

MODELING OF ICE SLURRY FORMATION BY DIRECT CONTACT HEAT TRANSFER

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ABSTRACT

Attempts have been made in recent years to use ice slurry for district cooling applications. By utilizing ice-slurry, the performance of air-conditioning systems can be improved. A system has been developed for the generation of ice slurry using direct contact heat transfer between water and the coolant, Fluorinert FC-84. A one-dimensional model for ice slurry growth around the injected supercooled coolant droplets in the ice generator is developed. The growth of ice and mushy region around the supercooled liquid droplets involve phase change and heat conduction between these layers. These conditions reduce the ice thickness problem to a differential equation, from which other quantities, such as, the mushy region thickness and temperature profiles in the layers can be determined. The simultaneous solutions of these equations for different injecting droplets temperatures predict the growth rate of ice and mushy regions, and the corresponding temperature profiles of these layers.

Keywords: Direct contact heat transfer, Ice-slurry, Growth of ice, Mushy layer thickness, Numerical predictions.

1. INTRODUCTION

Recently, attentions have been paid actively to develop an ice thermal energy storage system for district cooling for the purpose of energy saving and reducing electrical cost, specially during peak hours in summer [1,2]. In particular, it would be beneficial to utilize the large latent heat of ice for more efficient cooling purposes. In an ice storage systems, due to the insulating characteristics of ice, the freezing heat-transfer performance decreases due to buildup of ice layers on heat exchanger surfaces during storage of cool energy [3]. To resolve this problem, a novel thermal energy storage system with ice water slurry has been developed [4,5]. Ice slurry, which consists of fine ice crystals and liquid water, has a large latent heat of fusion. Moreover, cooling capacity of slurry ice is about four to six times higher than that of conventional chilled water system [6].

Several techniques for ice slurry formation by direct contact heat transfer have been developed recently. Kiatsiriroat et al. [7] studied an ice thermal storage having an injection of R12 refrigerant into the water to exchange heat directly. They found that the coefficient of performance was about 3.4-3.6 which is higher than that of the conventional system. Matsumoto et al. [8] studied a new method of forming ice, a water-oil emulsion was cooled with stirring in a vessel and changed into ice oil and water suspension. They found that ice-oil and water suspension (sluish ice), which has a good fluidity, can be

formed without adhering to the cooling surface. Kiatsiriorat et al. [9] studied the heat transfer characteristics of a direct contact evaporator using R12 and R22. Refrigerant (R12 or R22) injected at temperature lower than 0°C takes heat directly from the water in a storage tank and then gets evaporated. A correlation that relates the dimensionless parameters, such as Stanton number, Stephan number, Prandtl number and pressure ratio, was also developed. They found higher heat transfer coefficient with direct contact heat transfer technique. Chen et al. [10] investigated the nucleation probability of super cooled water inside cylindrical capsules. Song et al. [11] presented a simple model on direct contact heat transfer between two immiscible liquids in a countercurrent spray column and obtained the numerical solution with the variable step Runge-Kutta method. Wijesundera et al. [12] studies the generation of ice slurry using direct contact heat transfer between water and injecting super cooled liquid Fluorinert (FC-84) and estimates the heat transfer co-efficient between liquid droplets and water. These literatures reveal that several methods of producing ice slurries exist, though many of these need to be investigated in a great detail.

The main motivation of present study is to understand the physical phenomena of ice and mushy layers growth around the super cooled liquid droplet. The results obtained from this study and techniques employed will

help in the design and analysis of a system for the production and storage of ice slurry using direct contact heat transfer between water and coolant.

2. PROBLEM FORMULATION

Stefan's [13] one-dimensional, non-fluid-dynamical theory explains the mathematical approach to modeling the ice and unfrozen water layer growth. In the following analysis, one-dimensional model will be considered. The configuration is depicted in Fig. 1, where $B(t)$ and $b(t)$ are ice layer and mushy layer thickness, respectively.

Heat transfer in ice-slurry formation with coolant involves three-phase conditions, including multiphase flow with droplets, phase change and heat conduction in the ice and mushy layer.

The assumptions made in the following analyses are as follows: (i) the effects of turbulence, radiation, splashback of droplets are negligible; (ii) convection coefficient and thermophysical properties are constant; (iii) conduction of heat transfer through the both layers, ice and mushy, are considered.

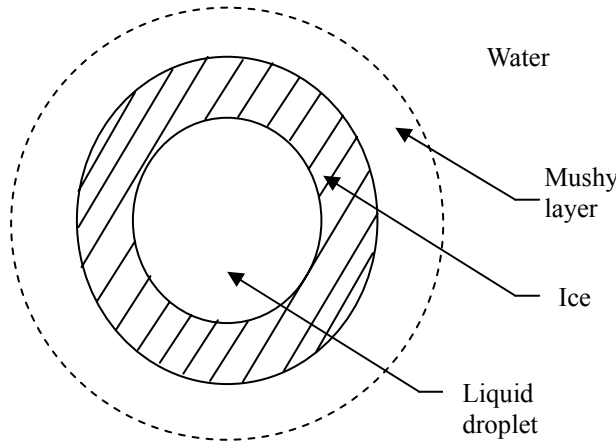


Fig 1(a). Schematic of Liquid droplet-ice-mushy-water layers

The growth process of ice layer and mushy layer around the impinging supercooled liquid droplets occurs in two distinct stages- at first stage, water comes in contact with the super cooled liquid droplets and freezes instantaneously, while at the second stage, both ice and mushy layer develop simultaneously. The initial incoming liquid droplet must, therefore, immediately adopt the sub-zero temperature and since there are nucleation site, freeze around the droplet.

Let the temperature in the liquid droplet be denoted by $T_d(t)$, while the temperatures in the ice and mushy layers are denoted by $T_i(y, t)$ and mushy layer is $T_m(y, t)$, respectively.

2.1 Governing Equations

The governing equations that describe heat transfer and ice formation are described in this section. In liquid droplet region, it is assumed that due to the formation of ice layer around the liquid droplet, the latent heat is conducted through the ice layer to the liquid droplet.

Then, the energy balance is given by, Energy gained by the liquid = Heat transfer by conduction to droplet during dt

$$(mC)_d dT_d = -k_i A \frac{dT}{dy} dt \quad (1)$$

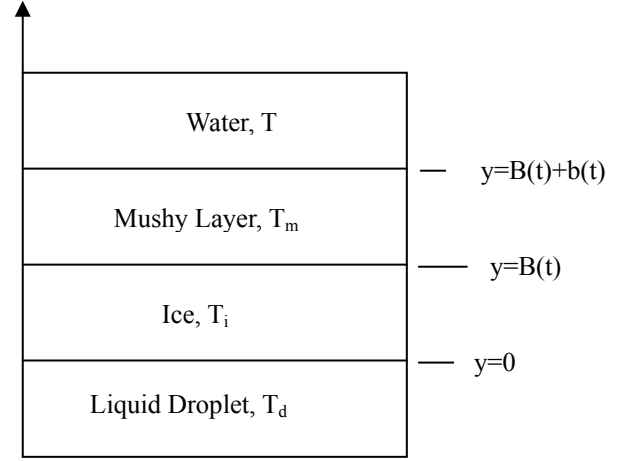


Fig 1(b). Cross-section of layers

where, m, C, A are the mass, specific heat and surface area, subscript d and i denote droplet and ice layer.

In the ice layer region, the heat diffusion equation is,

$$\frac{\partial T_i}{\partial t} = \frac{k_i}{\rho_i C_i} \frac{\partial^2 T_i}{\partial y^2} \quad (2)$$

In the mushy layer region, the heat diffusion equation is,

$$\frac{\partial T_m}{\partial t} = \frac{k_m}{\rho_m C_m} \frac{\partial^2 T_m}{\partial y^2} \quad (3)$$

where, k, ρ, C are the thermal conductivity, density and specific heat, subscript i and m denote ice and mushy layer.

Again, from the Stefan condition, the velocity of the moving ice/mushy layers interfaces, $B(t)$, is proportional to the heat flux across it.

$$\rho_i L \frac{\partial B}{\partial t} = k_i \frac{\partial T_i}{\partial y} - k_m \frac{\partial T_m}{\partial y} \quad (4)$$

which states that the velocity of the interface is proportional to the heat flux across it; L is the latent heat of fusion.

Furthermore, the growth of ice and mushy layers depend on the incoming liquid droplets velocity, cooling and geometry factor [14].

$$\rho_i \frac{\partial B}{\partial t} + \rho_m \frac{\partial b}{\partial t} = \alpha \gamma N G \quad (5)$$

2.2 Boundary Conditions

During the initial stage ($t = 0$), when there is only ice growth around the liquid droplet, the initial ice layer thickness can be found by integrating equation (5), with mushy layer thickness set to zero, to give,

$$B = \left(\frac{\alpha \gamma \mathcal{N} G}{\rho_i} \right) t \quad (6)$$

The liquid droplet temperature profile can be calculated by integrating equation (1) subject to T_{di} be initial droplet temperature and T_d be the final temperature.

$$T_d = T_i + (T_{di} - T_i) e^{-\frac{\lambda t}{B}} \quad (7)$$

$$\text{where, } \lambda = \frac{6k_i}{\rho_d C_d D}$$

The time taken for an incremental ice thickness growth is smaller than the time taken for the heat to be fully conducted through the newly formed layer. It is then approximated that the layer growth is sufficiently slow that heat losses/gains at the phase interface are fully conducted throughout the solid.

Then, heat conduction within the ice layer as well as the mushy layer, can be approximated as quasi-steady process [15]. The governing heat equation (2) for the ice then becomes,

$$\frac{\partial^2 T_i}{\partial y^2} = 0 \quad (8)$$

subject to boundary conditions of $T_i(0, t) = T_d$ at $y = 0$, $t > 0$ and at interfacial condition of $T_i(B, t) = T_f$ at $y = B$, $t > 0$. Solving Eq. (8) subject to the boundary conditions,

$$T_i(y, t) = \left(\frac{T_f - T_d}{B} \right) y + T_d \quad (9)$$

This equation represents a quasi-steady result since the ice thickness, B , varies with time.

In an analogous manner for the mushy layer, equation (3) reduces to

$$\frac{\partial^2 T_m}{\partial y^2} = 0 \quad (10)$$

subject to boundary conditions of $T_m(B, t) = T_f$ at $y = B$, $t > 0$. An energy balance at the ice/mushy layer interface, $y = B + b$, $t > 0$, yields the following interfacial constraint,

$$k_m \frac{\partial T_m}{\partial y} \Big|_{y=B+b} = h(T_m \Big|_{y=B+b} - T) \quad (11)$$

where, the term represents conduction through the mushy layer (left hand side) is balanced by the the heat flux by convection (right hand side) through this region.

The expression for heat conduction in Eq.(11) depends on the slope of the temperature profile in the mushy layer. However, since the mushy layer temperature is expected to be very close to T_f , a single slope of temperature profile for the layer can be assumed in the present analysis. There exists a lack of relevant experimental data due to the difficulty of reliable temperature measurements over such a small distance within the mushy layer. For this reason, it is considered that the equilibrium freezing temperature in the mushy layer. Then, consider the temperature is at the freezing point,

$$T_i(B, t) = T_m(0, t) = T_f \quad (12)$$

Solving Eq. (11), subject to the boundary conditions,

$$T_m = T_f + \frac{(T - T_f)}{\left(b - \frac{k_m}{h}\right)} (y - B) \quad (13)$$

This represents a quasi-steady temperature profile of the mushy layer, whereby the mushy layer temperature profile varies with time since the growth of both ice layer, $B(t)$ and mushy layer, $b(t)$, are time dependent.

Now, the unknowns, $B(t)$ and $b(t)$, can be determined by the simultaneous solutions of Eq.(4) and Eq.(5).

To calculate $B(t)$, solving the energy balance equation, Eq.(4), at the ice/mushy layer interface, $y = B$, gives to

$$\frac{dB}{dt} = \frac{u_0}{B} - \frac{u_1}{b - \frac{k_m}{h}} \quad (14)$$

where,

$$u_0 = \frac{k_i(T_f - T_d)}{\rho_i L}$$

$$u_1 = \frac{k_m(T - T_f)}{\rho_i L}$$

First term on the right hand side represents the heat conduction through the ice. This indicates that the ice growth rate decreases with the increase of the temperature of the liquid droplet and the increase of the ice layer thickness. The second term on the right hand

side is proportional to u_1 which incorporates the mushy layer. It indicates that the mushy layer thickness plays to the ice growth rate; with the increase of the mushy layer, ice growth rate decreases. Again, as h decreases, the denominator of the second term becomes negligible and the ice growth rate decreases significantly. On the other hand, increase of h significantly has a little effect to the ice growth rate. This indicates the relation to the ice growth and the mushy layer.

Calculating $b(t)$, solving the equation, Eq.(4), gives to

$$\frac{db}{dt} = \frac{\alpha\gamma VG}{\rho_m} - \frac{\rho_i}{\rho_m} \frac{dB}{dt} \quad (15)$$

This equation shows that the mushy layer growth rate depends on the velocity of the liquid droplet and the growth rate of the ice layer. When ice growth rate increases, mushy layer growth rate decreases.

The results in Eq.(15) can now be substituted into Eq.(14) and subsequently solved for $B(t)$. In this way, Eqs.(14) and (15) are solved for $B(t)$ and $b(t)$ sequentially. Since the equations are non-linear, coupled differential equations, the solution will be obtained numerically.

3. RESULTS AND DISCUSSION

The variables considered in the study and their ranges are given in Table 1. The practical importance of this current study lies in its application to the ice slurry generator. The formation of ice and mushy layers are observed on super cooled liquid droplet injected under different temperatures. The development of ice and mushy layers around the liquid droplet depend on the initial injecting temperature, T_{di} . The following three cases of initial droplet temperatures are considered in this paper.

The results obtained for these three cases are illustrated in Figs. 2-5.

Case1:

$$T_{di} = -5^\circ C, h = 250 W / m^2 K, V = 0.21 m / s$$

Case2:

$$T_{di} = -10^\circ C, h = 250 W / m^2 K, V = 0.21 m / s$$

Case3:

$$T_{di} = -15^\circ C, h = 250 W / m^2 K, V = 0.21 m / s$$

Fig. 2 shows the growth of ice layer over a period of time when the initial liquid droplet temperatures, T_{di} are $-5^\circ C$, $-10^\circ C$, $-15^\circ C$ respectively.

For case 1, curve shows a rapid growth of ice layer for the first 0.5 seconds, after which the growth rate becomes almost steady. It continues to grow until the ice thickness is 0.828 mm at 3.5 sec. It is also observed that like case 1, ice growth rate for the first 0.5 seconds after injection of coolant is very fast, though maximum ice thickness is 2.089 mm for case3, while for case2, it is 1.439mm.

Table 1: Parameter values used for Figs.2-5

α	0.9	
C_d	1050	$J kg^{-1} K^{-1}$
D	0.004	m
G	1730	$kg m^{-3}$
γ	1.0	
h	250	$W m^{-2} K^{-1}$
k_i	2.18	$W m^{-1} K^{-1}$
k_m	0.52	$W m^{-1} K^{-1}$
ρ_d	1730	$kg m^{-3}$
ρ_i	917	$kg m^{-3}$
ρ_m	1000	$kg m^{-3}$
L	3.344×10^5	$J kg^{-1}$
T_f	-1	$^\circ C$
T	0	$^\circ C$
V	0.21	$m s^{-1}$

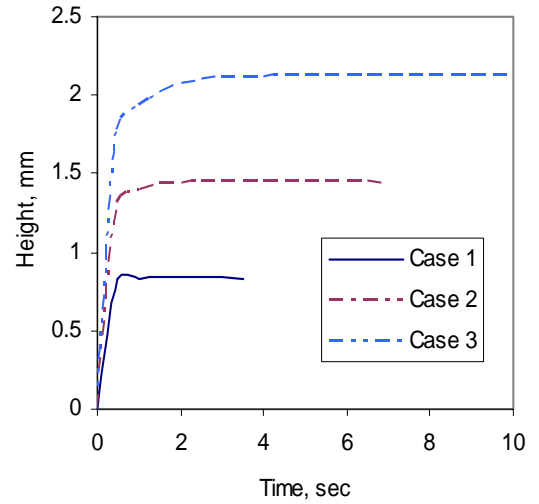


Fig 2. Ice layer growth for different T_d

Ice grows initially only around the injecting super cooled liquid droplet, after which both ice and mushy layer develops simultaneously. That is why ice growth rate becomes almost flat after few seconds of the operation.

Again, the differences among the ice thickness and growth rate time are due to the choice of different initial temperatures. As all other parameters are kept constant, ice growth rate is inversely proportional to the increase of liquid droplet temperature. However, this growth rate is not constant over temperatures. For decreasing temperature from $-5^\circ C$ to $-10^\circ C$, ice thickness increases by about 42%, whereas for temperature decreasing from $-10^\circ C$ to $-15^\circ C$, growth of ice thickness is about 31%.

Growth rate of mushy layer, consists of very fine ice particles and water, around the ice layer is shown in Fig.3. For all three cases, it is observed that growth of mushy layer increases linearly with time.

After initial ice growth, mushy layer begins to

develop around the ice layer. As a result, it is observed that ice layer growth rate becomes almost steady while the mushy layer grows linearly.

It is also observed from Fig. 3 that the thickness of the mushy layer is maximum (0.37mm) for the liquid droplet injecting at -15°C . It reveals that mushy layer thickness increases with the decrease of liquid droplet temperatures. In other words, mushy layer growth rate is proportional to the ice layer growth rate.

Fig. 4. shows that the liquid droplets temperatures increases exponentially, as stated in Eq.(1), which is due to the growth of ice layer and mushy layer simultaneously around it. For all three cases, there is a sharp decrease of liquid temperature after being steady for a while. This indicates that the termination of the surrounding ice and mushy layers growth around the liquid droplet at that time.

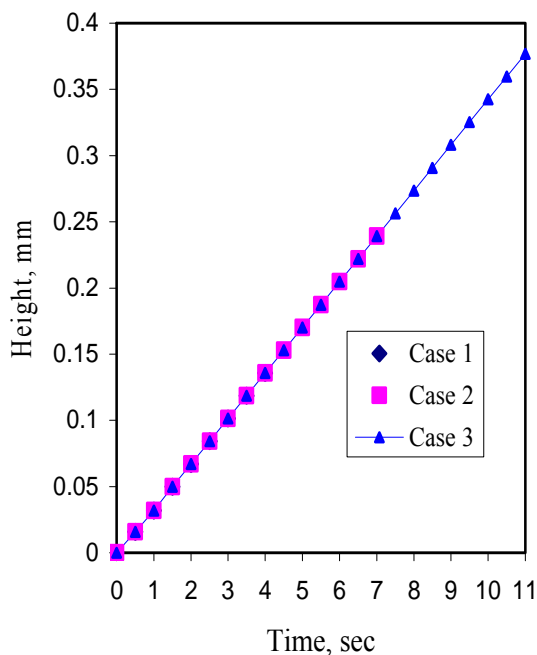


Fig 3. Meshy layer growth for different T_d

From the Fig. 4, it is also observed that the residence time is maximum (10 sec) for case 3, while it is minimum, 3.5 sec, for case 1. That is why it can be said that the residence times for growth of ice and mushy layers are proportional to the decreasing droplet temperatures.

Also, as observed from Fig. 5, there is a temperature gradient inside the mushy layer which ensures the presence of ice particles in that region.

Although this study has attempted to derive analytical model for ice layer and mushy layer buildup on the surface of liquid droplets, the sensitivity and dependence of this model on various coefficients should not be overlooked. For example, the convection coefficient (typically accurate within $\pm 25\%$). In spite of these limitations, this study gathers the influence of the initial temperature of the liquid droplet and generation of ice around it. As a result, a deeper physical insight of the growth of ice and mushy layers has been provided in this article.

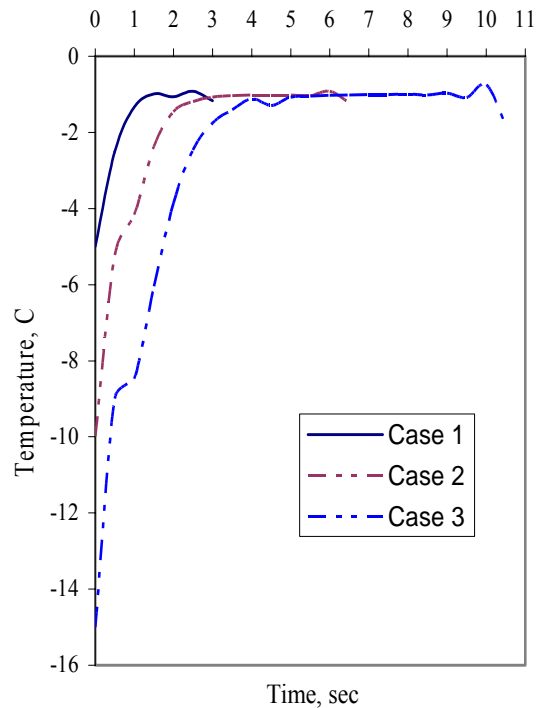


Fig 4. Liquid droplet temperatures at different conditions

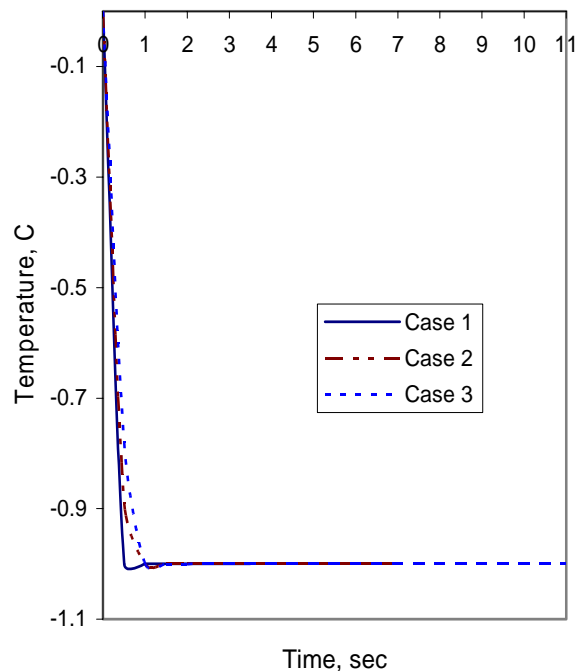


Fig 5. Mushy layer temperature

4. CONCLUSIONS

In this article, a simplified mathematical model has been developed to predict the simultaneous growth of ice and mushy layers around the surface of the liquid droplet. The analysis includes the solutions of lumped system inside the droplet, and conduction equations for thin ice and mushy layers. The results show that ice generation is faster just after dispersing from the nozzle while mushy layer growth around the ice layer is steady. Furthermore,

both ice and mushy layer thickness depend on the inlet coolant temperature. The ice slurry formation rate depends on the inlet coolant temperature. This model therefore provides significant information to understand the physical phenomena of ice slurry generation which may facilitate the development of improved design and operational guidelines for general implementations of the ice slurry cooling systems.

5. REFERENCES

1. Grumman DJ, Butkus AS., 1998, "The ice storage option", ASHRAE J, Vol.1, 29-33.
2. Braddy TW., 1994, "Achieving energy conservation with ice-based thermal storage", ASHRAE Trans, Vol.100(1), 1735-45.
3. Wijesundera N.E., and Jock L.C., 1996/97, "Design-simulation aspects of ice-based thermal energy storage systems", *DTG technology seminar series*, Singapore, 25-37.
4. Morikawa, H., Miyawaki, M., Fujimoto, T., and Aizawa, J., 1993, "Studies on Application of dynamic ice for district cooling (Part 1. Slurry ice transport system)", *Proceedings of JSME Conference*, No. 930-63, Vol. D, 94-96.
5. Snoek, C.W., 1994, "Ice-slurry based district cooling systems", *Proceeding of European Institute of Environmental Energy District Cooling Workshop*, 1-12.
6. Ure Z, Mashrae M., 1999, "Slurry ice based cooling systems", *Proc. of IIR 20th INT. CONF.*, 5: Paper No.3.
7. Kiatsiriroat T., Sirplubpla P., and Nuntaphan A., 1998, "Performance analysis of refrigeration cycle using a direct contact evaporator", *International Journal of Energy Research*, Vol. 22, 1179-1190.
8. Matsumoto K., Okada M., Kawagoe T., and Kang C., 2003, "Ice storage system with water-oil mixture formation of suspension with high IPF", *International Journal of Refrigeration*, Vol. 23, 36-344.
9. Kiatsiriroat T., Vithayasai S., Vorayos N., Nuntaphan A. and Vorayos N., 2003, "Heat Transfer prediction for a direct contact ice thermal energy storage", *Energy Conversion and Management*, Vol. 44, 497-508.
10. Chen S.L., Wang P.P., and Lee T.S., 1999, "An experimental investigation of nucleation probability of supercooled water inside cylindrical capsules", *Experimental thermal and fluid science*, Vol.18, 299-306.
11. Song M., Steiff A. and Weinspach P.M., 1996, "The analytical solution for a model of direct contact evaporation in spray columns", *Int. Comm. Heat Mass Transfer*, Vol.23, No. 2, 263-272.
12. Wijesundera N.E., M.N.A. Hawlader, C. W. B. Andy, M.K. Hossain, 2004, "Ice-slurry production using direct contact heat transfer", *Int. Journal of Refrigeration*, Vol. 27, 511-519.
13. J.M. Hill, 1987, *One-dimensional Stefan Problems: An Introduction*, Longman Sc.Tech. Press.
14. Naterer, G.F., 2003, "Coupled liquid film and solidified layer growth with impinging supercooled droplets and Joule heating", *Int. Journal of Heat and Fluid Flow*, Vol. 24, 223-235.
15. Myers, T.G., Hammond, D.W., 1999, "Ice layer film growth from incoming supercooled droplets", *International Journal of Heat and Mass Transfer* 42, 2233-2242.

6. NOMENCLATURE

Symbol	Meaning	Unit
B	thickness of ice layer	(m)
b	thickness of mushy layer	(m)
C	specific heat	(J kg ⁻¹ K ⁻¹)
D	drop diameter	(m)
G	liquid content	(kg m ⁻³)
h	convective heat transfer co-efficient	(W m ⁻² K ⁻¹)
k	thermal conductivity	(W m ⁻¹ K ⁻¹)
L	latent heat	(J kg ⁻¹)
T	temperature of layers	(°C)
t	time	(sec)
V	velocity of liquid droplet	(m s ⁻¹)
Greek symbols		
α	cooling factor	
γ	surface curvature factor	
λ	time constant	(s ⁻¹)
ρ	density	(kg m ⁻³)
Subscripts		
d	droplet	
m	mushy layer	
i	ice layer	
f	ice/mushy layer interface	