

# LACEMENT POTENTIAL SOLUTION OF A STIFFENED COLUMN OF COMPOSITE MATERIALS UNDER UNIFORM LOAD OVER A PART OF THE TIP

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## ABSTRACT

In this paper, a stiffened column of orthotropic composite materials is considered in order to analyze the elastic field due to a uniform compressive load over central half of the tip. Following the new approach of displacement potential, the mixed-boundary-value elastic problem is formulated in terms of a single displacement potential function. The solutions are obtained in the form of an infinite series. Some numerical results of different stress and displacement components at different critical sections of the stiffened column are presented in the form of graphs. The results appear to be quite reasonable and accurate, and thus establish the soundness as well as reliability of the present displacement potential solution

**Keywords:** Analytical solution, Stiffened column, Displacement potential function, Orthotropic materials.

## 1. INTRODUCTION

Elasticity problems are usually formulated either in terms of stress function or displacement parameters. Timoshenko and Goodier [1] considered stress function formulation and Uddin [2] considered displacement parameters to solve various boundary-value problems of elasticity. Successful application of the stress function formulation in conjunction with finite-difference technique has been reported by Chow *et al.* and Chapel and Smith [3-4] for the solution of plane elastic problems where all the conditions on the boundary are prescribed in terms of stresses only. Further, Conway and Ithaca [5] extended the stress function formulation in the form of Fourier integrals to the case where the material is orthotropic, and obtained analytical solutions for a number of ideal problems. The shortcoming of the stress function approach is that it treats boundary conditions in terms of loading only. Boundary restraints specified in terms of the displacement components cannot be satisfactorily treated by the stress function. As most of the practical problems of elasticity are of mixed boundary conditions, the stress function approach fails to provide any explicit understanding of the state of stresses at the critical regions of supports and loadings. The displacement formulation, on the other hand, involves finding two displacement functions simultaneously from the two second-order elliptical partial differential equations of equilibrium, which is extremely difficult, and the problem becomes more serious when the boundary conditions are mixed [2]. The difficulties involved in trying to solve practical stress problems

using the existing models are clearly pointed out by Durelli and Ranganayakuma [6].

As stated above, neither of the formulations is suitable for solving the problems of mixed boundary conditions. Hence a new mathematical model is an utmost necessary to solve the mixed boundary-value problems of elasticity. In three of our earlier papers [7, 8, 9], the displacement potential approach, a new technique to solve mixed boundary-value problems, was applied to analyze the elastic field in a stiffened composite bar subjected to loading in the direction parallel and perpendicular to the fibers. In that approach, the plane elasticity problems were formulated in terms of a single displacement potential function of space variables. The approach was verified to be efficient in managing mixed mode of boundary conditions as well as their zones of transition. The present paper demonstrates the application of the displacement potential approach to the analytical solution of a stiffened column of orthotropic composite material subjected to a compressive load over central half of the tip. The supporting edge of the column is assumed to be rigidly fixed and the two opposing edges are stiffened. The solutions are obtained in the form of an infinite series and the corresponding distributions of different stress and displacement components are presented mainly in the form of graphs.

## 2. ANALYTICAL MODEL OF THE PROBLEM

With reference to the Cartesian coordinate system  $x-y$ , a stiffened column of composite materials is shown in Fig. 1. The fibers are directed along the height of the

column. The bottom edge is rigidly fixed to a support and the opposing edges are stiffened. The height and the width of the column are designated by  $b$  and  $a$ , respectively. The central half of the tip of the column is subjected to a compressive axial load  $\sigma_{xx}$ , which is a function of  $y$  only.

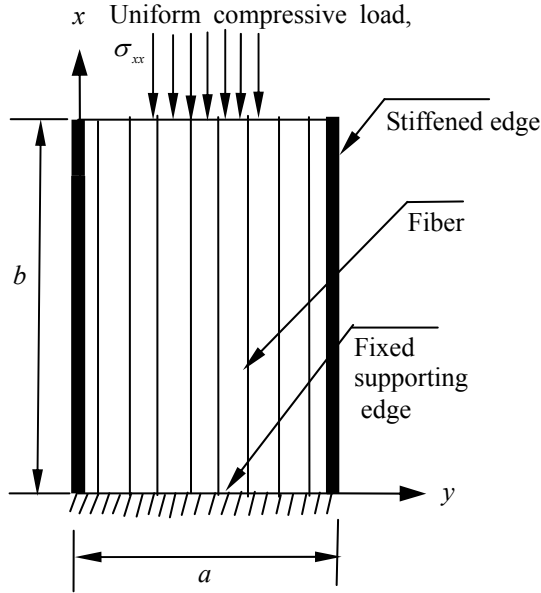


Fig 1. Analytical model of the problem

For this model of the problem, different stress and displacement components are calculated at different critical sections of the column using the method of single displacement potential function.

### 3. DISPLACEMENT POTENTIAL FORMULATION FOR THE PROBLEM

With reference to a rectangular Cartesian coordinate system and in the absence of body forces, the equilibrium equations are given by [1]

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0 \quad (1a)$$

$$\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} = 0 \quad (1b)$$

To express the equilibrium equations in terms of displacement components, we need to express the three stress components in terms of displacement parameters. The corresponding three stress-displacement relations for general orthotropic materials are obtained from the Hooke's law as follows [10]

$$\sigma_{xx} = \frac{E_1}{1 - \mu_{12} \mu_{21}} \left[ \frac{\partial u_x}{\partial x} + \mu_{21} \frac{\partial u_y}{\partial y} \right] \quad (2a)$$

$$\sigma_{yy} = \frac{E_2}{1 - \mu_{12} \mu_{21}} \left[ \frac{\partial u_y}{\partial y} + \mu_{12} \frac{\partial u_x}{\partial x} \right] \quad (2b)$$

$$\sigma_{xy} = G_{12} \left[ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right] \quad (2c)$$

Substituting the above stress-displacement relations into Eqs. (1a) and (1b) and using the reciprocal relation  $E_2 \mu_{12} = E_1 \mu_{21}$ , we obtain the two equilibrium equations for two-dimensional problems of orthotropic materials in terms of the two displacement components as

$$\left( \frac{E_1^2}{E_1 - \mu_{12}^2 E_2} \right) \frac{\partial^2 u_x}{\partial x^2} + \left( \frac{\mu_{12} E_1 E_2}{E_1 - \mu_{12}^2 E_2} + G_{12} \right) \frac{\partial^2 u_y}{\partial x \partial y} + G_{12} \frac{\partial^2 u_x}{\partial y^2} = 0 \quad (3a)$$

$$\left( \frac{E_1 E_2}{E_1 - \mu_{12}^2 E_2} \right) \frac{\partial^2 u_y}{\partial y^2} + \left( \frac{\mu_{12} E_1 E_2}{E_1 - \mu_{12}^2 E_2} + G_{12} \right) \frac{\partial^2 u_x}{\partial x \partial y} + G_{12} \frac{\partial^2 u_y}{\partial x^2} = 0 \quad (3b)$$

In the present study, a new displacement potential function  $\psi(x, y)$  is defined in terms of the two displacement components as follows:

$$u_x = \frac{\partial^2 \psi}{\partial x \partial y} \quad (4a)$$

$$u_y = -\frac{1}{Z_{11}} \left[ E_1^2 \frac{\partial^2 \psi}{\partial x^2} + G_{12} (E_1 - \mu_{12}^2 E_2) \frac{\partial^2 \psi}{\partial y^2} \right] \quad (4b)$$

$$\text{where } Z_{11} = \mu_{12} E_1 E_2 + G_{12} (E_1 - \mu_{12}^2 E_2)$$

With the above definition of  $\psi(x, y)$ , the first equilibrium equation 3(a) is automatically satisfied. Therefore,  $\psi$  has to satisfy the second equilibrium equation 3(b) only. Expressing Eq. 3(b) in terms of the displacement potential function  $\psi$ , the condition that  $\psi$  has to satisfy is

$$E_1 G_{12} \frac{\partial^4 \psi}{\partial x^4} + E_2 (E_1 - 2\mu_{12} G_{12}) \frac{\partial^4 \psi}{\partial x^2 \partial y^2} + E_2 G_{12} \frac{\partial^4 \psi}{\partial y^4} = 0 \quad (5)$$

### 4. SOLUTION OF THE PROBLEM

For the model shown in Fig.1, the stiffened column is considered to be of unit thickness and the potential function  $\psi$  is assumed to be

$$\psi = \sum_{m=1}^{\infty} X_m \cos \alpha y \quad (6)$$

where  $X_m$  is a function of  $x$  only and  $\alpha = m\pi/a$ . Thus,  $X_m$  has to satisfy the ordinary differential equation

$$X_m'''' - \left( \frac{E_2}{G_{12}} - \frac{2\mu_{12}E_2}{E_1} \right) \alpha^2 X_m'' + \frac{E_2}{E_1} \alpha^4 X_m = 0 \quad (7)$$

where the (') indicates differentiation with respect to  $x$ . The general solution of this differential equation can be given by:

$$X_m = A_m e^{r_1 x} + B_m e^{r_2 x} + C_m e^{r_3 x} + D_m e^{r_4 x} \quad (8)$$

where

$$r_1, r_2 = \alpha \left[ \frac{1}{2} \left\{ \left( \frac{E_2}{G_{12}} - \frac{2\mu_{12}E_2}{E_1} \right) \pm \sqrt{\left( \frac{E_2}{G_{12}} - \frac{2\mu_{12}E_2}{E_1} \right)^2 - \frac{4E_2}{E_1}} \right\} \right]^{1/2} \quad (9)$$

$$r_3, r_4 = -\alpha \left[ \frac{1}{2} \left\{ \left( \frac{E_2}{G_{12}} - \frac{2\mu_{12}E_2}{E_1} \right) \pm \sqrt{\left( \frac{E_2}{G_{12}} - \frac{2\mu_{12}E_2}{E_1} \right)^2 - \frac{4E_2}{E_1}} \right\} \right]^{1/2} \quad (10)$$

Here  $A_m, B_m, C_m,$  and  $D_m$  are constants. Now combining Eqs.(2), (4), (6), and (8), the expressions of stress and displacement components are obtained as follows:

$$u_x(x, y) = -\sum_{m=1}^{\infty} \alpha X_m' \sin \alpha y \quad (11)$$

$$u_y(x, y) = \frac{1}{Z_{11}} \sum_{m=1}^{\infty} \left[ -E_1 X_m'' + G_{12} (E_1 - \mu_{12}^2 E_2) X_m \alpha^2 \right] \cos \alpha y \quad (12)$$

$$\sigma_{xx}(x, y) = -\frac{E_1 G_{12}}{Z_{11}} \sum_{m=1}^{\infty} \left[ \alpha E_1 X_m'' + \alpha^3 E_2 \mu_{12} X_m \right] \sin \alpha y \quad (13)$$

$$\sigma_{yy}(x, y) = -\frac{E_1 G_{12}}{Z_{11}} \sum_{m=1}^{\infty} \left[ G_{12} \alpha^3 X_m + \alpha (\mu_{12} G_{12} - E_1) X_m'' \right] \sin \alpha y \quad (14)$$

$$\sigma_{xy}(x, y) = -\frac{E_1 G_{12}}{Z_{11}} \sum_{m=1}^{\infty} \left[ E_1 X_m'' + \alpha^2 \mu_{12} E_2 X_m' \right] \cos \alpha y \quad (15)$$

For the present problem, the following boundary conditions are available:

$$u_x = 0, \sigma_{yy} = 0 \text{ at } y=0 \text{ and } a \quad (16)$$

$$u_x(0, y) = 0, u_y(0, y) = 0 \quad (17)$$

It is seen that the boundary conditions given by Eq. (16) are satisfied automatically.

Now, the axial compressive loading on the tip,  $x = b$ , of the column can be expressed mathematically as

follows:

$$\sigma_{xx}(b, y) = P = \sum_{m=1}^{\infty} I_m \sin \alpha y \quad (18a)$$

where  $P$  is the axial compressive load distributed over the region from  $y=a/4$  to  $3a/4$ . From Fourier integral formula, it can be written that

$$I_m = -\frac{2P}{m\pi} [\cos(3m\pi/4) - \cos(m\pi/4)] \text{ for } m=1, 2, 3, \dots, \infty \quad (18b)$$

The shear stress at this boundary is

$$\sigma_{xy}(b, y) = 0 \quad (19)$$

By applying the associated boundary conditions in relevant equations, we get the following four equations in terms of the four unknowns  $A_m, B_m, C_m,$  and  $D_m$ :

$$r_1 A_m + r_2 B_m + r_3 C_m + r_4 D_m = 0 \quad (20a)$$

$$\left( \frac{Z_{12}}{Z_{11}} \alpha^2 - \frac{E_1^2}{Z_{11}} r_1^2 \right) A_m + \left( \frac{Z_{12}}{Z_{11}} \alpha^2 - \frac{E_1^2}{Z_{11}} r_2^2 \right) B_m + \left( \frac{Z_{12}}{Z_{11}} \alpha^2 - \frac{E_1^2}{Z_{11}} r_3^2 \right) C_m + \left( \frac{Z_{12}}{Z_{11}} \alpha^2 - \frac{E_1^2}{Z_{11}} r_4^2 \right) D_m = 0 \quad (20b)$$

$$\left( -\frac{E_1^2 G_{12} \alpha r_1^2}{Z_{11}} e^{r_1 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^3}{Z_{11}} e^{r_1 b} \right) A_m + \left( -\frac{E_1^2 G_{12} \alpha r_2^2}{Z_{11}} e^{r_2 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^3}{Z_{11}} e^{r_2 b} \right) B_m + \left( -\frac{E_1^2 G_{12} \alpha r_3^2}{Z_{11}} e^{r_3 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^3}{Z_{11}} e^{r_3 b} \right) C_m + \left( -\frac{E_1^2 G_{12} \alpha r_4^2}{Z_{11}} e^{r_4 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^3}{Z_{11}} e^{r_4 b} \right) D_m = I_m \quad (20c)$$

$$\left( -\frac{E_1^2 G_{12} \alpha r_1^3}{Z_{11}} e^{r_1 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_1}{Z_{11}} e^{r_1 b} \right) A_m + \left( -\frac{E_1^2 G_{12} \alpha r_2^3}{Z_{11}} e^{r_2 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_2}{Z_{11}} e^{r_2 b} \right) B_m + \left( -\frac{E_1^2 G_{12} \alpha r_3^3}{Z_{11}} e^{r_3 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_3}{Z_{11}} e^{r_3 b} \right) C_m + \left( -\frac{E_1^2 G_{12} \alpha r_4^3}{Z_{11}} e^{r_4 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_4}{Z_{11}} e^{r_4 b} \right) D_m = 0 \quad (20d)$$

$$\left( -\frac{E_1^2 G_{12} r_1^3}{Z_{11}} e^{r_1 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_1}{Z_{11}} e^{r_1 b} \right) A_m + \left( -\frac{E_1^2 G_{12} r_2^3}{Z_{11}} e^{r_2 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_2}{Z_{11}} e^{r_2 b} \right) B_m + \left( -\frac{E_1^2 G_{12} r_3^3}{Z_{11}} e^{r_3 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_3}{Z_{11}} e^{r_3 b} \right) C_m + \left( -\frac{E_1^2 G_{12} r_4^3}{Z_{11}} e^{r_4 b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_4}{Z_{11}} e^{r_4 b} \right) D_m = 0 \quad (20d)$$

where

$$Z_{12} = G_{12} (E_1 - \mu_{12}^2 E_2)$$

The above four simultaneous algebraic equations 20(a)-20(d) can further be realized in a simplified form for the solution of the unknowns as follows:

$$\begin{bmatrix} r_1 & r_2 & r_3 & r_4 \\ P_1 & P_2 & P_3 & P_4 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ R_1 & R_2 & R_3 & R_4 \end{bmatrix} \begin{bmatrix} A_m \\ B_m \\ C_m \\ D_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_m \\ 0 \end{bmatrix} \quad (21)$$

where

$$\begin{aligned} P_i &= \frac{Z_{12}}{Z_{11}} \alpha^2 - \frac{E_1^2}{Z_{11}} r_i^2 \\ Q_i &= -\frac{E_1^2 G_{12} \alpha r_i^2}{Z_{11}} e^{r_i b} - \frac{E_1 E_2 G_{12} \mu_{12} \alpha^3}{Z_{11}} e^{r_i b} \\ R_i &= \begin{pmatrix} -\frac{E_1^2 G_{12} r_i^3}{Z_{11}} e^{r_i b} \\ \frac{E_1 E_2 G_{12} \mu_{12} \alpha^2 r_i}{Z_{11}} e^{r_i b} \end{pmatrix} \end{aligned} \quad i = 1, 2, 3, 4.$$

## 5. RESULTS AND DISCUSSION

In this section, numerical results are presented for a Boron / Epoxy unidirectional composite column. The effective mechanical properties of the Boron / Epoxy composite are  $E_1=28.29 \times 10^4$  MPa,  $E_2=2.415 \times 10^4$  MPa,  $\mu_{12}=0.27$  and  $G_{12}=1.035 \times 10^4$  MPa. Furthermore, the aspect ratio of the column used in obtaining the results is taken as  $b/a=3.0$ .

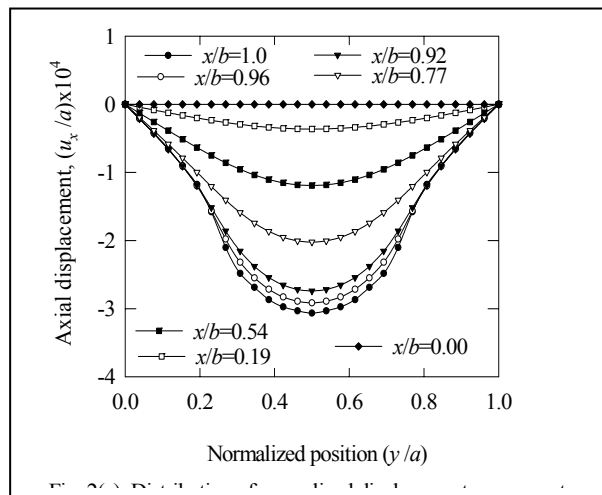


Fig 2(a). Distribution of normalized displacement component ( $u_x/a$ ) at different sections of the column.

The variation of the normalized axial displacement component with  $y$  is shown in Fig. 2(a). The axial displacement is negative and symmetric. At the supporting edge and stiffened edges of the column, the axial displacement is zero, which satisfies the physical boundary condition of the problem. For each section, the axial displacement is maximum at the center of the section i.e. at  $y/a=0.5$ . Furthermore, the axial displacement decreases as  $x/b$  decreases for a fixed value of  $y/a$ .

Figure 2(b) illustrates the variation of normalized

lateral displacement component with  $y$  at different sections of the column. Except the tip and its adjoining few sections of the column, the lateral displacement is zero for all of the sections. The lateral displacement is antisymmetric.

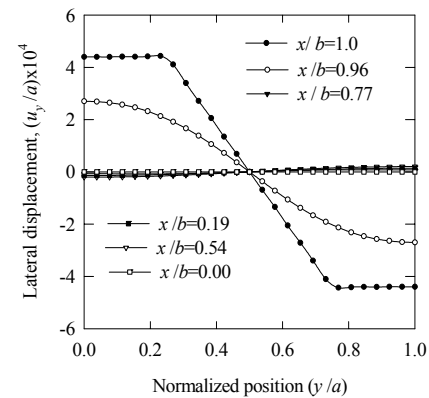


Fig 2(b). Distribution of normalized lateral displacement component ( $u_y/a$ ) at different sections of the column.

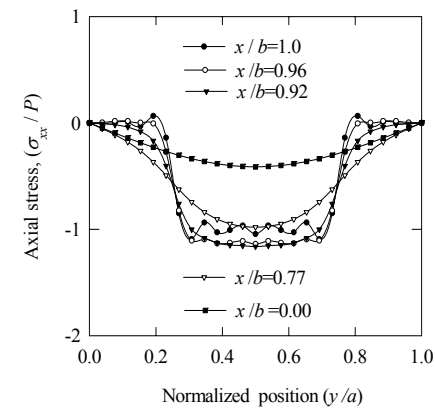


Fig 3(a). Distribution of normalized axial stress component ( $\sigma_{xx}/P$ ) at different sections of the column.

Figure 3(a) illustrates the distribution of normalized axial stress component with the variation of  $y$  at different sections of the column. At and near the tip of the column, the stress distribution is almost uniform over the central half of the tip. There is a small fluctuation in the stress distribution for  $x/b=1.0$ , which may occur due to the error in numerical computations. At the tip ( $x/b=1.0$ ) and its few adjoining sections, the axial stress sharply reduces to zero after the central half of the tip. For the sections at and near the fixed support, the axial stress gradually reduces from a maximum value at the center of the tip to zero at the stiffened edges.

Figure 3(b) is the distribution of normalized lateral stress vs.  $y$  at different sections of the column. This stress distribution is also symmetric with respect to  $y$ . Further, it is noted that at the two stiffened edges ( $y/a=0$  and  $y/a=1.0$ ), the lateral stress is zero, which satisfies the physical boundary conditions. Towards the supporting edge of the column, the lateral stress decreases.

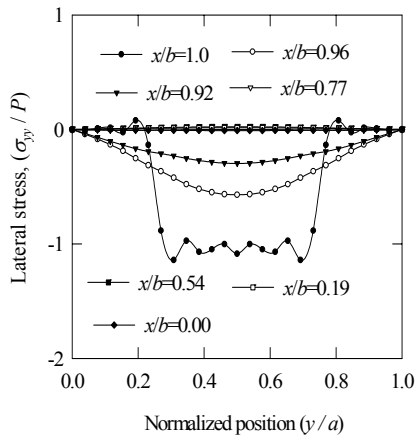


Fig 3(b). Distribution of normalized lateral stress components ( $\sigma_{yy}/P$ ) at different sections of the column.

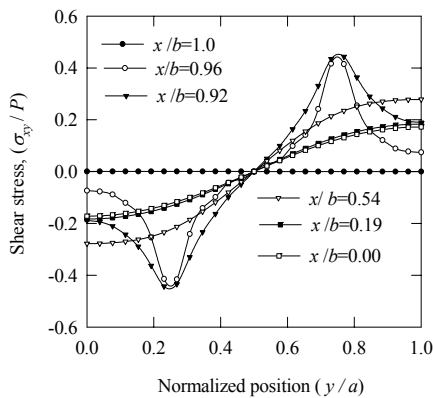


Fig 3(c). Distribution of normalized shear stress component ( $\sigma_{xy}/P$ ) at different sections of the column.

The distribution of normalized shearing stress as a function of  $x$  and  $y$  is shown in Fig.3(c). At  $x/b=1.0$ , i.e. at the tip of the column, the shearing stress is zero, which satisfies the physical boundary conditions of the problem. Except the tip ( $x/b=1.0$ ), there develops a considerable amount of shear stress which is distributed antisymmetrically along  $y$ . It is worthy to mention that although the axial and lateral stresses are zero at the fixed support ( $x/b=0.0$ ), there develops a significant amount of shear stress at this support.

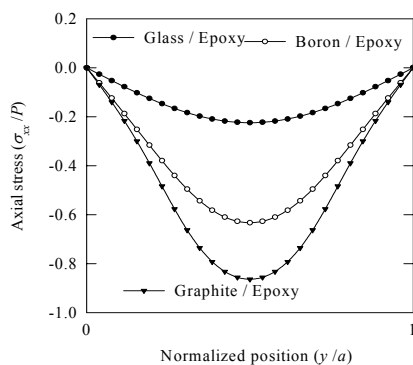


Fig 4. Distribution of normalized axial stress component ( $\sigma_{xx}/P$ ) at the section  $x/b=0.50$  of the column of different composites.

Figure 4 illustrates the normalized axial stress component at the section  $x/b=0.5$  of the column of different composites. The axial stress is maximum for the column of Graphite / Epoxy, minimum for the column of Glass / Epoxy, and medium for the column of Boron / Epoxy. Figure 5 illustrates the distribution of the lateral stress at the section  $x/b=0.5$  of the column of different composites. The lateral stress is maximum for the column of Glass / Epoxy, minimum for the column of Graphite / Epoxy, and medium for the column of Boron / Epoxy. Figure 6 illustrates the normalized shear stress component at the section  $x/b=0.5$  of the column of different composites. The shear stress is maximum for the column of Graphite / Epoxy, minimum for the column of Glass / Epoxy, and medium for the column of Boron/Epoxy.

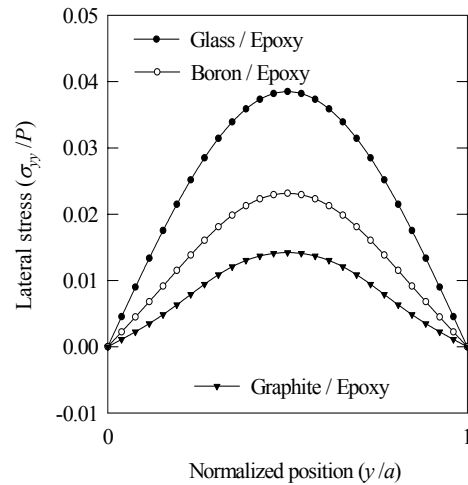


Fig 5. Distribution of normalized lateral stress component ( $\sigma_{yy}/P$ ) at section  $x/b=0.50$  of the column of different composites.

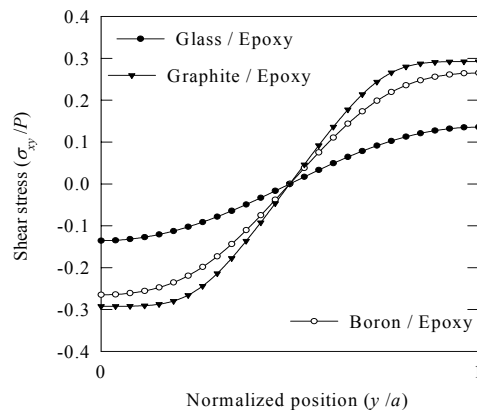


Fig 6. Distribution of normalized shear stress component ( $\sigma_{xy}/P$ ) at section  $x/b=0.50$  of the column of different composites.

## 6. CONCLUSIONS

A new displacement potential approach has been used to analyze the states of stresses and displacements in a

stiffened column of orthotropic composite material with mixed boundary conditions. The distinguishing feature of the present  $\psi$ -formulation over the existing approaches is that all modes of boundary conditions can be satisfied exactly, whether they are specified in terms of loading or physical restraints or any combination of them. The numerical results obtained by using the present approach conform to the physical phenomena of the problem. Thus, it is verified that the present approach is reliable and satisfactory in order to apply it in mixed boundary conditions of elastic problems.

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## 8. NOMENCLATURE

Symbol	Meaning	Unit
$\psi(x,y)$	Displacement potential function	-
$E_1$	Elastic modulus in longitudinal direction	MPa
$E_2$	Elastic modulus in transverse direction	MPa
$G_{12}$	Shear modulus	MPa
$\mu_{12}$	Major Poison's ratio	-
$\mu_{21}$	Minor Poison's ratio	-
$\sigma_{xx}$	Axial stress	kPa
$\sigma_{yy}$	Lateral stress	kPa
$\sigma_{xy}$	Shear stress	kPa
$P$	Axial compressive load	kPa
$I_0, I_m$	Constants	-
$A_m, B_m, C_m, D_m, B$	Constants	-
$u_x$	Axial displacement	m
$u_y$	Lateral displacement	m
$a$	Width of the column	m
$b$	Height of the column	m