

## EXPERIMENTAL ANALYSIS OF MIXED MODE FRACTURE: THE STRAIN ENERGY DENSITY CONCEPT

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### ABSTRACT

The main objective of the present study is to experimentally analyze the Mix Mode fracture in ductile material. MS sheets are used with a central crack of approximately 30 mm in length positioned at angle of  $\beta$  from  $0^0$  to  $90^0$  in increment of  $10^0$ . The Strain Energy Density concept has been used which marks fundamental departure from the classical and current concepts. All the tests are carried out in specimens containing an inclined crack. The measured values of  $\sigma a^{1/2}$  for different crack sizes and failure loads are plotted against the crack angles  $\beta$ . The agreement of experimental and theoretical results is good. The paper also includes the fracture analysis of an inclined crack under compression. Compressive force is applied to the specimens and critical loads are measured for cracks inclined at various angles with respect to the axis of loading.

**Keywords:** Fracture mechanics, Mixed mode fracture, Strain energy density factor.

### 1. INTRODUCTION

It is well known that the traditional failure criteria can not adequately explain failures which occur at a nominal stress level considering lower than the ultimate strength of the material. The current procedure for predicting the safe loads or safe useful life of a structure member has been evolved around the discipline of linear fracture mechanics. This approach introduces the concept of a crack extension force which can be used to rank materials in the same order of fracture resistance. The idea is to determine the largest crack that a material will tolerate without failure. Fracture mechanics is presently at a standstill until the basic problem of scaling from laboratory models to full size structures and Mixed Mode crack propagation are resolved. The current theory of fracture is inadequate for many reasons. First of all it can only treat idealized problems where the applied loads must be directed normal to the current plane. In this case, value of the Mode-I critical stress intensity factor,  $k_{Ic}$  or crack extension force  $G_{Ic}$  can be used to predict the applied loads that cause failure or conversely to find the allowable crack size that will not cause premature failure. However in a structural member the crack is seldom in a plane normal to the principle stress. As a rule, the crack follows a curved path. For two dimensional problems, the influence of Mode-II crack propagation is not always negligible. Complications arise here because the crack does not spread in a self similar manner and the classical treatment of the Strain Energy release rate concept brings

down. G, Chen., S, Rahman., Y, H. Park [1] present a new method for continuum based shape sensitivity analysis for a crack in a homogeneous, isotropic and linear elastic body subjected to mixed mode loading conditions. The method was based on the material derivative concept of continuum mechanics, domain integral representation of an interaction integral and direct differentiation. B, N. Rao., S, Rahman [2] present two methods for conducting a continuum shape sensitivity analysis of a crack. Unlike virtual crack extension techniques, no mesh perturbation is needed to calculate the sensitivity of stress intensity factors. B, N. Rao., S, Rahman [3] present a Finite Element Method, where the material properties are smooth functions of spatial coordinates and two newly developed interaction integrals for mixed mode fracture analysis. K, N. Shivakumar., M, Naiva., N, Adeyemi., V, S. Avva [4] have done their work on Evaluation of Modified Mixed Mode Fracture Test Apparatus. Overall objective of their research is to characterize Mode I, Mode II and Mixed Mode delamination fracture toughness of fabric composite manufactured through compression, autoclave and resin transfer molding processes under pure mode and mixed mode loading conditions. K, Sato., T, Hashida [5] worked on Development of Numerical Simulation Code for Hydraulic Fracturing Using Embedded Crack Element. The purpose of their study is the development of the numerical simulation code that can predict mixed mode crack propagation with a fracture process zone during

hydraulic fracturing in a deep rock mass. A finite element analysis with an embedded crack element was adapted to the simulation code.

## 2. THE MECHANICS OF FRACTURE

Fracture is an inhomogeneous process of deformation that causes regions of material to separate and load carrying capacity to decrease to zero. It can be viewed on many levels, depending on size of the fractured region that is of interest. At the atomistic level fracture occurs over regions whose dimensions are of the order of atomic spacing ( $10^{-8}$  in.); at the microscopic level fracture occurs over regions whose dimensions are of the order of the grain size (about  $5 \times 10^{-4}$  in.); and at the macroscopic level fracture occurs over dimensions that are of the order of the size of flaws or notches ( $10^{-1}$  in. or greater). At each level there are one or more criteria that describe the conditions under which fracture can occur. For example, at the atomistic level fracture occurs when bonds between atoms are broken across a fracture plane and new crack surface is created. This can occur by breaking bonds perpendicular to the fracture plane, (Fig 1), a process called cleavage, or by shearing bonds across the fracture plane, (fig 2), a process called shear.

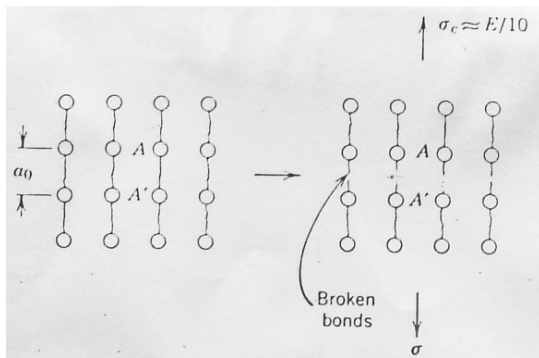


Fig 1. Cleavage

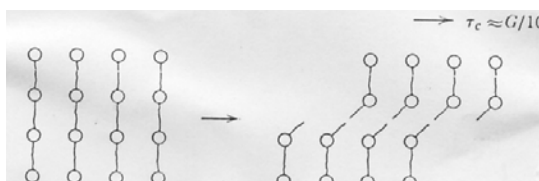


Fig 2. Shear

At this level the fracture criteria is simple; fracture occurs when the local stresses build up either to the theoretical cohesive strength  $\sigma_c \approx E/10$  or to the theoretical shear strength  $\tau_c \approx G/10$ , where  $E$  and  $G$  are the respective elastic and shear moduli.

## 3. STRAIN ENERGY DENSITY CONCEPT

Sih has proposed a theory of fracture based on the field strength of the local Strain Energy Density which marks a fundamental departure from the classical and current concepts. The theory requires no calculations on the energy release rate and thus possesses the inherent advantage of being able to treat all the Mixed Mode crack extension problems for the first time. Unlike the

conventional theory of  $G$  and  $k$ , which measures only the amplitude of the local stresses, the fundamental parameter in the new theory, the “Strain Energy Density factor”,  $S$ , is also direction sensitive. The difference between  $k$  or  $G$  and  $S$  is analogous to the difference between a scalar and a vector.

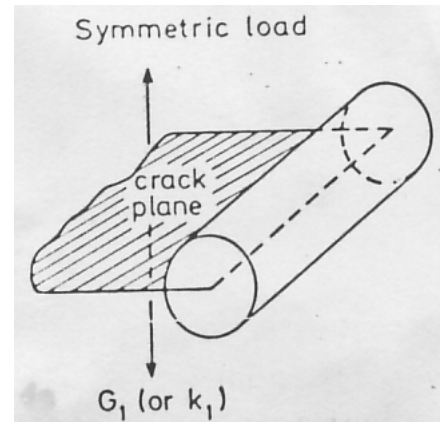


Fig 3. Scalar Theory

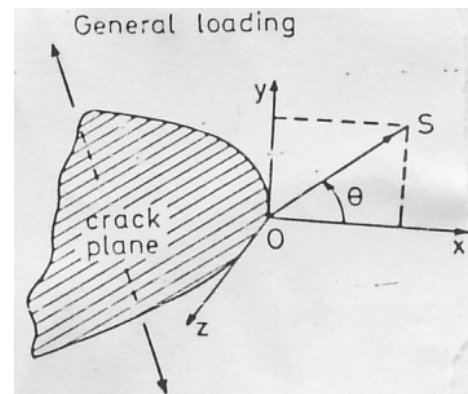


Fig 4. Director Theory

Referring to fig 3 and fig 4, the Griffith-Irwin theory can be viewed as a scalar theory in that it specifies only the critical value of a scalar  $G_{1c}$  or  $k_{1c}$  at incipient fracture. The direction of crack propagation is always presumed to be normal to the load. Moreover, the crack front must be straight such that  $G$  or  $k$  does not vary along the leading edge of the crack. In addition scalar theory can not yield the correct material parameter if two or more intensity factors are present along the crack border. The  $S$ -factor in the Sih theory behaves like a director. It senses the direction of least resistance by attaining a stationary value with respect to the angle  $\theta$ , as indicated in fig 4. As it will be shown, the stationary value of  $S_{min}$  can be used as an intrinsic material parameter whose value at the point of crack instability  $S_{cr}$  is independent of the crack geometry and loading.

### 3.1 Form of Local Energy Density

Consider the three dimensional case of a crack in a combined stress field and focus attention on a co-ordinate system  $(x, y, z)$  in figure 5, with the  $x$ -axis normal to the crack,  $y$ -axis perpendicular to the crack and  $z$ -axis tangent to the crack border. The stress values are

varied according to the following equations

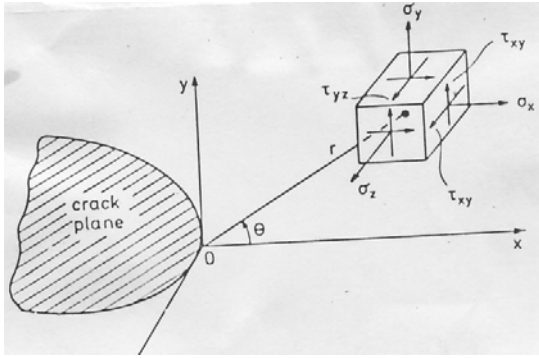


Fig 5. Combined stress field

$$\begin{aligned}
 \sigma_x &= \frac{k_1}{(2r)^{1/2}} \cos \frac{\theta}{2} \left[ 1 - \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2} \right] \\
 &\quad - \frac{k_2}{(2r)^{1/2}} \sin \frac{\theta}{2} \left[ 2 + \cos \frac{\theta}{2} \cdot \cos \frac{3\theta}{2} \right] + \dots \\
 \sigma_y &= \frac{k_1}{(2r)^{1/2}} \cos \frac{\theta}{2} \left[ 1 + \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2} \right] \\
 &\quad + \frac{k_2}{(2r)^{1/2}} \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \cdot \cos \frac{3\theta}{2} \cdot + \dots \\
 \sigma_z &= \frac{k_1}{(2r)^{1/2}} \cos \frac{\theta}{2} - 2 \cdot \vartheta \cdot \frac{k_2}{(2r)^{1/2}} \sin \frac{\theta}{2} \\
 \tau_{xy} &= \frac{k_1}{(2r)^{1/2}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \\
 &\quad + \frac{k_2}{(2r)^{1/2}} \cos \frac{\theta}{2} \left[ 1 - \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2} \right] + \dots \\
 \tau_{xz} &= \frac{-k_3}{(2r)^{1/2}} \sin \frac{\theta}{2} + \dots \\
 \tau_{yz} &= \frac{k_3}{(2r)^{1/2}} \cos \frac{\theta}{2} + \dots
 \end{aligned} \tag{1}$$

Where the non singularity terms have been dropped and  $r$ ,  $\theta$  are the polar components in the  $yz$  plane. For an elastic material the strain energy stored in the element  $dv = dx \cdot dy \cdot dz$  under a general three dimensional stress system is

$$\begin{aligned}
 dw &= \left[ \frac{1}{2E} (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) \right. \\
 &\quad \left. - \frac{\vartheta}{E} (\sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x) \right] dv \\
 &\quad + \frac{1}{2\mu} [\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2] dv
 \end{aligned} \tag{2}$$

Substituting equation (1) in to equation (2)

$$\frac{dw}{dv} = \frac{1}{r} [a_{11} \cdot k_1^2 + 2a_{22} k_1 k_2 + a_{22} \cdot k_2^2 + a_{33} \cdot k_3^2] + \dots \tag{3}$$

higher order terms in  $r$  have been neglected and that the strain energy density function near the crack poses a  $1/r$  energy singularity. Therefore

$$S = [a_{11} \cdot k_1^2 + 2a_{22} k_1 k_2 + a_{22} \cdot k_2^2 + a_{33} \cdot k_3^2] \tag{4}$$

represents the amplitude or the intensity of the strain energy density field and it varies with polar angle  $\theta$ . Where

$$\begin{aligned}
 a_{11} &= \frac{1}{16\mu} [(3 - 4\nu - \cos \theta)(1 + \cos \theta)] \\
 a_{12} &= \frac{1}{16\mu} \cdot 2 \sin \theta [\cos \theta - (1 - 2\nu)] \\
 a_{22} &= \frac{1}{16\mu} [4(1 - \nu)(1 - \cos \theta)(1 + \cos \theta)(3 \cos \theta - 1)] \\
 a_{33} &= \frac{1}{4\mu}
 \end{aligned} \tag{5}$$

Equation (3) represents the general form of the crack border stress field involving the three intensity factors  $k_1$ ,  $k_2$ ,  $k_3$ . For 2-D problems where the crack extends in  $xy$  plane only, the stress intensity factor does not vary along the crack front and  $S$  depends only on one variable, namely the angle  $\theta$ .

#### 4. EXPERIMENTAL ANALYSIS

The first study of the initial direction of crack growth in the presence of both  $k_1$  and  $k_2$  for the problem of a crack of length  $2a$  inclined at an angle  $\beta$  with the loading axis as shown in figure 7. The stress intensity factors for  $k_1$  and  $k_2$  are

$$\begin{aligned}
 k_1 &= \sigma \cdot a^{1/2} \cdot \sin^2 \beta \\
 k_2 &= \sigma \cdot a^{1/2} \cdot \sin \beta \cdot \cos \beta
 \end{aligned} \tag{6}$$

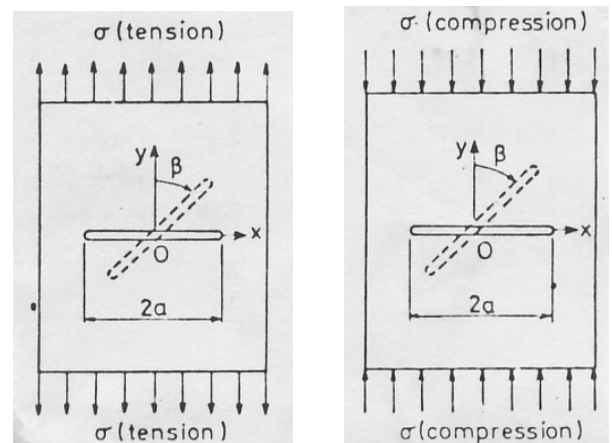


Fig 6 and 7. Tensile and compressive loading

Strain energy density theory states that crack propagates in the direction of

$$S = \sigma^2 \cdot a \left[ \begin{array}{l} a_{11} \sin^2 \beta + 2a_{12} \sin \beta \cdot \cos \beta \\ + a_{22} \cos^2 \beta \end{array} \right] \sin^2 \beta \quad (7)$$

being a minimum.

Differentiating equation (7) with respect to  $\theta$  and setting the result to zero, the fracture angle  $\theta_0$  for a given position of the crack specified by  $\beta$  can be calculated from

$$2(1-2\nu)\sin(\theta_0 - 2\beta) - 2\sin[2(\theta_0 - \beta)] - \sin 2\theta_0 = 0 \quad (8)$$

The validity of the prediction can be checked with the results of a series of experiments performed on the specimens on Universal Testing Machine (UTM). MS plates of approximately  $100 \times 50 \times 2 \text{ mm}^3$  were used with a central crack of approximately 30 mm in length positioned at angles of  $\beta$  from  $30^\circ$  to  $90^\circ$  in increment of  $10^\circ$ . The initial fracture angles for both ends of the crack were measured.

## 5. RESULTS

One of the principal aims of the fracture mechanics is to characterize the behavior of material in the presence of flows or cracks. In the vicinity of the crack tip  $S$  may be regarded as a crack resistance force with the interpretation of the crack tends to run in the direction of least resistance that corresponds to  $S$  reaching a minimum. Once  $S$  has attained a critical value of  $S_{cr}$  at the point of incipient fracture it may be regarded as a crack extension force which should be independent of loading condition and crack configuration.  $S_{cr}$  can be used as material constant that serves as an indication of the fracture toughness of the material.

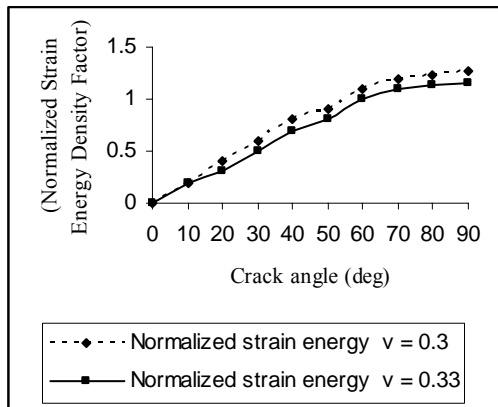


Fig 8. Variation of Density factor with crack angle for tensile loading

Fig (8) shows a plot of Normalized density factor against  $\beta$  for various Poisson's ratio. In general the factor increases with the crack angle  $\beta$  reaching a maximum along the axis of Mode-I crack extension. As  $S_{min}$  will be used as a material constant, the above statement implies that the lowest value of the applied stress to initiate the crack propagation occurs at  $\beta = 90^\circ$ .

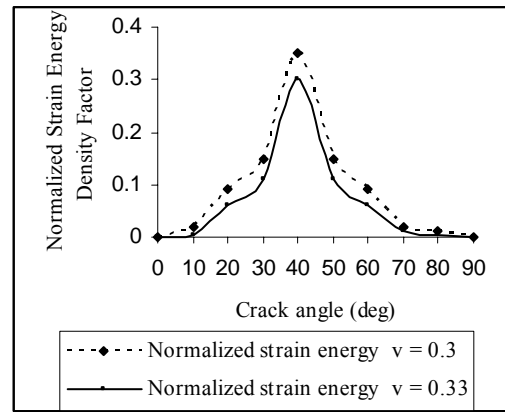


Fig 9. Variation of Density factor with crack angle for compressive loading

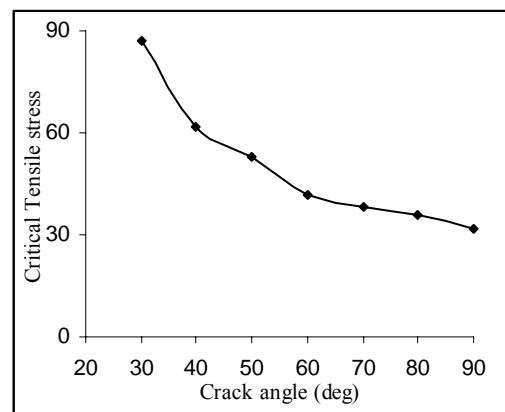


Fig 10. Critical Tensile stress versus crack angle

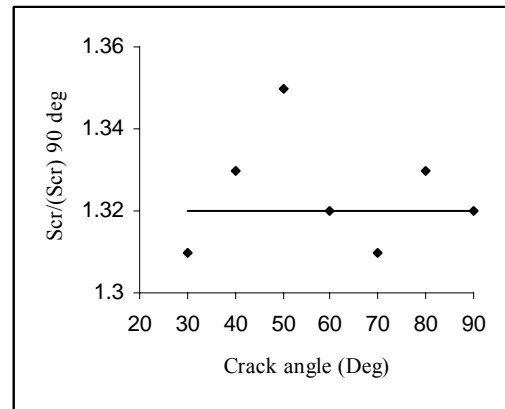


Fig 11. Critical Density factor as a material constant

A similar graph for uni-axial compression is shown in fig (9). An interesting point to be observed here is that the factor increases first with the increase of crack angle  $\beta$ , reaching a peak and then decreases in magnitude. The peak value is a function of the Poisson's ratio. This suggests that for density factor = constant, there exists a critical angle  $\beta_0$  at which the critical applied compressive stress is a minimum.

All the tests were carried out in specimen containing an inclined crack. The measured values of  $\sigma_{cr} \cdot a^{1/2}$  for different crack size and failure loads are plotted against

the crack angle  $\beta$  in fig (10). The same data is given in fig (11) with the critical stress density factor  $S_{cr}$  normalized with respect to its value  $(S_{cr})_{\pi/2}$  corresponding to Mode-I crack extension and  $S_{cr}$  remains constant.

## 6. CONCLUSIONS

The stationary value of the density factor can predict the direction of growth under mixed mode conditions. The critical value of  $S_{cr}$  has been shown to be independent of the crack geometry and loading hence it can be used as a material parameter for measuring the resistance against fracture. The future progress will depend largely on the willingness of the practitioner in the field to accept this new concept. The negligence of the mixed mode effect in design can lead to drastic errors on the prediction of the applied stress to cause the fracture.

## 7. REFERENCES

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## 8. NOMENCLATURE

Symbol	Meaning	Unit
$\sigma_{cr}$	Critical Stress	N/mm <sup>2</sup>
a	Crack length	mm
E	Modulus of Elasticity	N/mm <sup>2</sup>
$\mu$	Shear Modulus of Elasticity	N/mm <sup>2</sup>
$\beta$	Crack angle	Deg
$\theta$	Fracture angle	Deg
$\nu$	Poisson's ratio	
$k_1, k_2, k_3$	Stress intensity factors	
S	Strain energy density factor	
$16\mu S_{min} / \sigma^2 a$	Normalized strain energy density factor	