

EFFECT OF CRACK LENGTH ON THE STRESS INTENSITY FACTOR OF A RADIAL EDGE CRACK IN AN INFINITE PLATE

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ABSTRACT

In the present work the stress intensity factor of radial crack emanating from the edge of a circular hole in an infinite plate subjected to combined normal and shear loading is investigated numerically. Usually the stress intensity factor is determined by determining the shape factor, which depends on the crack geometry and the loading condition. The determination of shape factors require extensive numerical computation when large number of loading condition or crack geometry is considered. In this work stress intensity factor has been determined by calculating the shape factor in a more general form. The problem is considered as plane strain problem and accordingly the equations of elasticity are applied to this problem. After applying the boundary conditions and carrying out some manipulations, a Fredholm integral equation is obtained. Stress intensity factor is related to the Fredholm integral equation and thus it is obtained numerically. A computer program is written to obtain the numerical results. Stress intensity factor is calculated for different values of crack length and for different loading condition. It is found that Mode-I stress intensity factor first increases and then decreases with increasing the crack length and Mode-II stress intensity factor always increases as the crack length is increased. The stress intensity factors also vary with the changing of loading condition. For a specific crack length, the amplitude of Mode-I and Mode-II stress intensity factors depend on the ratio of normal to shear stress which is applied at the boundary.

Keywords: Stress intensity factor, Fredholm integral equation, Shape factor, Mode-I and Mode-II loading

1. INTRODUCTION

Stress intensity, K , as the name implies is a parameter that amplifies the magnitude of the applied stress and depends on the geometry and also on loading condition. These load types are categorized as Mode-I, Mode-II and Mode-III loading. Generally there are three modes to describe different crack surface displacement as in Fig. 1. Mode-I is opening or tensile mode where the crack surfaces move directly apart. Mode-II is sliding or in plane shear mode where the crack surfaces slide over one another in a direction perpendicular to the loading edge of the crack. Mode III is tearing and anti plane shear mode where the crack surfaces move relative to one another and parallel to the loading edge of the crack.

Analysis of stress intensity factor at geometric notches or holes has not been investigated to a great extent because of complexity of calculation and wide variety of parameters involved. The stress intensity factor for cracks at the edge of a notch can be expressed as [1]

$$K = F\sigma\sqrt{\pi l} \quad (1)$$

where l is the crack length, σ is the applied stress at the loading edge and F is called geometry factor. In

absence of an analytical solution of F , its determination requires extensive numerical computation for each particular notch profile, crack length and loading condition. Such a computational approach is not very efficient for problems in which large numbers of crack configurations and or loading conditions are to be considered. This has been demonstrated in Westgaard approach [2] to calculate the stress intensity factor for specific geometric configurations. The motivation of this work comes from the aim to establish a relatively simple method for calculating the stress intensity factors for cracks emanating from circular holes in an infinite elastic solid under arbitrary loading.

Broek [3] first suggested a simple engineering solution for estimating the stress intensity factors for cracks emanating from notches. His idea was to consider the crack length as including the notch depth. Smith and Miller [4] proposed a simple formula for the stress intensity factor of small cracks at the root of a circular notch of finite depth. But it was found by Kotousov [5] that these methods can lead to significant error when different crack configuration and loading conditions need to be considered.

Another approximation was suggested by Lukas in [6]. He suggested the method for calculating the stress

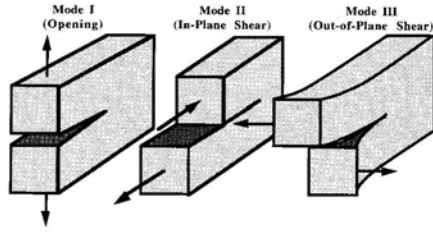


Fig 1. Modes of loading in fracture mechanics.

intensity factor for small cracks emanating from notches. Due to the origin of this approach, it is expected to provide good estimates only for the case where stress concentration factor is less than 3. Karlsson and Backlund in [7] used an analytical method to estimate K_I for small cracks. The method was based on the solution of an edge crack in a semi infinite solid with a linear distribution of the tensile stress on the crack edges.

Another general method for calculating the stress intensity factors is the weight function method [8]. The weight function have been obtained for a wide range of geometries and loading conditions in particular for normal loading. However, the case of shear loading has not been considered.

In general, the approximate approaches considered above work well if applied appropriately. However the application of these approaches to different crack configurations can lead to significant errors.

2. THEORETICAL EQUATION

The problem is approached by considering that the crack is subjected to the normal and shear loading conditions separately. Equations of elasticity for these cases are then applied. The problem is solved under the conditions of plane strain and the crack and the hole are defined in polar coordinates (r, θ) by the relations $1 \leq r \leq b$, $\theta = 0$ and $0 \leq r \leq 1$, $0 \leq \theta \leq 2\pi$ respectively (Fig. 2). As the loading is symmetric about the plane of the crack the problem can be reduced to that of finding a solution of the equations of elasticity for the region $1 < r_r < \infty$, $0 < \theta < \pi$.

After finding the solutions of the equations of elasticity [9] and after applying the boundary conditions, the following Fredholm integral equation is obtained [9].

$$P(t) - \int_1^b \frac{P(\rho)M(t, \rho)}{[(\rho-1)(Rb-2)]^{1/2}} d\rho = s(t) \quad (2)$$

where,

$$s(t) = \frac{t-1}{\pi} \int_1^b \left(\frac{Rb-y}{y-1} \right)^{1/2} \frac{yf(y)dy}{y-t} \quad (3)$$

$$M(t, \rho) = \frac{t-1}{\pi^2 t} \int_1^b \left(\frac{Rb-y}{y-1} \right)^{1/2} \frac{yK(y, \rho)dy}{y-t} \quad (4)$$

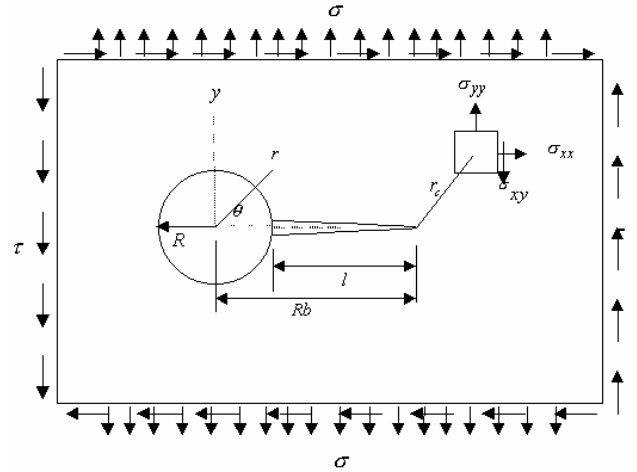


Fig 2. The crack geometry and loading condition.

$$k(r,t) = \frac{1(1-t^2)^2}{t(1-rt)^3} - \frac{t(1-t^2)}{(1-rt)^2} - \frac{1}{(1-rt)} + \frac{1-t^2}{r^2 t} - \frac{1}{r} \quad (5)$$

2.1 The Stress Intensity Factor

From the solutions of the equations of elasticity for the problem of a crack emanating from the edge of a circular hole under combined normal and shear loading, stresses and displacement fields around the crack tip can be expressed in terms of $P(t)$ and $q(t)$, which are the force parameters at crack faces. The stress intensity factor K_I and K_{II} can be expressed in terms of the displacement function. The stress intensity factor for both Mode-I and Mode-II is usually defined by the following expression [9]-

$$K_I = - \lim_{r \rightarrow Rb} \sqrt{2(Rb-r)} \frac{E}{2(1-\nu^2)} \frac{\partial u_\theta(r,0)}{\partial r} \quad (6)$$

$$K_{II} = \lim_{r \rightarrow Rb} \sqrt{2(Rb-r)} \frac{E}{2(1-\nu^2)} \frac{\partial u_r(r,0)}{\partial r} \quad (7)$$

Where the terms u_r , u_θ are displacement functions in r and θ directions and those can be expressed in terms of an integral equation of $P(t)$. After some manipulation, it can be expressed as [9]

$$u_r(r,0) \text{ or } u_\theta = \frac{2(1-\nu^2)}{E} \int_{r_r}^b \frac{P(t)}{\sqrt{(t-1)(b-1)}} dt; \quad 1 \leq r_r \leq b \quad (8)$$

Where $P(t)$ can be determined by solving the Fredholm equation (2). Accordingly the stress intensity factor assumes the following form,

$$K_{I,II} = \frac{-\sqrt{2}}{\sqrt{Rb-1}} P(b) \quad (9)$$

This is the generalized equation of stress intensity factor for a radial crack emanating from the edge of a circular hole under combined normal and shear loading.

Now the parameter $P(Rb)$ can be written in non dimensional form as $q(b)$,

$$q(b) = \frac{P(b)}{R\sigma} \text{ for normal loading and } q(b) = \frac{P(b)}{R\tau} \text{ for shear loading}$$

3. NUMERICAL FORMULATION

By using the Gauss- Chebyshev quadrature formula and using the conception of non dimensional form of $P(Rb)$ as $q(b)$, it is possible to replace the Fredholm integral equation (2) by the simultaneous linear equations as below

$$q(t_k) - \frac{\pi}{n} \sum_{m=1}^n q(t_m) M(t_k, t_m) = s(t_k) \quad (10)$$

where

$$t_k = \frac{b+1}{2} + \frac{b-1}{2} \cos \left[\frac{(2k-1)\pi}{2n} \right] \quad (11)$$

$$k = 1, 2, 3, \dots, n \quad [n = \text{no. of integration points}]$$

Using the non dimensional form of $P(b)$, the stress intensity factors can be re-written as

$$K_I = - \frac{\sqrt{2}\sigma R}{(Rb-1)^{1/2}} q(b) \text{ and } K_{II} = - \frac{\sqrt{2}\tau R}{(Rb-1)^{1/2}} q(b)$$

Where,

$$q(b) = \frac{\pi}{n} \sum_{m=1}^n q(t_m) M(b, t_m) = s(b). \quad (12)$$

So finally the following equations of stress intensity factors can be written

$$\frac{K_I}{\sigma\sqrt{R}} = - \frac{\sqrt{2}R}{(Rb-1)^{1/2}} \left\{ s(b) + \frac{\pi}{n} \sum_{m=1}^n q(t_m) M(b, t_m) \right\} \quad (13)$$

$$\frac{K_{II}}{\tau\sqrt{R}} = - \frac{\sqrt{2}R}{(Rb-1)^{1/2}} \left\{ s(b) + \frac{\pi}{n} \sum_{m=1}^n q(t_m) M(b, t_m) \right\} \quad (14)$$

4. RESULTS AND DISCUSSION

Figure 3 shows the effect of normalized crack length on the normalized stress intensity factor for normal to shear loading ratio, $S=2$ at the boundary of the plate. The figure shows that with the increase of normalized crack length b , stress intensity factor increases initially. When the value of b is about 1.25, the stress intensity factor reaches its maximum value of about 1.1. After this value

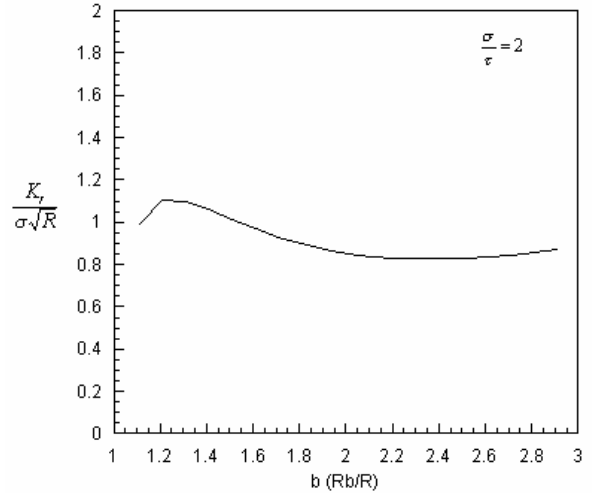


Fig 3. Effect of normalized crack length on the mode-I stress intensity factor.

of b , $\left(\frac{K_I}{\sigma\sqrt{R}} \right)$ starts decreasing and at large values of b ,

it finally achieves a value of 0.82. So under the above loading condition, it can be said that Mode-I stress intensity factor increases with increasing the crack length up to the value where the crack length is approximately 25% of the hole radius. Figure 4 shows that mode-II stress intensity factor for the above loading condition increases continuously as normalized crack length or b factor increases. The rate at which K_{II} increases is higher for the lower values of b . Unlike K_I , K_{II} always increases with the crack length and achieves maximum value at the maximum value of the crack length. Furthermore, this figure shows that until b equals to about 1.8, the rate of increasing the stress intensity factor is higher.

Figure 5 shows the variation of the mode-I stress intensity factor for different values of boundary stress ratio, S and for different values of crack length. As the normal to shear stress ratio at boundary is increased from 0.1 to 2.35, overall magnitude of K_I also increases. Moreover, for very smaller values of the stress ratio, K_I does not significantly vary with b . But at higher values of the boundary stress ratio, K_I has significant variation with b which is true in the lower range of b . But for all the cases K_I assumes the maximum value at about $b=1.25$. The effect of boundary stress ratio on normalized

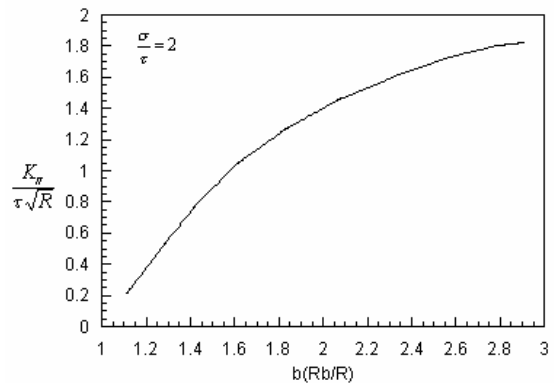


Fig 4. Effect of normalized crack length on mode-II stress intensity factor.

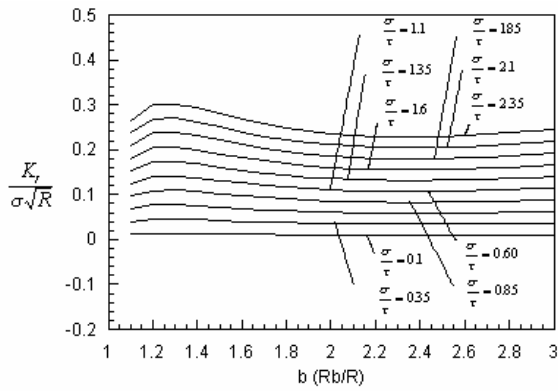


Fig 5. Effect of normal to shear stress ratio at boundary on the mode-I stress intensity factor.

mode-II stress intensity factor can be observed from Fig. 6. K_{II} increases as the boundary stress ratio is decreased. There is small variation between the values of K_{II} for different values of boundary stress ratio in the lower range of the crack length. The variation of K_{II} for different values of stress ratio becomes greater at higher values of crack length. Again, for higher values of stress ratio or in other words, at lower values of boundary shear stress, stress intensity factor changes very significantly with the increase of the factor b . But for lower values of the shear stress, variation of K_{II} is comparatively less. For b approximately equals to 1.2, K_{II} values are 0.1 and 0.2 for stress ratios 2.35 and 0.1 respectively. But for b equals to 3, K_{II} values are 0.9 and 4.8 respectively. So, large variation in K_{II} is present here.

In Fig. 7 the effect of factor b on (K_I / K_{II}) has been plotted for the boundary stress ratio 0.25. It is seen from the figure that the stress intensity factor ratio decreases as b increases. Up to b about 1.2, K_I is greater than K_{II} , and after $b=1.2$, K_{II} dominates. Again after $b=2$, the ratio (K_I / K_{II}) assumes a constant value. So for higher values of b , there is no effect of b on the stress intensity factor ratio for a particular loading condition. Fig. 8 shows the effect of boundary stress ratio on (K_I / K_{II}) for different b . It is clear from this figure that as the load ratio increases, (K_I / K_{II}) ratio also increases. Variation of (K_I / K_{II}) in lower range of S is greater than the variation in its upper range..

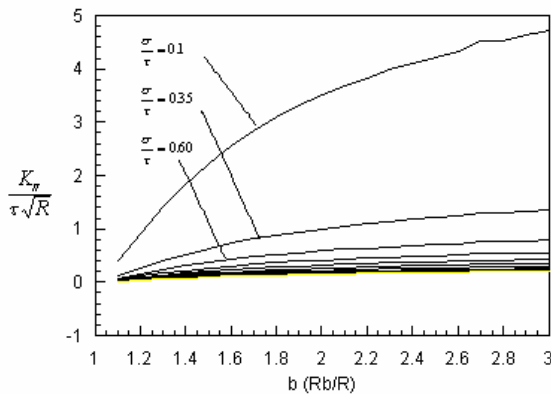


Fig 6. Effect of normal to shear stress ratio at boundary on the mode-II stress intensity factor.

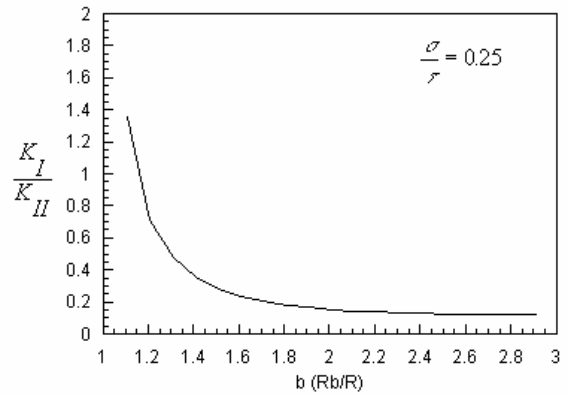


Fig 7(a). Effect of normalized crack length on the stress intensity factors ratio (K_I / K_{II}) for the boundary stress ratio of 0.25.

For example, if we change the load ratio from 0.25 to 0.75, (K_I / K_{II}) increases by about 50%. But in the upper range, if we change the load ratio from 1.75 to 2.25, (K_I / K_{II}) increases by only 12%. So from the point of view of (K_I / K_{II}) variation, a plate containing very small crack is more sensitive when it is mainly loaded with shear stress (load ratio in that case is smaller).

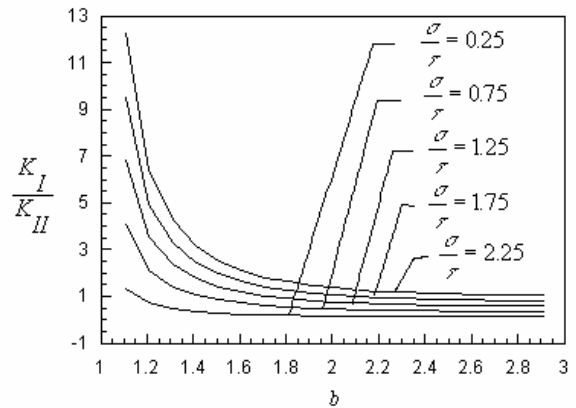


Fig 7(b). Effect of normalized crack length on the stress intensity factors ratio (K_I / K_{II}) for different boundary stress ratio.

5. CONCLUSIONS

The effect of various crack length on the stress intensity factor for a radial crack emanating from the edge of a circular hole under combined normal and shear loading condition is investigated in the present study. The numerical results of the present investigation are already shown. The following points can be noted from this study:

- i) The mode-I stress intensity factor K_I increases with the increase of crack length up to a value where $\frac{l}{R}$ is approximately equal to 1.25. At this value crack length is 25% of the hole radius.
- ii) Mode-II stress intensity factor K_{II} increases continually with the crack length.
- iii) With the increase of load ratio $\frac{\sigma}{\tau}$, K_I and K_{II} also

vary. Value of K_I increases with the increase of $\frac{\sigma}{\tau}$, while the value of K_{II} decreases. Again the change of K_I with crack length is sharp for higher values of $\frac{\sigma}{\tau}$.

But this tendency is reverse for K_{II} .

- iv) The sensitivity of the ratio K_I / K_{II} to the change of load ratio is higher when a small crack is present in the body. A small change in load ratio can cause a large change of K_I / K_{II} ratio.

6. REFERENCES

1. T. L. Anderson, *Fracture Mechanics*, Second edition, CRC press, New York, pp. 50-61.
2. Westgaard, H. M., 1939, " Bearing pressure and cracks". *Journal of Applied Mechanics*, Vol.6, pp. 49-53
3. Broek, D., 1927, "The propagation of fatigue cracks emanating from holes", National Aerospace laboratory, Report NLR TR 72134 U, Amsterdam.
4. Smith, R. A., Miller K., J., 1977, "Fatigue cracks at notches", *International Journal of Mechanical Science*, Vol.19, pp.11-22.
5. Kotousov, A. and Jones, R., 2001, "The stress intensity factor of a radial crack at the edge of a circular hole under shear loading", *Journal of Theoretical and Applied Fracture Mechanics*, in print.
6. Lucas, P., 1987, "Stress intensity factors of small cracks at notches", *Engineering Fracture Mechanics*, Vol. 26, pp.471-473.
7. Karlsson, A. and Backlund, J. 1978, "Summary of SIF graphs for cracks emanating from circular holes", *International Journal of Fracture*, Vol.14, pp. 585-596.
8. Fett. T. and Munz, D., 1997. "Stress intensity factors

and weight functions", Book craft Ltd.

9. Motaleb, A. M., 2005, "Study of stress intensity factor of radial crack developed from the edge of a circular hole under combined normal and shear loading"., M. Sc. Dissertation, Mech. Engg. Dept., BUET, Dhaka.

7. NOMENCLATURE

Symbol	Meaning	Unit
R	radius of the hole	(m)
Rb	distance between crack tip and hole center	(m)
<i>l</i>	crack length	(m)
b	non dimensional crack length (Rb/R)	
r	distance from the hole center in radial direction	(m)
<i>r_r</i>	Normalized dimension (r/R)	
θ	angle from crack plane	(Rad)
K_I	mode-I stress intensity factor	$MPa\sqrt{m}$
K_{II}	mode-II stress intensity factor	$MPa\sqrt{m}$
σ	normal stress component at boundary	(N/m ²)
τ	shear stress component at boundary	(N/m ²)
S	normal to shear stress ratio at boundary	(N/m ² /N/m ²)
ρ	radius of curvature at notch tip	(m)
P(t)	a stress function	(N/m)
q(t)	normalized stress function	
s(t)	normalized stress function	
ν	Poission's ratio	