Proceedings of the
International Conference on Mechanical Engineering 2005
(ICME2005) 28- 30 December 2005, Dhaka, Bangladesh

ICME05-AM-11

GEOMETRICAL AND MATHEMATICAL MODEL OF FIVE-BAR SPHERICAL LINKAGE METAMORPHIC ROBOT PALM

Jian S. Dai[1] and Mujibur Rahman Khan [2]

[1]Kings College University of London, UK.
[2]Dept. of Mechanical Engineering, BUET, Bangladesh

ABSTRACT

This paper will describe the geometrical configurations and develop a mathematical kinematic model of the five-bar spherical mechanism metamorphic robot palm. To develop the generalised mathematical model, the co-ordinate transformation matrices of the joint axis position and the finger position has been derived. The transformation matrices derived in this chapter are 4x4 matrices. From these 4x4 by matrices both the position and orientation of the links and fingers can be described. So the co-ordinate points of a point of interest in the three dimensional space can be described at different configuration state of the mechanism. The detail analysis has been done through this mathematical and geometrical modelling of five-bar spherical metamorphic linkage robotic palm. This mathematical and geometrical model can be used to inspect the feasibility of the mechanism design for the multi fingered robot palm.

Keywords: Five bar spherical mechanism, Metamorphic , Kinematic model, Robot palm

1. INTRODUCTION

Grasping is the main function performed by the mechanical hand. Mechanisms with more functionality are needed for secure grasping of different and complex object. With the advancement of the field of robotics it is important to think of mechanisms of multifingered robot hand with added functionality. The added functionality will have the ability to grasp as well as manipulate will be useful to perform more complex tasks.

Multi fingered hand with human like metamorphic palm have the potential to increase the number of contact points with an objects for achieving more stable grasp. In addition, the ability to reach a wide range of grasp configurations will allow many more types of objects to be grasped properly.

There are other important features of multifingered robotic palm. The ability of manipulating objects with in the hand. If the object can be manipulated with in the hand , however, the palm, fingers, and the object it self must move. This approach of executing fine motion leads to better control. This kind of robotic hand will also be able to change the orientation of the grasped objects. Fingers can reach into restricted enviournments where arms cannot reach.

All these unique features and capabilities of multifingered robotic palm motivated the author to study, analysis and develop the mathematical model and geometrical formation of this kind of robot hand.

2. PREVIOUS WORKS

Romdhane and Duffy [1] presents a kinestic analysis of multifingered hands with secure grasp of general object plus the manipulation of this object by the fingers without motion of the arm on which it is mounted. A pair of matrices is introduced that relate the small motion of the joints to the relative displacement of the objects and the fingertips. In the work done by Salisbury [2], which was subsequently extended by Kobayashi [3], a pair of matrices describing the kinestatics (instantaneous motions and static) of hand was presented. Kerr and Sanger [4] introduce a stiffness matrix for a grasp with elastic contacts. They deduced the internal force on the grasped object and determine weather it would slip.

In the PhD thesis of Kerr [5], it has been presented that, multifingered hands will increase the abilities and overall flexibility of robot systems. The kinematic issues examined are the determination of the finger tip and finger joint motions required to create a desire motion of the object, identifying situations where objects cannot be manipulated with complete generality and determining the workspace of the hand.

Kerr and Roth [6] discuss three of the fundamental problems encountered in grasping and manipulation of objects with in a multifingered hand. The first of these problems is that of determining how hard to squeeze an object with the fingers in order to ensure that the object is grasped properly. This part of work follows closely on Salisbury and Craig [7].

Dai and Rees Jones [8] gives a novel view of new metamorphic mechanisms and some development in deployable mechanisms and can be developed into multifingered robot hand, reconfigurable robot arms. The metamorphic mechanisms [9] takes the concept of metamorphosis and changes its structure, shape and subsequently mobility [10] to adapt to the enviourments and to meet the demands. Dai and Rees Jones [8,9] discusses about the characteristics of metamorphic mechanism, the facilities of changing the number of effective links and the change of configurations. They also discuss the different classes of mechanisms and focuses on the consequence of changing structures when erected or folded [10]. McCarthy [12] and Joseph duffy [13] analysis the spherical linkage mechanism. These linkages have the property that every link in the system rotates about the same fixed point. Thus trajectories of points in each link lie on concentric sphere with this point as centre.

3. GEOMETRICAL DESIGN

A generalized geometric model of a five bar spherical metamorphic mechanism is shown in the figure 1. Point o is considered as the centre of the sphere. Position 1, 2, 3, 4 & 5 are the revolute joints of the spherical linkages. o1, o 2, o 3, o 4 and o 5 represents the axis of the revolute joints. f1, f2, f3 are the three finger position in the spherical link L_{12}, L_{45} & L_{51}. Link L_{51} is considered as reference or fixed link. The angle subtended by the links L_{12}, L_{23}, L_{34}, L_{45}, L_{51} are respectively α, β, γ, δ, η. ρ_{1f} represents the angle created in the centre by the axis of the revolute joint 1 and

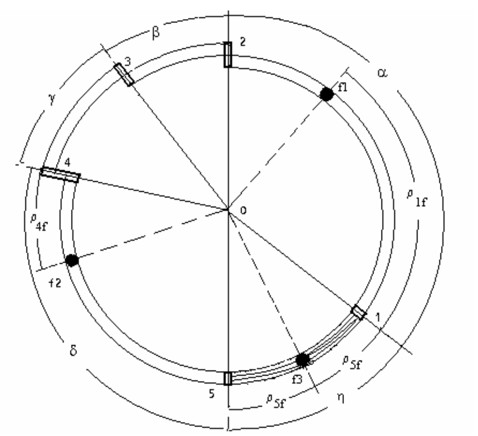

Fig 1. Geometric configuration of five bar spherical linkage metamorphic mechanism

the finger position f1. Similarly ρ_{4f} is the angle subtended by the joint axis 4 and the finger position f2 and ρ_{5f} is the angle between joint axis 5 and finger position f3. The mechanism structure and its capability of achieving a good range of configurations will depend on the selection of the angles α, β, γ, δ, η, ρ_{1f}, ρ_{4f}, ρ_{5f} . Figure 2 shows the linear distance between the joints of each link L_{12}, L_{23}, L_{34}, L_{45}, L_{51} termed as l_1, l_2, l_3, l_4 and l_5. These linear distances will be used later on for developing the transformation matrices.

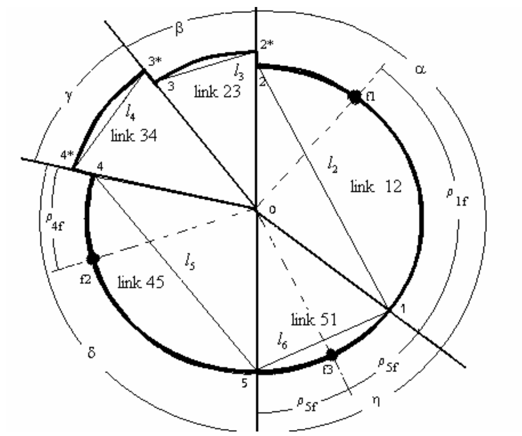

Fig 2. Linear distance between the joints of the spherical mechanism

4. KINEMATIC MODEL

A fixed co-ordinate frame F (Xo, Yo, Zo) has been considered at the centre of the sphere. The origin of the fixed co-ordinate frame F is o, which is also the centre of the sphere. The x-axis has been placed along the joint axis of joint 1. This set up of the co-ordinate frame is shown in the figure 3. To define the coordinate rotation and transformation, moving frames are attached to the position 1, f1, 2, 3, 4, f2, 5 and f3 . Now through the development of the transformation matrices the kinematics of the links can be described.The transformation matrix of the fixed frame F (X_o, Yo, Zo) at the origin is

$$T_0^{\,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which is the form of homogenous transformation matrix

$$\begin{bmatrix} (R) & & & (d) \\ & & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where (R) is the 3 x 3 orientation matrix and (d) is the 3 x 1 position matrix.The position matrix will define the x,y,z co-ordinate of that particular point in the three dimensional space. The coordinate transformation matrix of the joint position 1 relative to the fixed frame F (X_o, Yo, Zo) will be denoted as $T_0^{\,1}$. The relative coordinate transformation matrix at different joint and finger position will be derived.

4.1 Coordinate Transformation Matrix at Joint Axis Position 1

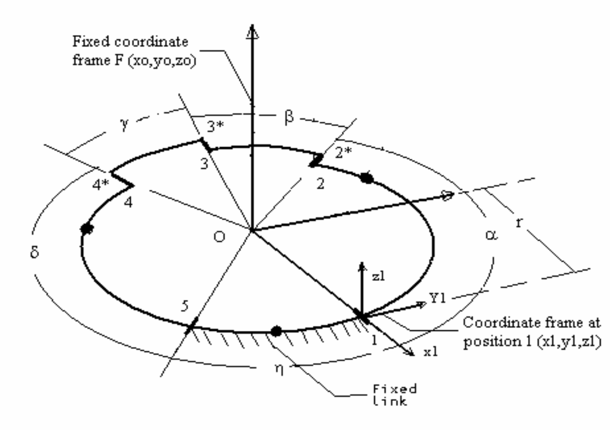

Fig 3. Coordinate transformation at joint position 1

To define the transformation matrix both the translation and the rotation of the moving frame have to be considered. figure 3 shows the co-ordinate frame at joint position1. To derive the transformation matrix at joint position 1 with respect to fixed frame, a translation of r along the x-axis with respect to fixed frame has been considered. Where r is the radius of the circle . But there is no rotation of the co-ordinate frame with respect to the fixed frame.

T_0^1 = Translation of (r, 0, 0, 1) and no rotation with respect to fixed frame F.

$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where T_0^1 represents the transformation matrix of joint 1 with respect to the fixed frame F. and

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \qquad (1)$$

Where X_1, Y_1, Z_1 represents the x,y,z co-ordinate in the space with respect to fixed frame.

4.2 Coordinate Transformation Matrix of Finger Position F1

The link configuration changes as the link L_{12} start to rotate about the joint axis 1. Figure 4 shows the angular displacement θ_1 of the link L_{12} with respect to the reference link L_{51}. The point f1 lies on the moving link L_{12}.

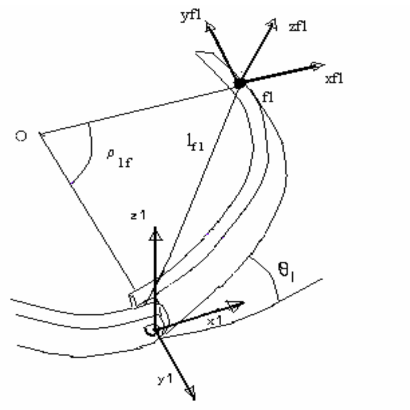

Fig 4. Relative angular rotation of the link

To define the transformation matrix at finger position f1, a translation along the x, y and z axis and a rotation about the x and z axis with respect to frame at position 1 has to be considered. Figure5 shows the geometric configuration to determine the translation in the x, y and z-axis with respect to co-ordinate at point 1.

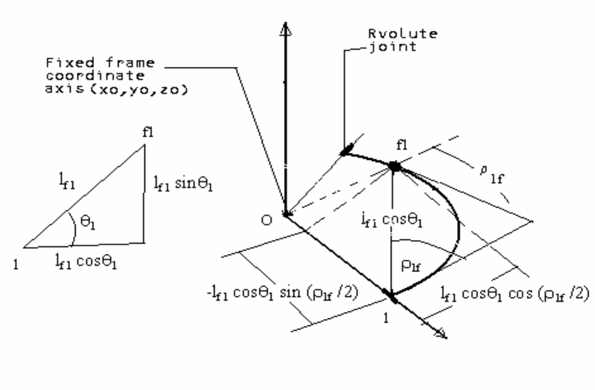

Fig 5. Translation of position f1 relative to position 1

The translation along the x-axis is $-l_{f1}\cos\theta_1 \sin(\rho_{1f}/2)$, along the y-axis is $l_{f1}\cos\theta_1 \cos(\rho_{1f}/2)$ and along the z-axis is $l_{f1}\sin\theta_1$ with respect to co-ordinate at point 1. The transformation matrix of point f1 with respect to point 1 termed as T_1^{f1} can be defined by the composition of translation along x, y and z axis and rotation of θ_1 about x-axis and ρ_{1f} about z-axis.

$$T_1^{f1} = \begin{bmatrix} C\rho_{1f} & -S\rho_{1f} & 0 & -l_{f1}C\theta_1 S(\rho_{1f}/2) \\ S\rho_{1f}C\theta_1 & C\rho_{1f}C\theta_1 & -S\theta_1 & l_{f1}C\theta_1 C(\rho_{1f}/2) \\ S\rho_{1f}S\theta_1 & C\rho_{1f}S\theta_1 & C\theta_1 & l_{f1}S\theta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where sin = S and cos = C , therefore

By applying the composition law of transformation matrices we can derive the co-ordinate transformation matrix T_0^{f1} of point f1 with respect to the (origin) fixed frame F.

$$T_0^{f1} = T_0^1 T_1^{f1}$$

$$= \begin{bmatrix}1&0&0&r\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix} C\rho_{1f} & -S\rho_{1f} & 0 & -l_{f1}C\theta_1 S(\rho_{1f}/2)\\ S\rho_{1f}C\theta_1 & C\rho_{1f}C\theta_1 & -S\theta_1 & l_{f1}C\theta_1 C(\rho_{1f}/2)\\ S\rho_{1f}S\theta_1 & C\rho_{1f}S\theta_1 & C\theta_1 & l_{f1}S\theta_1\\ 0&0&0&1\end{bmatrix}$$

$$= \begin{bmatrix} C\rho_{1f} & -S\rho_{1f} & 0 & -l_{f1}C\theta_1 S(\rho_{1f}/2)+r\\ S\rho_{1f}C\theta_1 & C\rho_{1f}C\theta_1 & -S\theta_1 & l_{f1}C\theta_1 C(\rho_{1f}/2)\\ S\rho_{1f}S\theta_1 & C\rho_{1f}S\theta_1 & C\theta_1 & l_{f1}S\theta_1\\ 0&0&0&1\end{bmatrix}$$

From the transformation matrix the position of the point f1 with respect to the fixed frame can be derived as equation (2)

$$\begin{bmatrix} X_{f_1}\\ Yf_1\\ Z_{f_1}\end{bmatrix} = \begin{bmatrix} -l_{f1}C\theta_1 S(\rho_{1f}/2)+r\\ l_{f1}C\theta_1 C(\rho_{1f}/2)\\ l_{f1}S\theta_1\end{bmatrix} \quad (2)$$

Where X_{f1}, Y_{f1}, Z_{f1} are the coordinate positions in the three dimensional space.

As the value l_{f1} and ρ_{1f} can be fixed according to geometric design parameter equation (2) can be describe as a function of θ_1.

$X_{f1}= f(\theta_1)$, $Y_{f1}= f(\theta_1)$ and $Z_{f1}= f(\theta_1)$

4.3. Coordinate Transformation Matrix of Position 2 And 2*

The coordinate transformation matrix T_1^2 of joint position 2 with respect to joint position 1, can be derived in the similar manner as described earlier. Figure 6 shows the transformed co-ordinate at point 2 and 2* with respect to point 1 when link L_{12} has an angular displacement of θ_1 relative to link L_{51}.

The translation to be considered along x-axis is $l_2 C\theta_1 S(\alpha/2)$, along y-axis $l_2 C\theta_1 C(\alpha/2)$, and along z-axis $l_2 S\theta_1$ followed by the rotation of θ_1 about x-axis and α about z-axis.

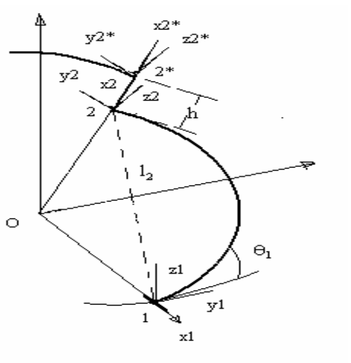

Fig 6. Coordinate transformation at position 2

$$T_1^2 = \begin{bmatrix} C\alpha & -S\alpha & 0 & -l_2 C\theta_1 S(\alpha/2)\\ S\alpha C\theta_1 & C\alpha C\theta_1 & -S\theta_1 & l_2 C\theta_1 C(\alpha/2)\\ S\alpha S\theta_1 & C\alpha S\theta_1 & C\theta_1 & l_2 S\theta_1\\ 0&0&0&1\end{bmatrix}$$

The co-ordinate transformation matrix T_0^2 of point 2 with respect to the (origin) fixed frame F,

$$T_0^2 = T_0^1 T_1^2 = \begin{bmatrix} C\alpha & -S\alpha & 0 & -l_2 C\theta_1 S(\alpha/2)+r\\ S\alpha C\theta_1 & C\alpha C\theta_1 & -S\theta_1 & l_2 C\theta_1 C(\alpha/2)\\ S\alpha S\theta_1 & C\alpha S\theta_1 & C\theta_1 & l_2 S\theta_1\\ 0&0&0&1\end{bmatrix}$$

From the transformation matrix the position of the point f1 with respect to the fixed frame can be derived as equation (3)

$$\begin{bmatrix} X_2\\ Y_2\\ Z_2\end{bmatrix} = \begin{bmatrix} -l_2 C\theta_1 S(\alpha/2)+r\\ l_2 C\theta_1 C(\alpha/2)\\ l_2 S\theta_1\end{bmatrix} \quad (3)$$

Where X_2, Y_2, Z_2 are the coordinate positions in the three dimensional space.

As the value l_2 and α can be set according to geometric design parameter, equation (4.8) can be describe as a function of θ_1.

$X_2= f(\theta_1)$, $Y_2= f(\theta_1)$ and $Z_2= f(\theta_1)$

The point 2 and 2* are on the same axis but of two different link. Point 2 is described as the joint position of link L_{12} and point 2* is the joint position of link L_{23} on the same joint axis as shown in the figure 4.8. The co-ordinate orientation matrix of these two points will same but there position matrix will differ as, h is the distance between the two points.

The transformation matrix T_2^{2*} of point 2* with respect two point 2, can be derived by considering only translation of h along x-axis and no rotation.

$$T_2^{2*} = \begin{bmatrix}1&0&0&h\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Co-ordinate transformation matrix T_0^{2*} of point 2* with respect to the (origin) fixed frame F, $T_0^{2*} = T_0^2 \; T_2^{2*}$

$$= \begin{bmatrix} C\alpha & -S\alpha & 0 & hC\alpha - l_2 C\theta_1 S(\alpha/2)+r\\ S\alpha C\theta_1 & C\alpha C\theta_1 & -S\theta_1 & hS\alpha C\theta_1 + l_2 C\theta_1 C(\alpha/2)\\ S\alpha S\theta_1 & C\alpha S\theta_1 & C\theta_1 & hS\alpha S\theta_1 + l_2 S\theta_1\\ 0&0&0&1\end{bmatrix}$$

From the above transformation matrix the position of the point 2* in the space with respect to the fixed frame can be derived as equation (4) as mentioned bellow

$$\begin{bmatrix} X_{2*}\\ Y_{2*}\\ Z_{2*}\end{bmatrix} = \begin{bmatrix} hC\alpha - l_2 C\theta_1 S(\alpha/2)+r\\ hS\alpha C\theta_1 + l_2 C\theta_1 C(\alpha/2)\\ hS\alpha S\theta_1 + l_2 S\theta_1\end{bmatrix} \quad (4)$$

where h is the linear distance between the point 2 and 2*. As the value l_2, h, α and r can be set according to geometric design parameter, equation (4) can be describe as a function of θ_1.

$X_{2*}= f(\theta_1)$, $Y_{2*}= f(\theta_1)$ and $Z_{2*}= f(\theta_1)$

4.4 Coordinate Transformation Matrix of Position 3 & 3*

According to the mechanism displacement, link L_{23} will have an angular displacement of θ_2 relative to link L_{12} which is shown in the figure 7.

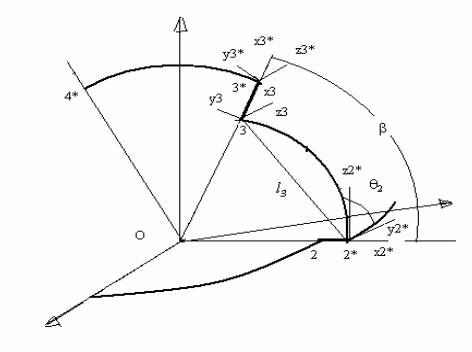

Fig 7. Coordinate transformation at position 3

Similarly link L_{34} will have an angular displacement of θ_3 relative to link L_{23} , link L_{45} will have an angular displacement of θ_4 relative to link L_{34} and θ_5 will be the angular displacement of link L_{51} with respect to link L_{45}

Similarly considering three dimensional translation and rotation and applying composition law of transformation matrix the co-ordinate transformation matrices T_0^3 T_0^{3*} T_0^{4*} T_0^4 T_0^{f2} T_0^5 &T_0^1 can be derived sequentially. From the derived transformation matrices the three dimensional co-ordinate points of joint position 3, 3*,4*,4,5,1 and finger point f2, f3 are determined as follows.

$$\begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix} = \begin{bmatrix} -l_3C\theta_2S(\beta/2)C\alpha - l_3C\theta_2C(\beta/2)S\alpha + hC\alpha - l_2C\theta_1S(\alpha/2) + r \\ -l_3C\theta_2S(\beta/2)S\alpha C\theta_1 + l_3C\theta_2C(\beta/2)C\alpha C\theta_1 - l_3S\theta_2S\theta_1 + hS\alpha C\theta_1 + l_2C\theta_1C(\alpha/2) \\ -l_3C\theta_2S(\beta/2)S\alpha S\theta_1 + l_3C\theta_2C(\beta/2)C\alpha S\theta_1 + l_3S\theta_2C\theta_1 + hS\alpha S\theta_1 + l_2S\theta_1 \end{bmatrix} \quad (5)$$

As the value l_3 , h, r , α and β will be set according to geometric design parameter, equation (5) can be describe as a function of θ_1 and θ_2.

$X_3= f(\theta_1,\theta_2)$, $Y_3= f(\theta_1,\theta_2)$ and $Z_3= f(\theta_1,\theta_2)$

Similarly

$X_{3*}= f(\theta_1,\theta_2)$, $Y_{3*}= f(\theta_1,\theta_2)$ and $Z_{3*}= f(\theta_1,\theta_2)$ (6)

$X_{4*}= f(\theta_1,\theta_2,\theta_3)$, $Y_{4*}= f(\theta_1,\theta_2,\theta_3)$ and $Z_{4*}= f(\theta_1,\theta_2,\theta_3)$ (7)

$X_4= f(\theta_1,\theta_2,\theta_3)$, $Y_4= f(\theta_1,\theta_2,\theta_3)$ and $Z_4= f(\theta_1,\theta_2,\theta_3)$ (8)

$X_{f2}= f(\theta_1,\theta_2,\theta_3,\theta_4)$, $Y_{f2}= f(\theta_1,\theta_2,\theta_3,\theta_4)$ and

$Z_{f2}= f(\theta_1,\theta_2,\theta_3,\theta_4)$ (9)

$X_5= f(\theta_1,\theta_2,\theta_3,\theta_4)$, $Y_5= f(\theta_1,\theta_2,\theta_3,\theta_4)$ and

$Z_5= f(\theta_1,\theta_2,\theta_3,\theta_4)$ (10)

$X_1= f(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)$, $Y_1= f(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)$ and

$Z_1= f(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)$ (11)

Co-ordinate points for finger position f3 is

$$\begin{bmatrix} X_{f3} \\ Y_{f3} \\ Z_{f3} \end{bmatrix} = \begin{bmatrix} -l_{f5}S(\rho_{5f}/2) + r \\ -l_{f5}C(\rho_{5f}/2) \\ 0 \end{bmatrix} \quad (12)$$

5. EQUATIONS FOR LINK CONFIGURATIONS

The different combinations of the relative angular displacements $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ of various links can achieve different configurations of the mechanism. Because of the close loop nature of the mechanism there will be certain constraints of the five-bar spherical mechanism. From the mathematical and geometrical analysis some equations have been generated which have to be satisfied by the various configuration stage of the mechanism.

Equation (1) and equation (10) both derived the same co-ordinate points at position 1 (X_1,Y_1,Z_1). So setting equation (1) = equation (10)

$f(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)=r$

$f(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)=0$

$f(\theta_1,\theta_2,\theta_3,\theta_4,\theta_5)=0$

From equation (3)

$X_2= f(\theta_1)$, $Y_2= f(\theta_1)$ and $Z_2= f(\theta_1)$

From equation (8)

$X_4= f(\theta_1,\theta_2,\theta_3)$, $Y_4= f(\theta_1,\theta_2,\theta_3)$ and $Z_4= f(\theta_1,\theta_2,\theta_3)$

As the mechanism is the metamorphic mechanism there will be a new configuration state of the mechanism when θ_3 , the relative angular displacement between the link L_{23} and L_{34} will be 180^0. At this the five bar mechanism will transform to four bar mechanism and the co-ordinates of point 2 and point 4 will coincide.

Therefore it can be said that

At $\theta_3 = 180^0$, equation (3) = equation (8)

$$f(\theta_1)=f(\theta_1,\theta_2,\theta_3) \quad (13)$$

$$f(\theta_1) = f(\theta_1,\theta_2,\theta_3) \quad (14)$$

$$f(\theta_1)=f(\theta_1,\theta_2,\theta_3) \quad (15)$$

Where $\theta_3 = 180^0$

The equations from (3) to (15) can be solved to find the relative angular displacements of the various link.

6. ONE LINK CONFIGURATION

There will be a certain configuration state when three links together will act as a single link. If the value of θ_2 and θ_3 is zero there will be no relative angular displacement between the links L_{12} and L_{23} and between the links L_{23} and L_{34}. That means at that state the three links L_{12}, L_{23} and L_{34} will move as an single link L_{14}. According to the design parameter L_{14} is a half circle as shown in the figure 8. At this stage the mechanism will act as a three-bar spherical mechanism.

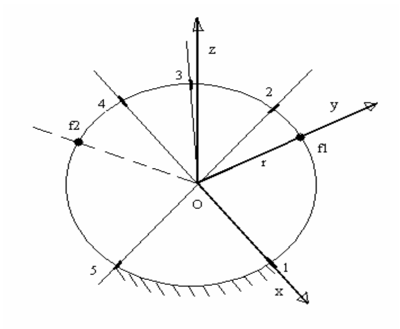

Fig 8. Three links together as a single link

Similarly link L_{23}; L_{34} and L_{45} can also take the form of a single link L_{25} when the value of relative angular displacement θ_3 and θ_4 will be zero. According to the design parameter L_{25} is also a half circle, which is shown in the figure 8.2.

Transformation matrix of finger position f1 will consist of translation of $r\cos\theta_1$ along the y-axis and $r\sin\theta_1$ along the z-axis and rotation of θ_1 about the x-axis.

$$T_o^{f1} = Trans[0, r\cos\theta_1, r\sin\theta_1, 1].Rot(x,\theta_1)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 & r\cos\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 & r\sin\theta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. CONCLUSIONS

Geometrical model and detail kinematic analysis of the five-bar spherical metamorphic mechanism has been presented through out this chapter. The mathematical formulation has been developed to achieve different link configurations and define the position and orientation of any point of the link in three-dimensional space. This mathematical and geometrical model can be used to design a metamorphic robot palm, which can achieve human like operational flexibility.

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