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DESIGN PARAMETERS OF A CIRCULAR PROVING RING OF UNIFORM STRENGTH

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ABSTRACT

A computer program based on C has been developed to design a circular proving ring of uniform strength having continuously variable cross-sectional area, with a view to increase its sensitivity through large diametral-deflection. The developed code calculates the thickness at any section of the ring to be designed ensuring that the maximum combined stress in any angular segment just reaches the allowable limit of stress, governed by the elastic limit of the ring material. Consequently, elastic flexural stiffness of the entire ring is fully utilized that fulfills the desired significant increase in diametral deflections. Extensive design data obtained using the code verify the fact that, in terms of sensitivity of the ring the best result is obtained for large values of width and mean radius. In terms of manufacturing cost, significant amount of material can be saved for such a designed ring. Moreover, given all similar conditions, the sensitivity can be increased significantly compared to a conventional proving ring thus eliminating the need of expensive vibrating reed mechanism

Keywords: Proving Ring, Diametral-deflections, Sensitivity, Uniform Strength, Allowable Stress.

1. INTRODUCTION

Proving ring is a force measuring instrument whose shape changes as diameter deflects elastically under load. Applied load is known from its characteristic load-deflection curve. As far as studies on ring structures are concerned, Reid and Bell [1] pointed out the fact that experiments in which metal rings are compressed to large deflections by a pair of opposed concentrated loads reveal a load-deflection characteristic which varies with the simple theory based upon rigid-perfectly plastic behaviour. Thus the influence of strain hardening on the deformation of thin rings subjected to opposed concentrated loads was investigated using a model in an approximate fashion and it is shown how the discrepancies between the experiments and the simple theory arise.

O'Dogherty [2] presented the fundamental formulae for the moment and strain distributions in circular, octagonal and extended octagonal rings. Expressions were also given for calculating ring deflections or stiffnesses. A design equation for determining ring thickness is derived, based on maximum strain criteria for the ring material and on data from measurements of strain gauge bridge sensitivities of six orthogonal ring dynamometers. A procedure was given for the design of extended octagonal rings in terms of geometrical parameters. Design curves were presented for the determination of an appropriate mean ring radius and the calculation of ring thickness. Formulae were also presented for the calculation of the strain gauge bridge sensitivity to the applied orthogonal forces and moment.

The website of National Institute of Standard and Technology [3] gives some note on the design and construction of constant cross section proving rings. It is noteworthy that all studies as listed in the reference deal with rings of constant cross sectional area.

Therefore, the present study is to develop a generalized computer program for a variable cross-section proving ring in order to comprehensively study the effect of maximum design load, mean radius and also width of the cross-section of the thin ring on its sensitivity for any isotropic engineering material.

2. ANALYSIS

The elastic diametral deflection equation is derived from the Castigliano's theorem, considering a circular ring that is loaded causing the diameter to change. Since the structure is symmetrical, one quadrant needs to be considered. The free body diagram is as shown in Fig. 1(a). Here the moment M_0 is statically indeterminate. As there is no rotation at point A during bending of the ring, therefore, $dU/dM_0=0$.
Where θ defines any section of the beam and

$$\theta = \frac{ML}{EI}, \theta = \frac{\partial U}{\partial M}, \text{ and}$$

strain energy $U = \int \frac{M^2}{2EI} dx$

The radial deflection at B is computed from the Castigliano theorem as

$$\delta_B = \frac{\partial U}{\partial P} = \frac{2}{EI}\int_0^{\pi/2} M \frac{\partial M}{\partial P} R\, d\theta \tag{1}$$

For any section, equation for the bending moment can be written as

$$M = M_0 - \left(\frac{P}{2}\right)\left(R - R\cos\theta\right)$$

and, $M_0 = \left(\frac{PR}{2}\right)\times\left(1-\frac{2}{\pi}\right)$

So the equation for the moment at any section becomes,

$$M = \left(\frac{PR}{2}\right)\times\left(\cos\theta - \frac{2}{\pi}\right) \tag{2}$$

For a constant cross section proving ring the net deflection is given by

$$\delta_B = 0.149\frac{PR^3}{EI} \tag{3}$$

For n numbers of variable cross-sections, however, the code uses equation (2) in the following form

$$\delta = \sum \frac{PR^3}{EI}\times\int_{\theta_1}^{\theta_2}(\cos\theta - \frac{2}{\pi})^2 d\theta \tag{4}$$

Taking into account the curvature effect, equations for total stress at inner and outer fibers are given by:

$$S_i = \frac{Mc_i}{Aer_i} + \frac{P\cos\theta}{2A} \tag{5}$$

$$S_0 = -\frac{Mc_0}{Aer_0} + \frac{P\cos\theta}{2A} \tag{6}$$

It should be noted here that while deriving the above equations (2)-(6), the terms like the potential energy due to normal and shear forces are ignored. It is justified because having the variable cross-section most of the angular segments are thin enough compared to the mean radius of the ring.

Since the designed cross-section is rectangular and width b is kept fixed, therefore the following variables are functions of h only.

$$R_n = \frac{h}{\ln(r_o / r_i)}$$

$e = R_n - R$

$c_i = R - (h/2)$

$c_o = R + (h/2)$

$A = (b.\ h)$

As seen the equations (3)-(6) are highly nonlinear in terms of h.

3. PROGRAM FEATURE

The code is able to generate the height of each segment following the width, mean diameter and other design parameters set by the user. It solves the evolved nonlinear equation using Secant method.

The whole proving ring is divided into 360 segments and each segment is further divided into 50 sub-segments. The moment along each sub-segment is determined from equation (2). Then these moments are sorted in ascending order, the maximum value of the moments is set as the design moment for the corresponding segment. After determining the moment of a segment, the bending stress is determined using the curved beam formula, this stress is then combined algebraically with the normal stress to determine the total stress for the segment using the equations (5) and (6).

All the moments and the stresses are functions of height of corresponding segments, that is why an initial guess is required to carry out the whole process stated above. The determining criterion is achieved as the combined stress becomes near about allowable stress limit (for instance 200 MPa for the present study) with a tolerance limit of 0.00001 mm for the height of the segment. Once the height 'h' is selected for all segments the net deflection is calculated using equation (3).

4. RESULTS AND DISCUSSION

Though the developed code is capable of handling any combination of parameters for a thin ring, to demonstrate its usefulness the following limiting values were chosen for designing a 5 tf capacity proving ring to be made from structural steel: a specific width of 30 mm and mean radius of 127 mm, with linear stress-strain assumption a yield strength of 400 MPa, and a Young's modulus of 200 GPa. The safety factor has been kept 1. The code generates the values of h continuously to generate the variable ring profile as shown in Fig. 1(b), and the corresponding net deflections of such rings with variable cross-sections as shown in Table 1. Using the same computer code, it was found that the sensitivity of the ring increases with the number of segments. Hence, for the same set of data (P=5 tf, R=127 mm, b=30 mm) if one quadrant of the ring is divided into 5,10,15,30,45 and 90 segments, the net deflections are, respectively, 0.49 mm, 0.56 mm, 0.60 mm, 0.64 mm, 0.67 mm and 0.6864 mm. From this trend, it is assumed that after 90 segments per quadrant, the deflections do not increase significantly. Moreover, the stress jumps will also be minimum for such a large of segments. Thus, in this study, all calculations are presented for 90 segments per quadrant of the proving rings of variable cross-sections.

It is important to note from Fig. 1(b), that unlike the cantilever beam of uniform strength that has continuous parabolic profile along its span and the minimum cross-section at the tip (Timoshenko, [4]), the proving ring of uniform strength has the minimum cross-section at the point of inflection. Because of large number of segments for the present study, a smooth and continuous contour of the ring profile can be obtained and there is no need for providing fillets at the sharp corners to avoid stress concentrations. For such a continuous profile the net deflection has also increased significantly (almost 2.5

times) in comparison to the conventional rings as seen from Table 1. However, slight modification to this profile is necessary because of fixing the necessary attachments and the dial gage with bolts. Moreover, if needed the ring profile may be made continuous by slightly increasing the calculated thickness at the inflection points to avoid any chance of stress concentrations. The sensitivity of the complete ring with these modifications would still likely to be high but should be checked by rigorous tests. For the ring as shown in Fig. 1(b), the bending moment changes its sign approximately at an angle of 50^0 as seen from Fig. 2. Moreover, the maximum value of height (h) is at $\theta = 90^0$ while its minimum value, near the point of inflection ($\theta = 50^0$), is only 1/8 th of the maximum value. The highest moment is 1982 N-m at $\theta = 90^0$ and is only 22.73 N-m at $\theta = 51^0$. The inner fiber is critically stressed allover the ring except for the angular segment 50^0 to 63^0. Stresses are, however, almost uniformly distributed either in the inner fiber or in the outer fiber, the maximum combined stresses being equal to the allowable stress of 200 MPa on all the segments according to curved beam formula. For a conventional ring having the constant cross-sectional area with h = 47.59mm, the stress would also be maximum at $\theta = 90^0$, but the distribution will not be that uniform, elastic flexural stiffness of most of the angular segments will be underutilized thus resulting in negligibly small diametral deflections as can be seen from Table 1. Although it is not demonstrated here for the sake of brevity, the designed ring (Fig. 1(b)) with variable cross-sections, originally designed for compressive loading, can also be safely used for tensile loading to show high sensitivity.

More results showing the influence of P, R and b on the sensitivity are obtained and presented in Figs. 3 and Table 1. As seen from Table 1, the conventional proving ring without varying cross-section and with a width of 30 mm, maximum allowable compressive load of 5 ton force, a mean radius of 5 inch (127 mm), the total deflection is only 0.27 mm, whereas with this newly designed variable cross section proving ring will give a total deflection of 0.6864 mm (can be seen also from Fig. 3). Since the main objective, that is, highly increased sensitivity has been achieved for the designed ring it does not need expensive vibrating reed mechanism as used by the conventional rings. Furthermore, Fig. 3 shows that the sensitivity of the proving rings increases with increasing width. This is because of the fact that from Table 1, the maximum height of cross-section decreases with increasing width that in turn ensures that the flexural rigidity of each cross-section is fully utilized by increasing stresses to their maximum allowable limits.
For increasing mean radius, sensitivity increases keeping all other parameters constant as seen from Table 1. As for instance, the net deflection is 0.4788 mm for a design load of 5 tf and a mean radius of 4 inch (101.6 mm). If the mean radius is increased to 5 inch (127 mm) the net deflection increases by 44% and if the mean radius is decreased to 3 inch (76.2 mm), the net deflection decreases by 38%.

On the other hand, the effect of change in the maximum designed load on the sensitivity can also be observed from Fig. 3. The net deflection increases at a faster rate for increasing width and decreasing maximum design load for the same mean radius. For example, from Fig. 3, if the designed load is decreased from 7.5 tf to only 2.5 tf, the sensitivity increases by twice for a width of 30 mm. The same parameter (that is sensitivity) increases by more than twice for a width of 75 mm.

The optimum design parameters can be selected for a given load capacity from Figs. 3. Therefore, it can be concluded that for the given maximum design load the best design in terms of high sensitivity can be achieved for increasing width and mean radius. The width is not a big problem but the mean radius should be kept small for space constraint during the application of the ring.

The present design schemes does not include the effects of potential energy terms due to normal and shear forces assuming all proving rings are thin. Practically, the rings' sensitivity may further increase because of those effects. Regarding reliability of the present study involving 360 variable segments, interested readers may refer to Rahman et al. [5] and Rahman et al. [6] where a proving ring of only 24 variable segments was constructed to demonstrate that deflection does increase in comparison to a conventional ring.

5. CONCLUSIONS

A computer program for designing proving rings with large number of variable cross-sections that is compact and ensures maximum elastic deflection with moderate elastic stresses has been developed and its usefulness has been demonstrated. Because of large number of variable cross-sections the profile of the ring has become practically continuous; the stress is almost uniformly distributed thus fully exploiting the strength of the ring material. Consequently, the net deflection has also increased significantly in comparison to a conventional ring. Comprehensive results obtained from the developed code ensure that, given the maximum design load the best design in terms of high sensitivity can be achieved for increasing width and mean radius. Therefore, according to this optimum design scheme the width should be large keeping the mean radius within limit for space constraint during the application of the ring. Proving rings made as per this developed code would be practically inexpensive, as it would require less material for the ring.

6. TABLES AND FIGURES

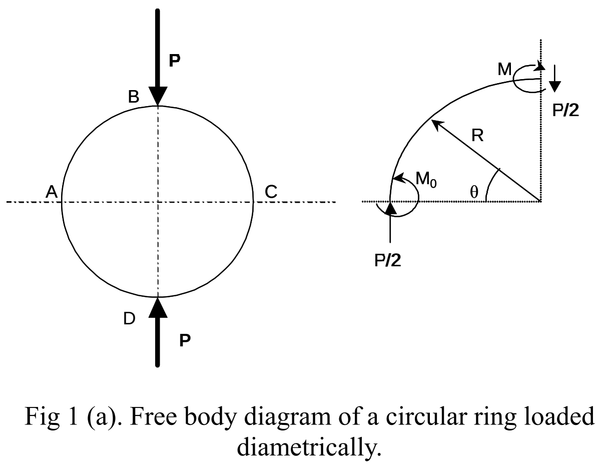

Fig 1 (a). Free body diagram of a circular ring loaded diametrically.

Table 1: Comparison of conventional ring with the present designed ring

Capa city (tf)	*R* (mm)	*b* (mm)	Deflection (mm)	Maximum *h* (mm)	Deflection of convention al ring (mm)*
5	127	30	-0.686425	47.59	-0.2778
5	127	40	-0.817394	40.80	-0.3306
5	127	50	-0.933088	36.25	-0.3771
5	127	60	-1.03787	32.94	-0.4190
5	127	70	-1.13435	30.38	-0.4575
5	127	80	-1.22424	28.34	-0.4934
5	101.6	30	-0.477866	42.96	-0.1934
5	101.6	40	-0.571322	36.77	-0.2312
5	101.6	50	-0.653915	32.64	-0.2645
5	101.6	60	-0.728743	29.63	-0.2946
5	101.6	70	-0.797659	27.32	-0.3222
5	101.6	80	-0.861878	25.47	-0.3478
5	76.2	30	-0.29795	37.74	-0.1203
5	76.2	40	-0.3584	32.22	-0.1450
5	76.2	50	-0.411855	28.55	-0.1667
5	76.2	60	-0.460307	25.89	-0.1863
5	76.2	70	-0.504947	23.85	-0.2042
5	76.2	80	-0.546557	22.22	-0.2209

[* Calculated taking constant height as in the 5th column and using equation (3)]

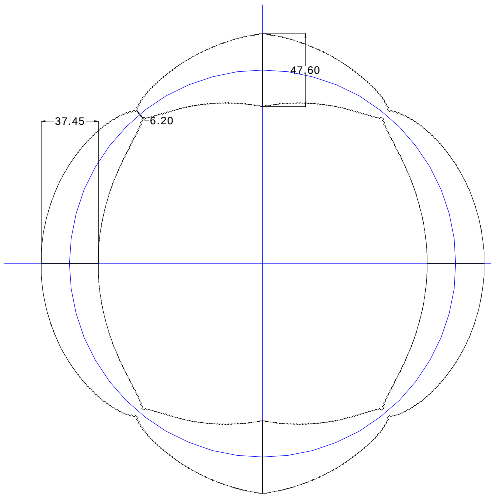

Fig 1(b). Complete designed proving ring with variable profile for P = 5tf, R = 127mm, b = 30mm.

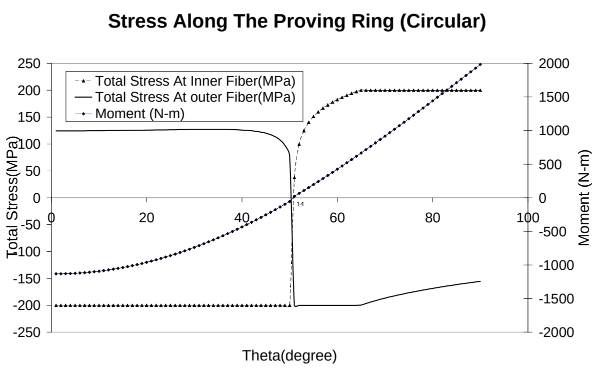

Fig 2. Stresses and moment along the segments of proving ring with variable profile for P=5tf, R=127mm, b=30mm.

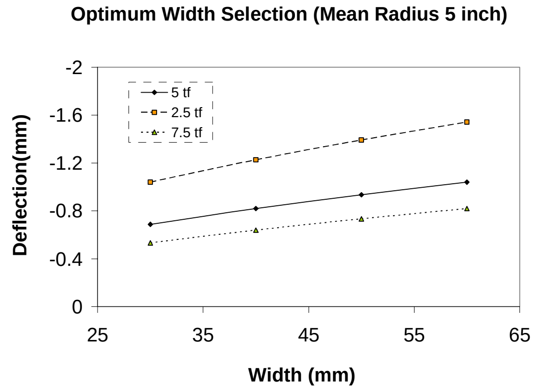

Fig 3. Optimum width selection of the proving ring with mean radius 127 mm.

7. REFERENCES

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8. NOMENCLATURE

Symbol	Meaning	Unit
A	Cross sectional area of the segment	
c_i	Distance from neutral axis to inner fiber.	
b	Width of the ring	
E	Modulus of elasticity	
R	Mean radius	
S_o	Stress at outer fiber according to curved beam theory	
S_{o_total}	Total normal stress at outer fiber.	
δ	Deflection	
c_o	Distance from neutral axis to outer fiber.	

e	Distance from centroidal axis to neutral axis
h	Thickness/height of the segment
I	Area Moment of inertia
P	Load capacity
S_i	Stress at inner fiber according to curved beam theory
S_{i_total}	Total normal stress at inner fiber
θ	Angle of segment