dot{b}_s \end{Bmatrix} \qquad (14)$$

Note: (i) this simplified relation also satisfies the consistency condition $\dot{\phi}=0$, and (ii) for non-frictional material $\varphi_i=0$, it simply leads to $\Delta \dot{t}_n = k_n \Delta b_n$ and $\Delta \dot{t}_s = -h \Delta b_s$.

4.1 Biaxial Test

To verify the implementation a simple biaxial test has been considered, which is also used to demonstrate the objectivity of the present approach. On top the vertical displacement u_y is prescribed, while the bottom of the specimen is fully fixed. The onset/propagation is according to the onset of plasticity. The objectivity of the softened force-displacement response with respect to different triggering points, mesh densities and mesh alignments has been examined using different meshes as shown in Figure 2(a)-(d).The force-displacement curves is shown in Figure 3.

4.2 Retaining Wall

The second example considers a soil retaining wall problem considering initial stresses from the gravity loading and K0-procedure. In this non-uniform stress field, localization in the continuum takes place progressively starting from the most critically stressed points and propagating in the direction favoring material bifurcations [16,17]. The force-displacement curve in the present passive (smooth) retaining wall problem is shown in Fig.4.
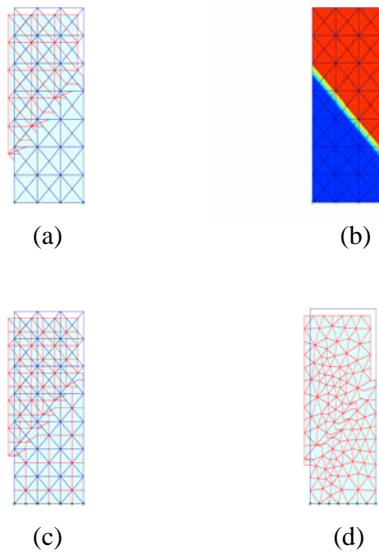
Fig 2. Deformed mesh in: (a) regular coarse mesh right triggering, (b) regular coarse mesh left triggering, (c) regular medium mesh right triggering, (d) irregular medium mesh right triggering.
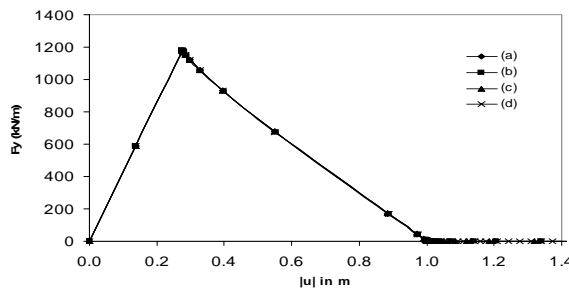

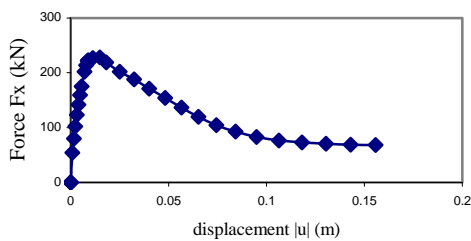Fig 3. Force-displacement curves ($E = 10^4$ kN/m^2, ν=0, c=100 kN/m^2, φ=ψ=0°, h=100 kN/m^3).

Fig 4. Force-displacement curve (c=30 kPa, φ=20°, γ = 16 kN/m^3, wedge depth is 2m); F_x at peak $\approx$ 230 kN, F_x residual (at c = 0) $\approx$ 70 kN, threshold value according to γ^p at peak.

5. CONCLUSIONS

An application of PUM/XFEM-based interface model has been presented. This type of interface elements can be inserted across the soil elements in a mesh-independent manner. The onset/propagation has been based on a threshold value of an equivalent plastic shear strain, while the interface orientations are based on the gradient of displacement- increments. The objectivity of the present PUM/XFEM-interface model has been

verified by considering biaxial tests using different meshes. The capability of the present interface model for describing progressive localization/discontinuity has been demonstrated by considering soil retaining wall problems. The corresponding peak and residual load are comparable to the classical wedge solution. Further, the present interface model is also capable of describing the progress of a curved slip-line/discontinuity in a passive rough-wall problem. More works will be performed in the near future to further evaluate the capability of the present PUM/XFEM-interface model. This research is supported by the Technology Foundation STW (project DCB 6368).
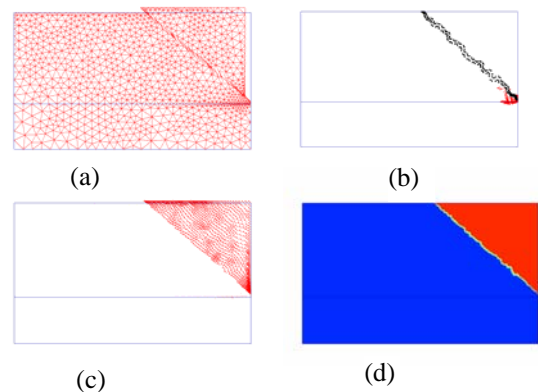

Fig 5. (a) Deformed mesh; (b) Shear-band path (red/grey: plastic points, black: "cracked" points); (c) Displacement plot/sliding mechanism; (d) Localized zone.

6. REFERENCES

1. Rots JG., 1991, "Smeared and discrete representations of localized fracture", International Journal of Fracture, 51:45-59.
2. Schellekens JCJ, de Borst R., 1992, "On the numerical integration of interface elements", International Journal for Numerical Methods in Engineering, 36:43-66.
3. Schellekens JCJ, de Borst R., 1993, "A non-linear finite element approach for the analysis of mode-I free edge delamination in composites", International Journal of Solids and Structures, 30:1239-1253.
4. Tijssens MGA, van der Giessen E, Sluys LJ., 2000, "Modeling of crazing using surface methodology", Mechanics of Materials, 32:19-35.
5. Tijssens MGA, Sluys LJ, van der Giessen E., 2000, "Numerical simulation of quasi-brittle fracture using damaging cohesive surfaces", European Journal of Mechanics A/Solids, 19:761-779.
6. Simo JC, Oliver J, Armero F., 1993, "An analysis of strong discontinuous induced by strain-softening in rate-independent inelastic solids", Computational Mechanics, 12:277-296.
7. Oliver J, Cervera M, Manzoli O., 1999, "Strong discontinuities and continuum plasticity models: the strong discontinuity approach", International Journal of Plasticity, 15(3):319-351.
8. Wells GN, Sluys LJ., 2001, "A new method for modeling cohesive cracks using finite elements",

International Journal for Numerical Methods in Engineering, 50(12):2667-2682.

9. Regueiro RA, Borja RI., 2001, "Plane strain finite element analysis of pressure sensitive plasticity with strong discontinuity", International Journal of Solids and Structures, 38:3647-3672.

10. Mosler J, Meschke G., 2003, "3D modeling of strong discontinuities in elastoplastic solids: Fixed and rotating localization formulations", International Journal for Numerical Methods in Engineering, 57:1553-1576.

11. Ortiz M, Leroy Y, Needleman A., 1987, "A finite element method for localized failure analysis", Computer Methods in Applied Mechanics and Engineering, 61:189-214.

12. Wells GN., 2001, "Discontinuous modeling of strain localization and failure", Ph.D. thesis, Delft University of Technology, Delft, The Netherlands.

13. Rudnicki JW, Rice JR., 1975, "Conditions for the localization of deformation in pressure sensitive dilatant materials", Journal of Mechanics and Physics of Solids, 23:371-394.

14. Vermeer PA, Vogler U, Septanika EG, Stelzer O. A., 2000, "Strong discontinuity method without locking", *Proc. 2nd Int. Symp. on Continuous and Discontinuous Modelling of Cohesive Frictional Materials,* Stuttgart, Germany.

15. Septanika EG, Brinkgreve RBJ., 2005, "Progressive shear band model for soil retaining wall problems", *Proc. 7th Int. Conf. on Analysis of Discontinuous Deformations (ICADD-7),* Honolulu, Hawaii.

16. Borja RI, Lai TY., 2002, "Propagation of Localization Instability under Active and Passive Loading", Journal of Geotechnical and Geoenvironmental Engineering, 128(1):64-75.

17. Borja RI, Regueiro RA., 2001, "Strain localization in frictional materials exhibiting displacement jumps", Computational Methods in Applied Mechanics and Engineering, 190(20-21): 2555-2580.

7. NOMENCLATURE

Symbol	Meaning	Unit
$\mathbf{N}$	Unit normal vector	-
$\mathbf{Nst}$	Orthogonal axes (n,s,t)	-
$\mathbf{G}$	Gravity vector	(m/s^2)
$\mathbf{N}$	Shape function matrix	-
$\mathbf{u}$	Total displacement vector	(m)
$\boldsymbol{\varepsilon}$	Strain vector	-
$\boldsymbol{\varepsilon}^p$	Plastic strain vector	-
$\mathbf{a}$	Regular displacement vector	(m)
$\mathbf{b}$	Displacement-jump vector	(m)
$\mathbf{B}$	B-matrix	-
$\mathbf{R}$	Orientation matrix	-
$\boldsymbol{\sigma}$	Cauchy stress vector	(Pa)
$\mathbf{t}$	Tractions at discontinuity	(N/m)
$\mathbf{t}_u$	Boundary tractions	(N/m)
$\mathbf{D}^e$	Elastic stiffness of soil	(Pa)
$\mathbf{D}^{ep}$	Tangent stiffness of soil	(Pa)
$\mathbf{T}^e$	Elastic stiffness discontinuity	(N/m)
$\mathbf{T}^{ep}$	Tangent stiffness discontinuity	(N/m)

$\mathbf{b}_\Gamma$	Jump at discontinuity	(m)
$\mathbf{K}$	System matrix	(N/m)
$\Delta\mathbf{a}$	Increment of $\mathbf{a}$	(m)
$\Delta\mathbf{b}$	Increment of $\mathbf{b}$	(m)
$\mathbf{f}^{ext}$	External forces	(N)
$\mathbf{f}^{int}$	Internal forces	(N)
F	Yield function	-
G	Plastic potential function	-
λ	Plastic multiplier	-
ϕ	Yield function discontinuity	-
μ	Potential function discontinuity	-
λ_Γ	Plastic multiplier discontinuity	-
w	Gaussian weight function	-
l	Length parameter	-
r	Distance from the tip	
$\tilde{\gamma}^p_{eq}$	Threshold for plastic strain	-
ΔV	Incremental function	(m)
ΔV_i	Incremental at the nodes i	(m)
(ξ,η,ζ)	Gaussian point coordinate	-
J	Jacobean matrix	-
σ_1	Major compressive stress	(Pa)
σ_3	Major tensile stress	(Pa)
φ	Frictional angle	-
ψ	Dilatational angle	-
c	Cohesion	(Pa)
φ_i	Frictional angle discontinuity	-
ψ_i	Dilatational angle discontinuity	-
c_i	Cohesion at discontinuity	(N/m)
h	Softening modulus	(N/m^2)
t_n	Normal traction	(N/m)
t_s	Shear traction	(N/m)
k_n	Normal stiffness discontinuity	(N/m)
b_n	Normal jump at discontinuity	(m)
b_s	Shear jump at discontinuity	(m)
E	Young's modulus	(Pa)
ν	Poisson's ratio	-
F_x	Horizontal force at the wall	(N)

Note:

$(\dot{\ })$ is a symbol for the rate of parameter ( )

$\delta(\ )/\delta\mathbf{x}$ is a symbol for the gradient of ( ) over $\mathbf{x}$