Proceedings of the
International Conference on Mechanical Engineering 2005
(ICME2005) 28- 30 December 2005, Dhaka, Bangladesh

ICME05-FL-13

DEPTH OF INVESTIGATION AROUND A CIRCULAR WELLBORE IN AN ELLIPTICAL DOMAIN

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ABSTRACT
This study presents an expression of depth of investigation (DOI) around a circular wellbore in an elliptical flow domain. A steady state elliptical flow equation around a circular wellbore is considered as the basis of the derivation. With the help of continuous succession of steady state method, the steady state equation is applied in unsteady state situation to find the DOI. The DOI is expressed as a function of the eccentricity of the elliptical domain and compared with the radius of investigation of radial system considering the value of eccentricity equal to zero. Results are presented in graphical forms.

Keywords: Depth of investigation, Elliptical domain, Porous media.

1. INTRODUCTION
For radial flow problems, the concept of "radius of investigation" has been established and widely used in well test design and analysis. When a change in production rate of a well of a reservoir takes place, it can be considered as a disturbance or a pulse and it will propagate throughout the reservoir to be adjusted with time. The distance over which the pressure transient has moved into a formation following a rate change in a well can be defined as the radius of investigation.

For radial flow systems, as the disturbance propagates radially so the investigated area is circular. A circle should have a radius and so "radius of investigation" concept is applicable for radial system. In case of linear or elliptical flow problems, the scenario is different. For linear flow cases investigated area is rectangular and for elliptical cases it is elliptical. So the term "radius of investigation" is not suitable for the latter systems. It is better to use the term "depth of investigation" instead of "radius of investigation" for these cases.

The study of elliptical flow is more recent than the study of radial or linear flow. Probably the earliest discussion on elliptical flow is attributed to Muaskat[1] in 1937. He presented a steady state elliptical flow equation from a finite-length line source into an infinitely large reservoir. Elliptical flow situations are discussed afterwards by many authors[2]–[4] mostly to model a reservoir with a vertical fracture at the center of the wellbore. In absence of vertical fracture elliptical flow situation may occur in anisotropic reservoir. During the production phase of cyclic steam stimulation due to the altered permeability of shear zone created by high pressure steam at the injection period around the wellbore, elliptical flow situation is also occurred.

Van der Ploeg *et al.*[5] proposed a closed form solution for flow into circular wellbore in an elliptic drainage system. Their work was related to water flow in a confined elliptical aquifer. They assumed a free surface at the water head. They presented steady state solution considering gravity flow.

Sarker and Tamim[6] used the model of van der Ploeg *et al.* and applied it to reservoir engineering scenario. They modified it and found a steady flow equation around a circular wellbore in a reservoir with elliptical domain.

Liao and Lee[8] investigated depth of investigation for elliptical flow problems of hydraulically fractured wells. They considered elliptical wellbore to derive the flow equation. They extended their model to a fractured well considering a very small length of minor axes of the elliptical wellbore. With the help of the method of continuous succession of steady states, they derived the expression of depth of investigation for fractured wells.

2. MODEL DESCRIPTION
In groundwater problems van der Ploeg *et al*[5] found the flow equation as –

$$q = -\frac{2\pi K h A_{NO} \Delta\phi}{\ln(a/r_w)} \tag{1}$$

Sarker and Tamim[6] used the modified form of this groundwater flow equation to apply in petroleum engineering. Their final form of the flow equation is given by –

$$q = -\frac{2\pi k h A_{N0} \Delta(p + \rho g z)}{\mu \ln(a/r_w)} \tag{2}$$

This equation is applicable to steady state situation around a circular wellbore at the center of the elliptical flow system under constant pressure at the boundary of the reservoir. If flow is considered horizontal then Δz becomes zero and Eq. (2) becomes –

$$q = -\frac{2\pi k h A_{N0} \Delta p}{\mu \ln(a/r_w)} \tag{3}$$

Eq. (3) is used as the basis of the derivation of the depth of investigation equation around a circular wellbore in an elliptical domain. The steady state equation, Eq. (3), which is not a function of time, is applied to transient state condition by using the method of continuous succession of steady states (CSSS). This method considers a transient unsteady-state flow as continuous succession of steady state flow pattern. CSSS method was first used by Muskat [1]. Liao and Lee [8] also used CSSS method in their derivation of radius of investigation.

The following equation of DOI has been derived for the elliptical system.

$$t_D = \frac{2}{\pi A_{NOi}} \left[\frac{\pi}{4} \ln(a_{Di}) \left(a_{Di}^2 \sqrt{1-e^2} - 1 \right) - A_{NOi} \times I_2 \right] \tag{4}$$

where,

$$I_2 = \int_0^{\pi/2} \int_1^{a_{Di} f(\theta)} \frac{1}{A_{NO}} \ln\left(\frac{r_D}{f(\theta)} \right) r_D \, dr_D \, d\theta \tag{5}$$

$$f(\theta) = \frac{\sqrt{(1-e^2)}}{\sqrt{\sin^2\theta + (1-e^2)\cos^2\theta}} \tag{6}$$

t_D is dimensionless time and a_{Di} is the dimensionless major axis of the elliptical drainage area i.e. the dimensionless DOI in terms of major axis. They have the following expressions –

$$t_D = \frac{kt}{\phi \mu c_t r_w^2} \text{ and}$$

$$a_{Di} = \frac{a_i}{r_w}$$

Appendix A gives the detail derivation of DOI around a circular wellbore in an elliptical domain. Eq. (4) is an implicit equation of depth of investigation, a_{Di}. The equation can readily determine the dimensionless time required to drain a given value of depth of investigation.

3. COMPARISON OF THE MODEL

The implicit expression of depth of investigation, Eq. (4) is derived for elliptical domain with circular wellbore at the center. Different eccentricity will represent different shape of the elliptical flow system. Circle is a special case of ellipse whose eccentricity is zero. Expression of depth of investigation for radial flow system whose drainage area is circular is available in the literature. Liao and Lee [8] discussed the depth of investigation in details for radial flow systems as well as elliptical flow systems with vertical fracture at the middle. For radial flow systems, their equation for radius of investigation is given by –

$$t_D = \frac{1}{4}\left(r_D^2 - 1 - 2\ln r_D \right) \tag{7}$$

It is also an implicit expression for depth of investigation r_D. When the eccentricity of the elliptical domain is zero it becomes circular. In this case i.e. at e = 0, Eq. (4) should produce the depth of investigation similar to the radius of investigation in circular domain as found from Eq. (7). Both the equations are plotted in Fig 1. It is found that the curves are overlapping each other giving the same values of depth of investigation or radius of investigation. This shows that the equation developed in the present study can be successfully used to determine the depth of investigation in elliptical flow domain with a circular wellbore in the middle.

4. RESULT

The depth of investigation for elliptical system is expressed in terms of the length of major axes. The elliptical drainage boundary can be obtained from major axes and eccentricity of the elliptical domain. In Fig 2, dimensionless depth of investigation is plotted against the dimensionless time at different eccentricities. From Fig 2 it is found that the depths of investigation or the lengths of the major axes of the elliptical propagation increases with increase in dimensionless time. With the increase in eccentricities, major axes of the elliptical domain increases. This is in consistence with a radial system as well.

5. REFERENCES

1 Muskat, M.: "The Flow of Homogeneous Fluids Through Porous Media", IHRDC, Boston, 1937.

2 Hale, B. W., Evers, J. F.: "Elliptical Flow Equation for Vertically Fractured Gas Wells", JPT, December 1981, pp. 2489-2497.

3 Kuchuk, F., Brigham W. E.: "*Transient Flow in Elliptical System*", SPEJ, December 1979, pp. 401-410.

4 Kuchuk, F., Brigham W. E.: "Unsteady-State Water Influx in Elliptic and anisotropic Reservoir/Aquifer Systems", SPE of AIME, June 1981, pp. 309-314.

5 van der Ploeg, R. R., Kirkham, D., Boast, C.W.: "Steady state well flow theory for a confined elliptical aquifer", Water Resources Research, Vol. 7, No. 4, August 1971, pp. 942- 954

6 Sarker, M. R. H., Tamim, M.: "Steady Flow into a Circular Well at the Center of a Confined Elliptical Drainage System", Journal of Chemical Engineering, The Institute of Engineers, Bangladesh, Vol. ChE 1, No. 1, November, 2005, pp 52-56.

7 Sarker, M. R. H.: "Analytical Investigation of Steady Flow Around a Circular Wellbore in an Elliptical Domain", M.Sc. thesis, Department of Petroleum and mineral Resources Engineering,

Bangladesh University of Engineering and Technology, Dhaka, Bangladesh, June 2005.
8 Liao, Y., Lee, W. J.: "Depth of Investigation for Elliptical Flow Problems and Its Applications to Hydraulically Fractured Wells", SPE 27908, March 1994.

6. NOMENCLATURE

Symbol	Meaning	Unit
θ	Angular coordinate	--
ρ	Fluid density	kg/m^3
ϕ	Porosity	pu
a	Half of major axis of the elliptical domain	m
a_{Di}	Dimensionless major axis	--
A_{NO}	Geometric factor of the elliptical domain	--
c_t	Total compressibility	N^{-1}
t_D	Dimensionless time	--
r_D	Dimensionless	

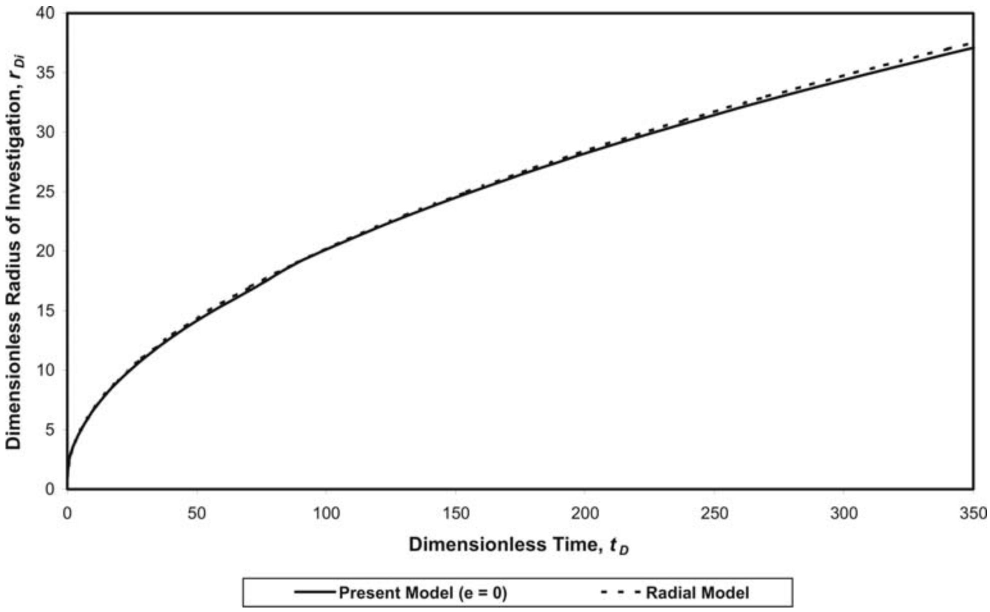

Fig 1. Comparison of Depth of Investigation from Elliptical and Radial Model

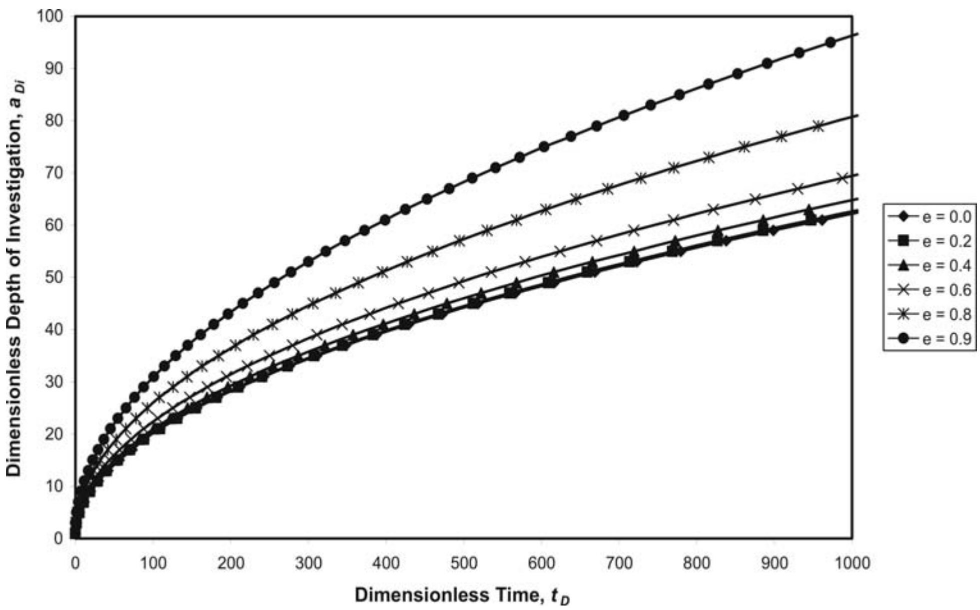

Fig 2. Depth of Investigation of Elliptical Domain (in Terms of Major Axis) with Different Eccentricities

APPENDIX A

Derivation of depth of Investigation

The steady-state flow in an elliptical domain with circular wellbore located at the center as expressed earlier by the Eq (3) can be written with slight modification as –

$$q = \frac{2\pi k h A_{NO}\left(p - p_w\right)}{\mu \ln\left(a/r_w\right)} \tag{A.1}$$

For a porous medium, Liao and Lee [8] found the expression of porosity as –

$$\phi = \phi_o e^{c_m\left(p - p_o\right)} \tag{A.2}$$

and for a fluid, the density is –

$$\rho = \rho_o e^{c_l\left(p - p_o\right)} \tag{A.3}$$

where,

ϕ = porosity at pressure p

ϕ_o = porosity at pressure p_o

c_m = pore compressibility and

ρ = fluid density at pressure p

ρ_o = fluid density at pressure p_o

c_l = fluid compressibility

If the compressibilities c_m and c_l are small then using the approximation of exponential series (*e*-series) one can write –

$$\phi\rho = \left(\phi\rho\right)_o + \left(\phi\rho\right)_o c_t\left(p - p_o\right) \tag{A.4}$$

Where, c_t = c_m + c_l, and c_t is called total compressibility.

Thus it can be written –

$$\left(\phi\rho\right)_i - \left(\phi\rho\right) = \left(\phi\rho\right)_o c_t\left(p_i - p\right) \tag{A.5}$$

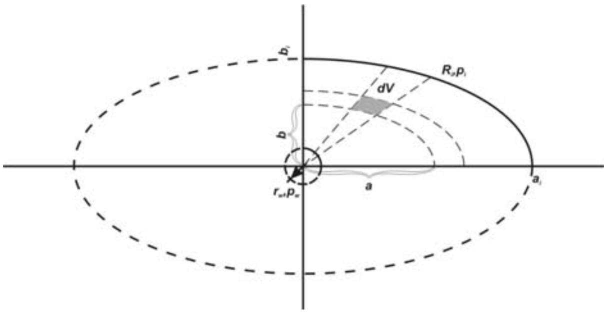

Fig 1. Elliptical Domain with Circular Wellbore at the Center

For a given time t, if it is assumed that the disturbance (pressure variation) has influenced an area from r_w to r_i where the pressures are p_w and p_i respectively, then the instantaneous pressure distribution at this time obeys the steady state distribution of Eq (A.1). The equation can be rewritten as –

$$p - p_w = \left(\frac{q\mu}{2\pi k h}\right)\frac{\ln\left(a/r_w\right)}{A_{NO}} \tag{A.6}$$

or, $$\ln\left(a/r_w\right) = \frac{2\pi k h}{q\mu}A_{NO}\left(p - p_w\right) \tag{A.7}$$

and,

$$p_i - p_w = \left(\frac{q\mu}{2\pi k h}\right)\frac{\ln\left(a_i/r_w\right)}{A_{NOi}} \tag{A.8}$$

or, $$\ln\left(a_i/r_w\right) = \frac{2\pi k h}{q\mu}A_{NOi}\left(p_i - p_w\right) \tag{A.9}$$

Now,

$$\ln\frac{a_i}{a} = \ln\left(\frac{a_i}{r_w}\right) - \ln\left(\frac{a}{r_w}\right) \tag{A.10}$$

or, $$\ln\frac{a_i}{a} = \frac{2\pi k h}{q\mu}\left[A_{NOi}\left(p_i - p_w\right) - A_{NO}\left(p - p_w\right)\right] \tag{A.11}$$

or, $$A_{NOi}\left(p_i - p_w\right) - A_{NO}\left(p - p_w\right) = \frac{q\mu}{2\pi k h}\ln\frac{a_i}{a} \tag{A.12}$$

Putting the value of $\frac{q\mu}{2\pi k h}$ in Eq (A.12) from Eq (A.8) one can write,

$$A_{NOi}\left(p_i - p_w\right) - A_{NO}\left(p - p_w\right) = \frac{p_i - p_w}{\ln\left(a_i/r_w\right)}A_{NOi}\ln\frac{a_i}{a} \tag{A.13}$$

or, $$p = p_w + \frac{A_{NOi}}{A_{NO}}\left[\left(p_i - p_w\right) - \frac{p_i - p_w}{\ln\left(a_i/r_w\right)}\ln\frac{a_i}{a}\right] \tag{A.14}$$

Subtracting p_i from both sides –

$$p - p_i = -p_i + p_w + \frac{A_{NOi}}{A_{NO}}\left[\left(p_i - p_w\right) - \frac{p_i - p_w}{\ln\left(a_i/r_w\right)}\ln\frac{a_i}{a}\right] \tag{A.15}$$

or, $$p - p_i = \left(\frac{A_{NOi}}{A_{NO}} - 1\right)\left(p_i - p_w\right) - \frac{A_{NOi}}{A_{NO}}\frac{p_i - p_w}{\ln\left(a_i/r_w\right)}\ln\frac{a_i}{a} \tag{A.16}$$

or, $$p = p_i - \left[\left(1 - \frac{A_{NOi}}{A_{NO}}\right)\left(p_i - p_w\right) + \frac{A_{NOi}}{A_{NO}}\frac{p_i - p_w}{\ln\left(a_i/r_w\right)}\ln\frac{a_i}{a}\right] \tag{A.17}$$

The instantaneous production rate is given by Eq (A.1) and it is rewritten as –

$$q(t) = \frac{2\pi k h A_{NO}\left(p - p_w\right)}{\mu \ln\left(a/r_w\right)} \tag{A.18}$$

An incremental volume within the drainage volume is dV.

$$dV = h\left(r\,dr\,d\theta\right) \tag{A.19}$$

Up to this time t, this small volume has produced an incremental liquid mass dG.

$$dG = [(\phi\rho)_i - (\phi\rho)]dV \tag{A.20}$$

$$dG = [(\phi\rho)_i - (\phi\rho)]h(r\,dr\,d\theta) \tag{A.21}$$

Thus up to this time t, the total liquid mass from this volume is –

$$G(t) = 4\int_0^{\pi/2}\int_{r_w}^{R}[(\phi\rho)_i - (\phi\rho)]h\,r\,dr\,d\theta \tag{A.22}$$

where,

$$R = \frac{ab}{\sqrt{a^2\sin^2\theta - b^2\cos^2\theta}} \tag{A.23}$$

From Eqs (A.26) and (A.5) one can write

$$G(t) = 4\int_0^{\pi/2}\int_{r_w}^{R}(\phi\rho)_o c_t(p_i - p)h\,r\,dr\,d\theta \tag{A.24}$$

$$= 4(\phi\rho)_o c_t h\int_0^{\pi/2}\int_{r_w}^{R}(p_i - p)r\,dr\,d\theta \tag{A.25}$$

$$= 4(\phi\rho)_o c_t h\int_0^{\pi/2}\int_{r_w}^{R}\left[\left(1-\frac{A_{NOi}}{A_{NO}}\right)(p_i - p_w) + \frac{A_{NOi}}{A_{NO}}\frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a}\right]r\,dr\,d\theta \tag{A.26}$$

$$= 4(\phi\rho)_o c_t h(p_i - p_w)\int_0^{\pi/2}\int_{r_w}^{R}\left[\left(1-\frac{A_{NOi}}{A_{NO}}\right) + \frac{A_{NOi}}{\ln(a_i/r_w)}\frac{\ln(a_i/a)}{A_{NO}}\right]r\,dr\,d\theta \tag{A.27}$$

$$= 4(\phi\rho)_o c_t h(p_i - p_w)\int_0^{\pi/2}\int_{r_w}^{R}\left[1-\frac{A_{NOi}}{A_{NO}}\left\{\frac{\ln(a_i/r_w) - \ln(a_i/a)}{\ln(a_i/r_w)}\right\}\right]r\,dr\,d\theta \tag{A.28}$$

$$= \frac{4(\phi\rho)_o c_t h(p_i - p_w)}{\ln(a_i/r_w)}\int_0^{\pi/2}\int_{r_w}^{R}\left[\ln(a_i/r_w) - \frac{A_{NOi}}{A_{NO}}\ln(a/r_w)\right]r\,dr\,d\theta \tag{A.29}$$

On the other hand, for constant rate production,

$$G(t) = qt\rho_o = \frac{2\pi khA_{NOi}(p_i - p_w)}{\mu\ln(a_i/r_w)}\rho_o t \tag{A.30}$$

Comparison of Eqs (A.29) and (A.30) yields

$$\frac{kt}{\phi_o c_t\mu}\frac{\pi A_{NOi}}{2} = \int_0^{\pi/2}\int_{r_w}^{R}\left[\ln(a_i/r_w) - \frac{A_{NOi}}{A_{NO}}\ln(a/r_w)\right]r\,dr\,d\theta \tag{A.31}$$

Now, defining dimensionless groups $r_D = \frac{r}{r_w}$ and $t_D = \frac{kt}{\phi\mu c_t r_w^2}$ we get

$$r^2 = r_w^2 r_D^2 \tag{A.32}$$

$$\text{or, } 2rdr = 2r_w^2 r_D dr_D \tag{A.33}$$

$$\text{or, } rdr = r_w^2 r_D dr_D \tag{A.34}$$

Introducing dimensionless groups in Eq (A.31) it is found

$$\frac{\pi A_{NOi}}{2}t_D = \int_0^{\pi/2}\int_1^{R_D}\left[\ln(a_{Di}) - \frac{A_{NOi}}{A_{NO}}\ln(a_D)\right]r_D\,dr_D\,d\theta \tag{A.35}$$

$$\text{or, } \frac{\pi A_{NOi}}{2}t_D = \int_0^{\pi/2}\int_1^{R_D}\ln(a_{Di})r_D\,dr_D\,d\theta - A_{NOi}\int_0^{\pi/2}\int_1^{R_D}\frac{\ln(a_D)}{A_{NO}}r_D\,dr_D\,d\theta \tag{A.36}$$

Now, from equation of an ellipse –

$$\frac{r^2\cos^2\theta}{a^2} + \frac{r^2\sin^2\theta}{b^2} = 1 \tag{A.37}$$

$$\text{and, } b^2 = a^2(1-e^2) \tag{A.38}$$

where, e is the eccentricity of the ellipse

From Eqs (A.37) and (A.38) it can be deduced that

$$\frac{r^2\cos^2\theta}{a^2} + \frac{r^2\sin^2\theta}{a^2(1-e^2)} = 1 \tag{A.39}$$

$$\text{or, } a^2 = r^2\cos^2\theta + \frac{r^2\sin^2\theta}{(1-e^2)} \tag{A.40}$$

$$\text{or, } a = r\sqrt{\cos^2\theta + \frac{\sin^2\theta}{(1-e^2)}} \tag{A.41}$$

$$\text{or, } r = a\frac{\sqrt{(1-e^2)}}{\sqrt{\sin^2\theta + (1-e^2)\cos^2\theta}} \tag{A.42}$$

$$\text{if, } f(\theta) = \frac{\sqrt{(1-e^2)}}{\sqrt{\sin^2\theta + (1-e^2)\cos^2\theta}}, \tag{A.43}$$

then

$$r = a\,f(\theta) \tag{A.44}$$

$$\text{and } a = r/f(\theta) \tag{A.45}$$

So, Eq (A.44) becomes

$$\frac{\pi A_{NOi}}{2}t_D = \ln(a_{Di})\int_0^{\pi/2}\int_1^{a_{Di}f(\theta)}r_D\,dr_D\,d\theta - A_{NOi}\int_0^{\pi/2}\int_1^{a_{Di}f(\theta)}\frac{1}{A_{NO}}\ln\left(\frac{r_D}{f(\theta)}\right)r_D\,dr_D\,d\theta \tag{A.46}$$

In the left hand side of Eq (A.54) the there are two parts. These are –

$$I_1 = \ln(a_{Di})\int_0^{\pi/2}\int_1^{a_{Di}f(\theta)}r_D\,dr_D\,d\theta \tag{A.47}$$

And

$$I_2 = \int_0^{\pi/2}\int_1^{a_{Di}f(\theta)}\frac{1}{A_{NO}}\ln\left(\frac{r_D}{f(\theta)}\right)r_D\,dr_D\,d\theta \tag{A.48}$$

I_1 can be evaluated by integration and it can be written as –

$$I_1 = \frac{\pi\ln(a_{Di})}{4}\left[a_{Di}^2\sqrt{1-e^2} - 1\right] \tag{A.49}$$

On the other hand, I_2 is evaluated numerically. So the final expression becomes –

$$t_D = \frac{2}{\pi A_{NOi}}\left[\frac{\pi}{4}\ln(a_{Di})\left(a_{Di}^2\sqrt{1-e^2} - 1\right) - A_{NOi}\times I_2\right] \tag{A.46}$$