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SIMULATION OF SUPERSONIC GAS-PARTICLE FLOW USING EULERIAN-EULERIAN APPROACH

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ABSTRACT
The present work deals with the effects of solid particles on the thermo-fluid characteristic of supersonic flows. Supersonic gas-particle flows have practical applications in many scientific and technical fields, Reentry vehicles flying in the atmosphere where they enter through a dust or rain cloud, ablation cooling, solid propellant rockets and blast waves moving over the earth's surface are examples of such applications. Simulation of the flows is carried out using numerical method. The flow is considered two-dimensional, compressible; laminar of a dilute dusty gas and full Navier-Stokes equations coupled with particle-phase equations are the governing equations of the flows. Since the individual motion of each particle is of no interest in this work, the particle phase is considered as the continuum (or Eulerian approach is employed). Van Leer scheme is employed to approximate inviscid fluxes of gas phase decretized equations and MUSCL scheme with Van Albada limiter is applied to improve the accuracy of the approximation. The Particle-phase is considered as an inviscid continuum media and solution of the Riemann problem is used to approximate the inviscid fluxes. Two test cases are carried out to evaluate the performance of the algorithm and the results are compared with the results of other references.

Keywords: Gas-solid, Compressible, Supersonic, Dusty gas, Two-phase, Van Leer.

1. INTRODUCTION
During recent dictates a lot of studies have been done in gas-particle two phase [1]-[11]. In 1980 Osiptison[12] studied incompressible gas-particle two phase flow over a semi-infinite flat plate. In 1988, Wang and Glass [13] simulated a laminar compressible dilute gas-particle flow over a semi-infinite flat plate. They employed a combination of asymptotic expansion analytical method and a numerical method. In 1994 Chamkha[14] studied the effects of particle diffusion on thermal boundary layer of compressible gas-particle flows. In the same year Daniel and et al.[15] presented a second order numerical method based on Van leer scheme in compressible gas-particle flows study. Chamkha [16] extended the work of Wang and Glass[13] in 1996. They included viscous effects of particle phase. In 2001 Reddy and Padmapriya[17] modeled two dimensional dilute gas-particle flows at high Mach number using Eulerian-Eulerian approach and in the same year. Park and Baek [18] studied the interaction of gas-particle behind a moving shock wave.

The propose of this research is to investigate the influences of solid particles on the flow characteristics of supersonic high Mach number flows.

2. PROBLEM FORMULATION
In order to formulate the problem mathematically, some assumptions are made these assumptions are: particles are spheres with equal radius, material density of the particles is much greater than gas density, The particles have no Brownian motion, the interaction between the particles is neglected, the volume occupied by the particles is negligible, the number density of the particles is high enough to be considered as a pseudo gas, the temperature distribution within the particles is uniform, the gas phase is considered as perfect gas, and moreover, radiation heat transfer, chemical reactions, coagulations, phase change and deposition in this system are neglected.

The governing equations of each phase are introduced briefly in the following subsections.

2.1 Gas phase
According to the previous discussions viscous effects have been take in to account in the present work. So full Navier-Stokes equations are considered as the governing equations of gas phase. These equations in conservative form are as follow:

$$\frac{\partial U}{\partial t}+\frac{\partial F_I}{\partial x}+\frac{\partial G_I}{\partial y}-\frac{\partial F_v}{\partial x}-\frac{\partial G_v}{\partial y}=H \qquad (1)$$

In this equation U represents conservative variables vector, F and G are flux vectors. Subscripts of v and I denote viscous and inviscid fluxes respectively. More detail about the viscous and inviscid fluxes can be found in Ref [17]. H is a vector containing particle phase influence terms which is defined as:

$$H=\begin{bmatrix} 0 \\ F_{Px} \\ F_{py} \\ uF_{P\mathrm{x}}+vF_{P\mathrm{y}}+Q_P \end{bmatrix}$$

Where F_{px}, F_{py}, and Q_P are force in x direction, force in y direction and heat transfer rate between two phases respectively. These interaction terms are computed as:

$$F_{px}=\mu\rho_p\,(u_P-u\ )/\tau_v$$

$$F_{py}=\mu\rho_p\,(v_P-v)/\tau_v$$

$$Q_p=\frac{C_{P_p}\mu\rho_p}{C_P M^2{}_\infty\ (\gamma-1)\tau_T}Nu\ (T_P-T\ )$$

In these relations:

$$\tau_T=\frac{C_{P_p}\,\mathrm{Pr}\,\sigma_p d_p^2 U_\infty}{6\mu_\infty\lambda_\infty C_P},\qquad \tau_V=\frac{\sigma_P d^2{}_p}{18\mu_\infty},$$

$$\lambda_\infty=\frac{\sigma_p d_p^2 U_\infty}{18\mu_\infty}$$

Nu is the normalized Nusselt number based on the particle diameter and depends on the gas-particle interaction mechanism. The drag coefficient and the Nusselt number for a single sphere in a viscous flow are assumed to be valid for the particle cloud, so they are defined as:

$$Cd=\frac{24}{\mathrm{Re}_r}\qquad Nu=2.+0.6\,\mathrm{Re}_{\mathrm{r}}^{\frac{1}{2}}\,Pr^{\frac{1}{3}}$$

Re_r is the slip Reynolds number of the particles which is defined as:

$$\mathrm{Re}_r=\frac{\rho\ \ (\mathrm{u}_P-\mathrm{u})\ D_p}{\mu}$$

In the present work the viscosity–temperature relation for the gas phase is evaluated from Sutherland's law.

2.2 Particle phase

The Eulerian approach is employed for particle phase. The conservative form of the governing equations is:

$$\frac{\partial U_p}{\partial t}+\frac{\partial F_p}{\partial x}+\frac{\partial G_p}{\partial y}=H_p \qquad (2)$$

In these equations:

$$U_p=\begin{bmatrix}\rho_p\\ \rho_{\mathrm{p}}u_{\mathrm{p}}\\ \rho_{\mathrm{p}}\upsilon_{\mathrm{p}}\\ \rho_{\mathrm{p}}E_p\end{bmatrix},\ F_p=\begin{bmatrix}\rho_{\mathrm{p}}u_p\\ \rho_{\mathrm{p}}u_p^2\\ \rho_{\mathrm{p}}u_p\upsilon_p\\ \rho_{\mathrm{p}}u_pE_p\end{bmatrix},\ G_p=\begin{bmatrix}\rho_{\mathrm{p}}\upsilon_p\\ \rho_{\mathrm{p}}u_p\upsilon_p\\ \rho_{\mathrm{p}}\upsilon^2\\ \rho_{\mathrm{p}}\upsilon_{\mathrm{p}}E_p\end{bmatrix}$$

$$H_p=-H$$

Note that the solid phase does not have pressure term, and in fact the pressure of the gas phase P is considered as the pressure of the mixture.

3. NUMERICAL SCHEME

Finite volume technique is employed to discretize the governing equations of two phases. The governing equations are integrated in physical domain and inviscid fluxes on surface cells are approximated using upwind method. These methods are compatible with the nature of the governing equations of compressible two phase flows. The set of the particle phase governing equations is a semi-parabolic system and such a system has characteristics only in one direction. For this system the inviscid fluxes are approximated based on flux difference splitting method and Riemann problem is solved for six possible conditions including [15]:

a) Expansion Conditions:

1) $u_{pl}>\circ$; $u_{pr}>\circ$; $u_{pl}<u_{pr}$, $u_p=u_{pl}$, $\rho_{\mathrm{p}}=\rho_{pl}$

2) $u_{pl}<\circ$; $u_{pr}>\circ$; $u_{pl}<u_{pr}$, $\rho_p=\circ$

3) $u_{pl}<\circ$; $u_{pr}<\circ$; $u_{pl}<u_{pr}$, $\rho_{\mathrm{p}}=\rho_{\mathrm{pr}}$, $u_p=u_{pr}$

b) Compression conditions:

1) $u_{pl}>\circ$; $u_{pr}>\circ$; $u_{pl}>u_{pr}$, $\rho_p=\rho_{pl}$, $u_p=u_{pl}$

2) $u_{pl}>\circ$; $u_{pr}<\circ$; $u_{pl}>u_{pr}$, $\rho_p=\rho_{pl}+\rho_{\mathrm{pr}}$

$$\rho_p u_p=\rho_{pl}u_{pl}+\rho_{pr}u_{pr}\quad ,\quad u_p=\frac{\rho_{pl}u_{pl}+\rho_{pr}u_{pr}}{\rho_{pl}+\rho_{pr}}$$

3) $u_{pl}<\circ$; $u_{pr}<\circ$; $u_{pl}>u_{pr}$, $\rho_p=\rho_{p\mathrm{r}}$, $u_p=u_{pr}$

Subscripts l and r represent left and right side of the cell face respectively.

The set of gas phase governing equations has hyperbolic nature. This set has three real characteristics and for inviscid fluxes approximation Vanleer method has been employed [19]. This method belongs to the flux vector splitting methods, and has first order degree of accuracy. Since the numerical diffusion of this first order method is high, it is modified using MUSCL method with Van Albada limiter. Central is used to approximate viscous fluxes of the gas equations. This approximation is compatible with physical nature of the diffusion terms.

After spatial integration, time dependent terms are discretized employing Euler explicit method.

3.1 Boundary conditions

One of the most important boundary conditions in gas-particle flows is conditions of the particle phase near the solid boundaries; experimental researches indicate that particles experience slight slip at solid boundaries. Since the solid phase is assumed to be dilute, its behavior may be predicted via rarefied gas dynamics principles. Accordingly particle tangential velocity is controlled by friction and particle collision to the wall, so here we have considered a sliding coefficient depending on axial location from the leading edge of the body [13]:

$$u_p(x,0)=0 \qquad \text{for} \qquad x \geq x_{cr}$$
$$u_p(x,0)=1-\mu_w x \qquad \text{for} \qquad x \leq x_{cr}$$

Where x_{cr} is determined from the following compatibility equations:

$$\frac{\partial u_p}{\partial x}=-(\mu D)_w \qquad\qquad u_{pw}=0$$

For the other variables of the particle phase the following conditions are employed on solid boundaries:

$$T_p(x,0)=T_w \text{ ، } v_p(x,0)=0$$

For gas phase wall boundary conditions are considered as:

$$u(x,\circ)=\circ \ \ ; \ \ \mathrm{v(x,0)=0} \ \ ; \ \ \mathrm{T(x,0)=T_w}$$

And finally free stream boundary conditions are:

$$u(x,\infty)=1. \text{ ، } T(\mathrm{x},\infty)=1.$$
$$T_p(x,\infty)=1.,\ \rho_p(x,\infty)=1.,\ u_p(x,\infty)=1.$$

4. RESULTS AND DISCUSSION

In order to evaluate the performance of the Present procedure, several test cases are carried out.

The first test case is flow over a flat plate with the following conditions:

$$M_\infty=6, \quad \mathrm{Re}_\infty=10^6, \quad T_w=2.$$

Figs. 1 to 3 give the results of this test case. In these figures the results of the present calculations are compared with those of Ref. [17]. Fig. 1 shows velocity profiles of pure gas and dusty gas (gas with solid particle) flows the agreement between the presentresults and those of Ref. [17] is excellent. More over it can be seen that the boundary layer thickness of pure gas is more than that of dusty gas flow. This is due to that the density of the dusty gas is more than that of pure one. Similar behavior for temperature profiles may be seen in figure 2. Figure 3 shows variation of friction coefficient parameter ($Cf\sqrt{\mathrm{Re}_x}$) along the flat plate. In this figure one way coupling results (including only the effects of particle phase on gas phase) and two way coupling results (including the interaction between the two phase) are compared with the results of pure gas calculation and those of Ref.[17]. According to this figure, the effects of the gas phase on particle phase are very important. More over since the overshoot of the present friction factor distribution is located closer to the leading edge of plate (In comparison with distribution of ref. [17])

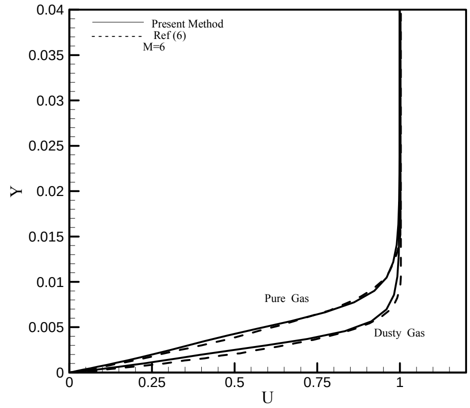

Fig 1. Velocity Profiles of Pure gas and dusty gas

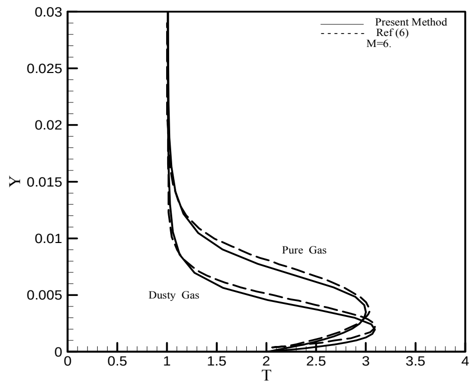

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Fig 2. Temperature Profiles of Pure gas and dusty gas at x=1.2

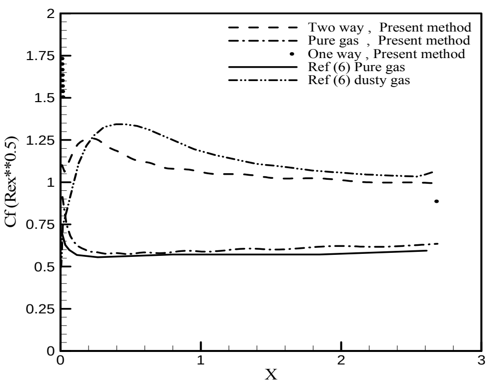
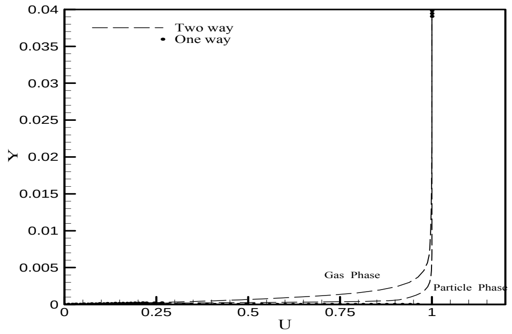


Fig 3. Friction coefficient($Cf\sqrt{\mathrm{Re}_x}$) distribution for test case 1

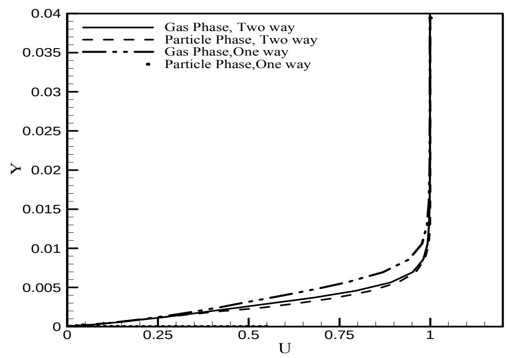
It seems that the present distribution is more accurate than the distribution of Ref. [17]. According to Fig.1 and Fig.2, the two phase interaction will increase velocity and temperature gradients inside the boundary layer, and this fact leads to increasing the skin friction force and surface heat transfer rate. Since in one-way interaction, only the influence of the gas phase on the particle phase is considered, variation of boundary layer parameters is less than those of the two way interaction. This can be seen in Fig.3.

The second test case is a gas-particle supersonic flow with the following conditions:

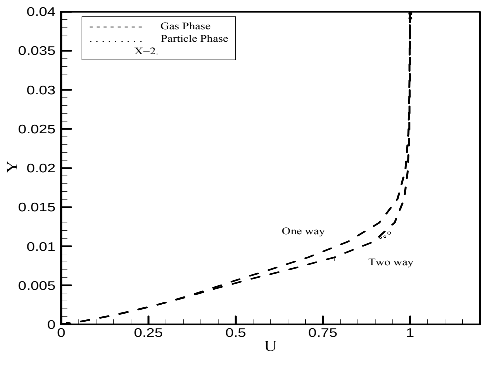
$$M_\infty = 10. \ , \ \mathrm{Re}_\infty = 10^6, \ T_W = 2.$$



(a)



(b)

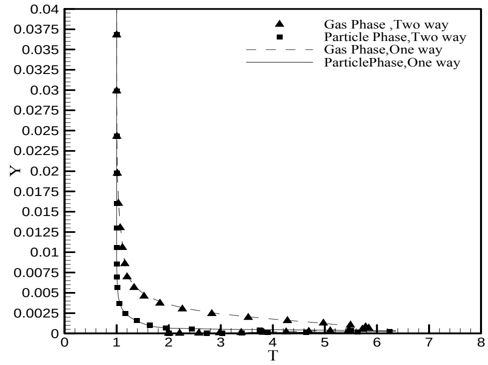


(c)

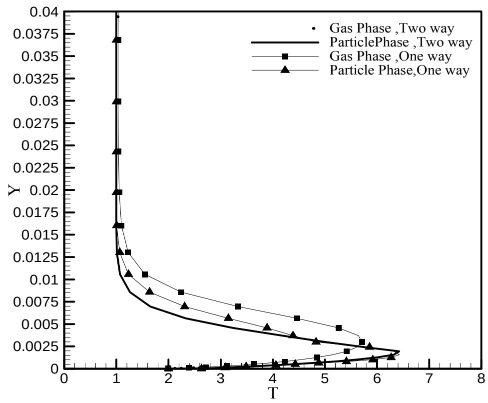
Fig 4. Gas and particle phase Velocity profiles for one way and two way coupling at a) $x = 0.09$, b) $x = 0.7$, $x = 2.0$

Figs.4 gives the results of the present calculations at three different locations from the leading edge. In this figure the results of one-way and two-way assumptions are compared. At x=0.09 (high slipping region) the two way coupling and one way coupling results are nearly the same. At x=0.7 (intermediate slipping region), at x=2.0 (low slipping region) the results of these two approach are partly different. Moving form the leading edge along the flat plate

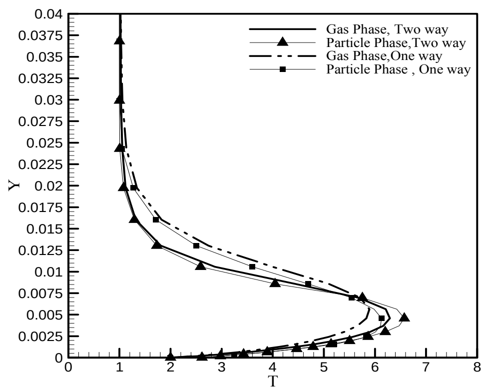
Due to reduction of slipping, velocity particle phase approaches to velocity and temperature of gas. Similar trend may be seen for temperature profiles in Fig.5.



(a)



(b)



(c)

Fig 5. Gas and particle phase temperature profiles for one way and two way coupling at a) $x = 0.09$, b) $x = 0.7$, c) $x = 2.0$

Boundary layer thickness distribution variations with Reynolds number is given in Fig. 6

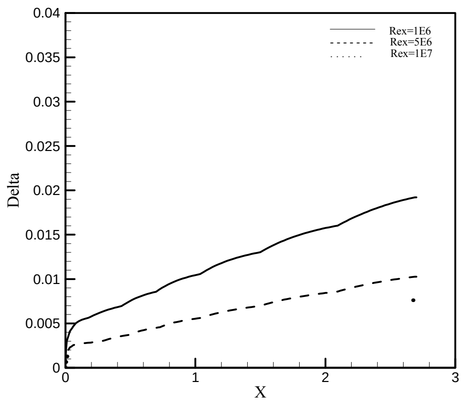


Fig 6. Boundary Layer thickness distribution variations with Reynolds number for the 2nd case

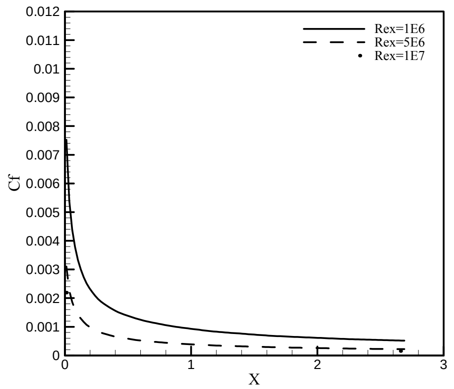


Fig 7. Friction factor distribution variations with Reynolds number for the 2nd case

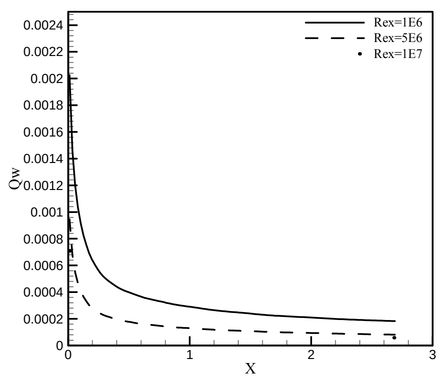


Fig 8. Heat transfer coefficient distribution variations with Reynolds number for the 2nd case

It can be seen that at every location along the plate the boundary layer thickness is proportional to $(\mathrm{Re})^{-1/2}$ this fact is predicted by self similarity boundary layer. Skin friction and heat transfer coefficient distributions along the plate are shown in figures 7 and 8 respectively. In these figures the coefficients distributions are given at three different Reynolds number. According to these figures, as well as the boundary layer thickness, friction factor and heat transfer coefficient are inversely proportional to the $\sqrt{\mathrm{Re}}$

5. CONCLUSIONS

In this paper, two dimensional, supersonic, laminar flows of gas- particle were numerically simulated. An algorithm based on Eulerian-Eulerian approached was developed, and the governing equations (Navier-Stokes equations for gas phase and Euler equations for particle phase) solved using flux splitting method. The algorithm has a capability to solve supersonic gas-particle flows with Mach even at hypersonic range. Two test cases were carried out to evaluate the performance of the simulation The fist test case was a flow with M=6, and its were compared with those of Ref. [17], excellent agreement was found out between the results. The second test case was a hypersonic flow with Mach number of 10, for this case the Reynolds number effects on boundary layer thickness, friction factor, and heat transfer coefficient were investigated.

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