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# WALL EFFECT ON HEAT TRANSFER PAST A CIRCULAR CYLINDER - A NUMERICAL STUDY

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## ABSTRACT
This paper describes a numerical study on the unsteady laminar flow of an incompressible Newtonian fluid past a circular cylinder near a plane boundary using vorticity - stream function formulation. Thompson, Thames, Mastin (TTM) method of generation of automatic boundary fitted co-ordinate generation system is used to construct the grids of the physical flow field. The vorticity-transport equation, stream function equation and energy equations are first non dimentionalized and then transformed to computational domain In this study, finite difference implicit method is used to solve the transformed governing equations. The solutions are carried out for Reynolds numbers in the range between 20 to150 and gap ratio G/D (ratio of gap to diameter) in the range between 0.3 and 1.0. The flow parameters such as vorticity, stream function, velocity and temperature are calculated by providing appropriate boundary conditions for the same for different Reynolds numbers and different gap ratios. With these flow parameters, effect of flow field on the heat transfer around the cylinder is investigated. Nusselt number around the cylinder and average Nusselt number are shown as the function of Reynolds number and gap ratio. From this study it is found that heat transfer increases with increase in Reynolds Number. Mean Nusselt number increases with decrease in gap ratio.

**Keywords:** Incompressible Flow, Vorticity, Stream Function, Circular Cylinder, Gap Ratio, Finite Difference Method, Plane Wall

## 1. INTRODUCTION
The incompressible flow of Newtonian fluids past a circular cylinder represents classical problem in fluid mechanics. Several works have been carried out both numerically and experimentally for flow past an isolated cylinder in infinite flow field. A vast body of literature containing experimental, numerical and analytical studies is available on the flow past a circular cylinder [1-2].

For flow past a circular cylinder, the flow is characterized by the cylinder diameter (D), the free stream velocity (U) and the Reynolds number (Re). It is well established that the separation of boundary layer on the cylinder surface begins at Reynolds number equal to 5 [3]. Between Reynolds numbers 10 to 40 a pair of steady symmetric vortices develop behind the cylinder and the re-circulation zone length grows linearly with increase in Reynolds number. The vortex shedding occurs for Reynolds number above 49. The vortex shedding flow remains laminar for Reynolds number up to around 150 [4]. Transition to three -dimensional flow starts at Reynolds number of around 180-194 depending on experimental condition and ends at Reynolds number equal to about 260 at which fine scale three - dimensional eddies appear [5].

The flow of viscous incompressible fluid past a circular cylinder in an unconfined domain was first studied systematically by Kawaguti and Jain [6] for Reynolds number ranges from 1 to 100. The steady state solution was numerically obtained up to a Reynolds number of 50. In their results the streamlines and iso-vorticity contours along with the drag co-efficient and the angle of separation, length of the standing vortex and surface pressure distribution were computed as a function of Reynolds number and time.

Karniadakis [7] investigated forced convection heat transfer from an isolated cylinder in cross flow for Reynolds numbers up to 200 by direct numerical solution. In their study they presented spatial structure of von karman vortex street, the unsteady lift and drag co-efficient and unsteady local heat transfer co-efficient.

However, when a circular cylinder is placed near a plane wall, the separation and wake development will depend on the Reynolds number, the gap ratio and the characteristics of the wall boundary layer of the wall. Various studies mostly experimental have been done over the past few decades [8-9]. Most of the experiments were carried out at Reynolds number in the sub critical regime. The flow regime that is relatively insensitive to Reynolds number is selected for the experiments. The effect was investigated thoroughly without complicating influence of the Reynolds number. Taneda [10] carried out an experiment at low Reynolds number up to 170.

At the present time, very few numerical studies have been reported for the flow past a circular cylinder near a plane wall particularly during the transient process. Lei et al [11] investigated vortex shedding near a plane wall for different gap ratios and different Reynolds numbers ranging from 80 up to 1000.They observed vortex shedding phenomenon using various methods. They showed the relation of Reynolds number and gap ratio at which suppression of vortex shedding occurs.

In summary, majority number of works done as discussed above is made of the isothermal flow past a circular cylinder. More work is needed to be done to achieve a better understanding regarding heat transfer from a cylinder near a plane wall in cross flow. In our present study a non isothermal case is considered for flow past a circular cylinder in the vicinity of a plane wall. In this study a hot fluid is considered to flow past a cold circular cylinder near a plane wall. In this study, the two dimensional Navior-Stokes equations for time-dependent, viscous and incompressible flows along with continuity equation and energy equation are solved using vorticity - stream function formulation by finite difference method. Velocity components are obtained from stream function in each time step. The temperature field is obtained after solving Energy equation. The local Nusselt number and average Nusselt number around the cylinder is calculated form temperature field. The grid in this study is generated numerically by solving Laplace equations as the generating system. The numerical method developed in this study has been validated against the bench mark problem of an uniform flow past an isolated circular cylinder. With this numerical procedure the vortex shedding flow across circular cylinder near a plane wall at different gap ratios is investigated for different Reynolds numbers up to 150.The effect of Reynolds numbers in fluid flow and heat transfer is analyzed and results are presented for various gap ratios.

## 2. GOVERNING EQUATIONS AND NUMERICAL METHOD

The fundamental equations for two dimensional incompressible unsteady viscous flow with no body forces in Cartesian co-ordinate system are,

(A) Continuity equation:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (1)$$

(B) X-momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{x}} + \gamma\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) \quad (2)$$

(C) Y-momentum equation:

$$\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{y}} + \gamma\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) \quad (3)$$

(D) Energy equation:

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho c_p}\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right) \qquad (4)$$

From equation (2) & (3) pressure term is eliminated with the help of equation (1).

Again by defining vorticity (ω) as

$$\bar{\omega} = \frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}} \qquad (5)$$

Vorticity - transport equation can be written as

$$\frac{\partial \bar{\omega}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{\omega}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{\omega}}{\partial \bar{y}} = \nu\left(\frac{\partial^2 \bar{\omega}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{\omega}}{\partial \bar{y}^2}\right) \qquad (6)$$

Now by defining stream function (ψ) by

$$\frac{\partial \bar{\Psi}}{\partial \bar{y}} = \bar{u} \qquad (7a)$$

$$\frac{\partial \bar{\Psi}}{\partial \bar{x}} = -\bar{v} \qquad (7b)$$

and with the help of equation (5) stream function equation is derived as

$$\frac{\partial^2 \bar{\Psi}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{\Psi}}{\partial \bar{y}^2} = -\bar{\omega} \qquad (8)$$

Therefore the equations which are solved in this study for unsteady incompressible viscous flow are equations (6), (8) & (4).

The non-dimensional form of these equations in Cartesian co-ordinates are written as

Vorticity-Transport equation:

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) \qquad (9)$$

Stream function equation:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\omega \qquad (10)$$

Velocities:

$$\frac{\partial \Psi}{\partial y} = u \qquad (11a)$$

$$\frac{\partial \Psi}{\partial x} = -v \qquad (11b)$$

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Re\ Pr}}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (12)$$

In order to solve equations (9), (10), (11) & (12) in a curvilinear co-ordinate system, the co-ordinate transformation considered is

$$\xi = \xi(x, y) \qquad (13a)$$

$$\eta = \eta(x, y) \qquad (13b)$$

Here $(\xi,\eta)$ is the co-ordinate system in the computational plane. With the above transformation the non dimensional equations can be transformed into the computational co-ordinates $(\xi,\eta)$ as follows.

Vorticity-transport equation in the computational plane is rewritten as

$$\alpha\omega_{\xi\xi}+2\beta\omega_{\xi\eta}+\gamma\omega_{\eta\eta}+\delta\omega_{\xi}+\varepsilon\omega_{\eta}=\mathrm{Re}[u(\omega_{\xi}\xi_x+\omega_{\eta}\eta_x)+v(\omega_{\xi}\xi_y+\omega_{\eta}\eta_y)]+\mathrm{Re}\,\omega_t \qquad (14)$$

The stream function equation in the computational plane is rewritten as

$$\alpha\psi_{\xi\xi}+2\beta\psi_{\xi\eta}+\gamma\psi_{\eta\eta}+\delta\psi_{\xi}+\varepsilon\psi=-\omega \qquad (15)$$

Velocities are rewritten as

$$u=[\xi_y\psi_{\xi}+\eta_y\psi_{\eta}] \qquad (16a)$$
$$v=-[\xi_x\psi_{\xi}+\eta_x\psi_{\eta}] \qquad (16b)$$

and the energy equation in the computational plane is rewritten as

$$\alpha T_{\xi\xi}+2\beta T_{\xi\eta}+\gamma T_{\eta\eta}+\delta T_{\xi}+\varepsilon T_{\eta}=\mathrm{Re\ Pr}[u(T_{\xi}\xi_x+T_{\eta}\eta_x)+v(T_{\xi}\xi_y+T_{\eta}\eta_y)] \qquad (17)$$

Where subscripts denote differentiation.
The co-efficients are given by

$$\alpha = \xi_x^2+\xi_y^2 \qquad (18)$$
$$\beta = \xi_x\eta_x+\xi_y\eta_y \qquad (19)$$
$$\gamma = \eta_x^2+\eta_y^2 \qquad (20)$$
$$\delta = \xi_{xx}+\xi_{yy} \qquad (21)$$
$$\varepsilon = \eta_{xx}+\eta_{yy} \qquad (22)$$
$$u = (\xi_y\psi_{\xi}+\eta_y\psi_{\eta}) \qquad (23)$$
$$v = (\xi_x\psi_{\xi}+\eta_x\psi_{\eta}) \qquad (24)$$

The above equations are solved by finite difference implicit method.

## 3. PHYSICAL FLOW FIELD AND NUMERICAL PARAMETERS

In this study, a rectangular flow field is considered as shown in fig.1. Four types of G/D ratio, G/D= 0.3, 0.5, 0.8 and 1.0 is considered. Horizontal position of the cylinder is considered at a distance of 5D from inlet. Top boundary is set to 11D distance from bottom wall and outlet boundary is set at 27D distance from the cylinder centre.

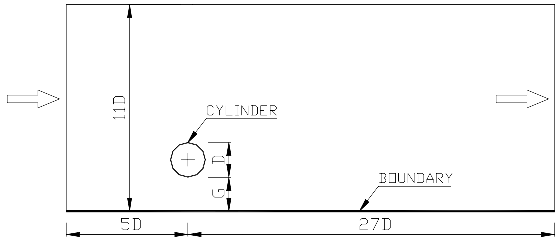


Fig 1. Layout of Physical Flow Field

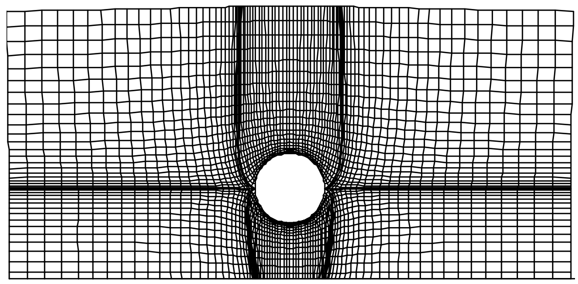

Fig 2. Mesh around the cylinder for G/D=0.8

In this study, 141x60 grids with 72 nodes on cylinder surface are considered as these grids are found to be adequate in solving the present problem. The grids near the cylinder is shown in fig. 2 for G/D=0.8.

In this study, calculations are done with a non-dimensional time step of t = 0.005. The vorticity transport equation, stream function equation and energy equation are solved through second order central difference.

## 4. BOUNDARY CONDITIONS

Vortcity-Stream function formulation using finite difference method needs careful representation of boundary conditions for vorticity and stream function at all boundaries. Both Dirichlet and Neumann boundary conditions are used in boundaries wherever applicable. At inlet uniform longitudinal velocity and zero transverse velocity is selected. At no-slip boundary of wall and cylinder surface zero velocity components are considered. Boundary conditions for vorticity and stream function as follows.

Boundary condition for vorticity: -

$\omega = 0$ at inlet, bottom and top boundary (25)

$\partial\omega/\partial x = 0$ at outflow boundary (26)

Boundary condition for stream function: -

$\psi_{i,j} = \int u dy$ at inlet boundary. (27)
$\partial^2\psi/\partial x^2 = 0$ at outlet boundary (28)
$\partial\psi/\partial y = u = 1$ at top boundary (29)
$\psi = 0$ at bottom boundary. (30)

Boundary condition for vorticity and stream function at cylinder surface and wall :
The no penetration boundary condition for u indicates that at cylinder surface $\psi_{cyl}$ = constant (31)
while no-slip boundary condition for u at cylinder surface shows that

$$\frac{\partial \psi}{\partial n} = 0 \text{.......... .......... .......... .......... ...} \quad (32)$$

and

$$\int_{cyl} \frac{\partial \omega}{\partial n} = 0 \text{.......... .......... .......... .......... ...} \quad (33)$$

the value of $\psi_{cyl}$ is constant , which is need to be updated at every time step.
Again since the Stream function is uniquely determined up to a constant, the constant at the no-slip wall can be set as zero. Thus
$\Psi_{wall} = 0$ (34)

Boundary condition for Temperature: -
T=1 at inlet boundary
T=0 at cylinder surface
$\partial T/\partial y = 0$ at top and bottom boundary
$\partial T/\partial x = 0$ at outlet boundary

## 5. NUSSELT NUMBER

The heat transfer co-efficient or local Nusselt number around the cylinder for each gap ratio and different Reynolds number is determined by the following expression,

Local Nusselt number $Nu_\theta = \partial T/\partial R_{(R,\theta)} \;_{R=D/2}$ (35)

The local Nusselt number varies with the angle around the cylinder . The angle is measured clockwise from the forward stagnation point as shown in fig.3.

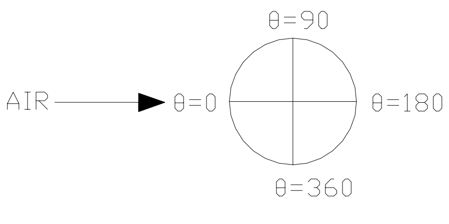


Fig 3.Position of angle on cylinder surface.

Mean Nusselt number around the cylinder is determined from the following expression :

$$Nu_m = \frac{1}{2\pi}\int_0^{2\pi} Nu \; d\theta \quad (36)$$

## 6. RESULTS AND DISCUSSION

From the governing equations it is seen that the heat transfer and fluid flow characteristics depends upon Reynolds number and Prandtl number. In this present paper the fluid is considered to be air with a Prandtl number 0.705. In the following section, it is proposed to initiate discussion first on the transient aspects of the flow past the cylinder in an unbounded domain and thereafter, the effect of the presence of a near-wall boundary on the same along with the flow structures will be considered for different gap ratios.

### 6.1 Vortex Shedding

At moderate Reynolds number greater than approximately 49 and up to 150 the flow remains laminar and two dimensional vortex shedding, also known as Karman vortex street occurs. At a Reynolds number of 150, typical streamline patterns of the unsteady flow for an isolated cylinder in a cross flow for a time cycle is shown in fig.4. It is seen that at every time period T, there is a natural shedding.

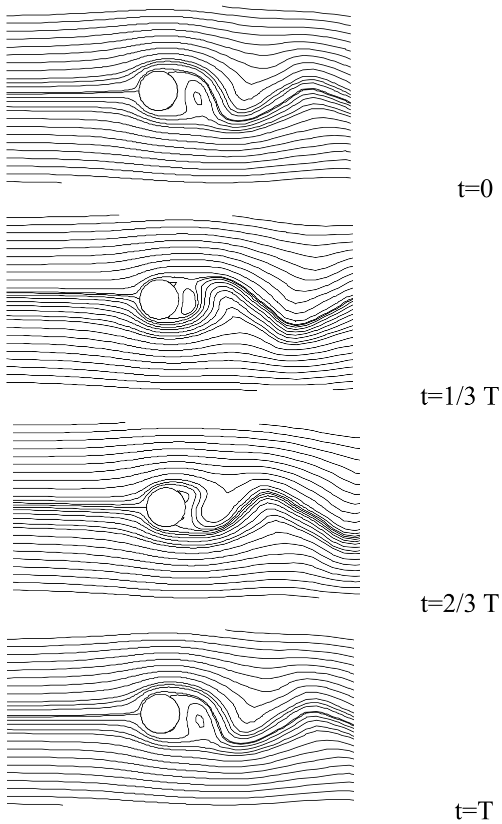


Fig 4. The vortex shedding in a time cycle for Re =150 for an isolated cylinder

Strouhal number which is a measure of the vortex shedding frequency is obtained by taking the inverse of the non-dimensional time period. In the present study, Strouhal number are deduced from the time-evaluation of the average Nusselt number.

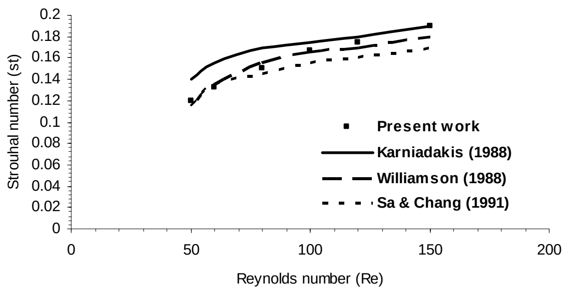


Fig 5. Strouhal number for an isolated cylinder in the of Reynolds number 50-150.

Fig.5 shows the Strouhal number St for various Reynolds number in the range of 50-150. The results are compared with the experimental results of Karniadakis [7], Williamson [12 ]and Sa and Chang[13]. Good agreement is observed between the results.

## 6.2 Transient Flow Field Development Around A Cylinder Near A Plane Wall

For the purpose of discussion a Reynolds number of 150 has been chosen. This is due to the fact that at still lower number i.e. say around 49, the vortex shedding at the wake of the cylinder does not occur even for an isolated cylinder and a steady state analysis of the flow regime appears to be sufficient. The result is obtained for G/D=0.5 where G is the gap between the cylinder and wall and D is the diameter of the cylinder. A typical streamline pattern of the unsteady flow for Reynolds number 150 and G/D = 0.5 is shown in fig. 6.

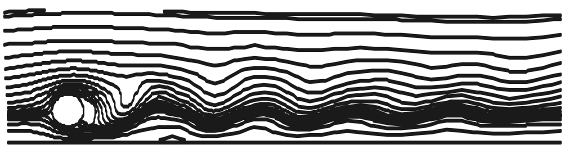

Fig 6. The von karman vortex street for Re=150 & G/D=0.5

Fig. 7 presents the time evaluation of the stream function values on the cylinder surface ($\psi_{cyl}$) at different gap ratios for the Reynolds number Re=150. It is seen that when the cylinder is located far from the wall (G/D=1.0), the surface stream function oscillates regularly. When the cylinder is moved closer to the wall (G/D<1), the amplitude of the surface stream function values decreases gradually, indicating the weakness of vortex shedding.

In fig. 8 variation of local Nusselt number around the cylinder surface is shown for different gap ratios for Re=150. It is observed that the local Nusselt number at upper half of the cylinder nearly constant for all gap ratios. The local Nusselt number at lower half of the cylinder varies as the cylinder comes close to the wall

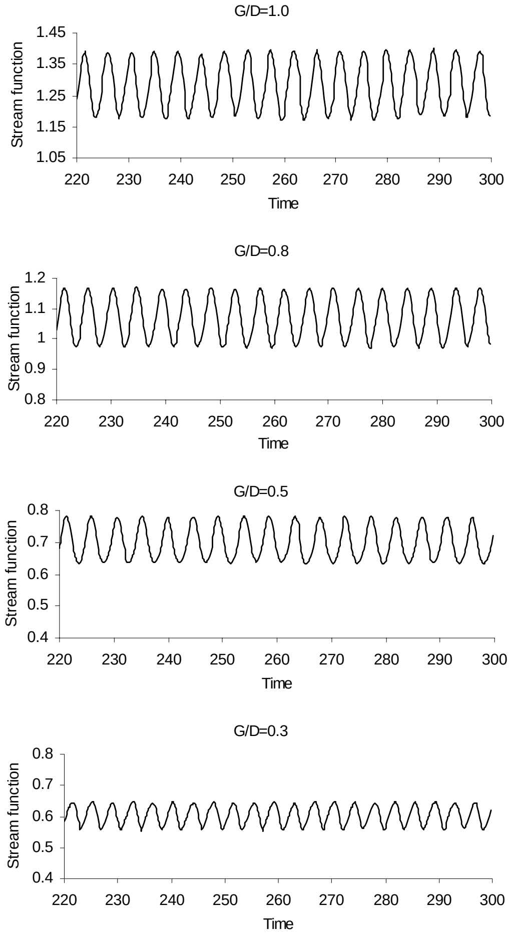


Fig 7. Time evaluation of the cylinder surface stream function at different gap ratios (Re=150).

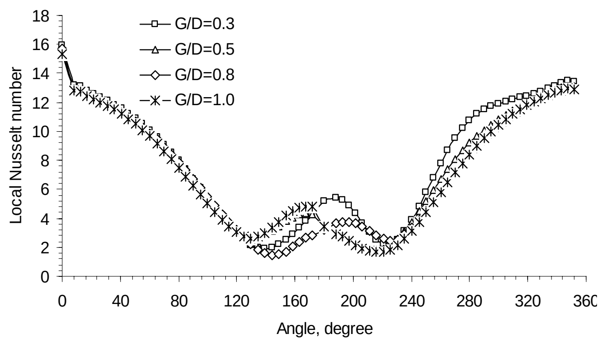


Fig 8. Variation of local Nusselt number around the cylinder surface for different gap ratios and Re=150.

The local Nusselt number ($Nu_\theta$) increases in the lower half of the cylinder surface as the gap ratio decreases. The influence of lateral wall seems to increase the velocity of flow and force it to pass through the gap. The increase in velocity results in an increase in heat transfer at lower half of the cylinder. The increase in Nusselt number is more prominent at angles between 240$^o$ to 320$^o$.

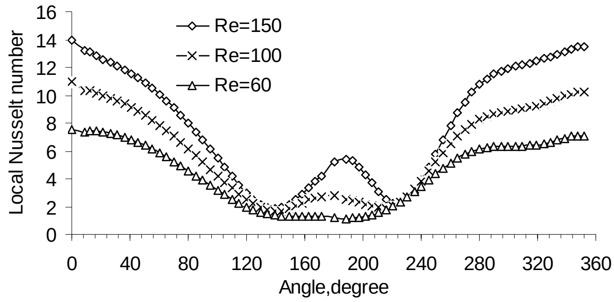


Fig 9. Variation of local Nusselt number around the cylinder surface for different Re at G/D=0.3.

Fig. 9 shows the local Nu around the cylinder for Re=60,100,150 at G/d=0.3. It is observed that variation of local Nu at upper half and lower half of the cylinder decrease as the Re decreases.

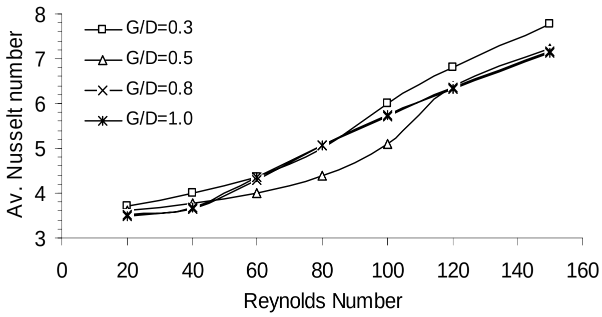


Fig 10. Variation of average nusselt number with Reynolds number for different gap-ratios.

Fig. 10 presents the variation of average Nusselt number in the range of Reynold number 20-150 for different gap-ratios G/D. It is seen that for the gap-ratio of 0.3, average Nusselt number increases prominently above Re=100. The variation Nusselt number almost same for G/D=0.8 and G/D=1.0. At this gap ratios variation is seen to be less sensitive to the Reynolds number.

For the gap-ratio of 0.5 and Reynolds number up to 120, the nusselt number variation is decreased than other gap ratios follows the similar trends to that of the gap-ratio for 0 .2. No vortex shedding is observed up to this Reynolds number. However, average nusselt number is found to increase sharply for Reynolds number greater than 100.

## 7. CONCLUSIONS

In this numerical study, heat transfer for non-isothermal flow past a circular cylinder in the vicinity of a plane wall is considered. An implicit finite difference method is used and solution to the two-dimensional Navier-Stokes equation and Energy equation for laminar unsteady incompressible viscous flows has been obtained. The flow is calculated for Reynolds Numbers range from 20 to 150 . Based on the numerical solutions , effect of Reynolds Numbers and gap ratio on heat transfer are investigated.

The major findings are summarized below.

**a.** Heat transfer increases with increase in Reynolds number.

**b.** Mean Nusselt number increases with decrease in gap ratio.

**c.** As the cylinder moves towards the wall, local Nusselt number at lower half of the cylinder shows higher values than that from the upper half.

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