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EFFECTS OF CONDUCTION AND CONVECTION ON MAGNETO HYDRODYNAMIC FLOW WITH VISCOUS DISSIPATION FROM A VERTICAL FLAT PLATE

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ABSTRACT
The effects of conduction and convection on magneto hydrodynamic (MHD) boundary layer flow with viscous dissipation from a vertical flat plate of thickness 'b' have been investigated. The governing equations are made dimensionless by using suitable transformations. The dimensionless equations are then solved numerically in the entire region starting from the lower part of the plate to the down stream by means of implicit finite difference method known as Keller box scheme. The dimensionless skin friction co-efficient, the surface temperature distribution, the velocity distribution and the temperature profile over the whole boundary layer are shown graphically for different values of the magnetic parameter *M*, the viscous dissipation parameter *N* and the Prandtl number *Pr*.

Keywords: Magneto hydrodynamic, Free convection, Viscous dissipation.

1. INTRODUCTION

Free convection flow is often encountered in cooling of nuclear reactors or in the study of the structure of stars and planets. Along with the free convection flow the phenomenon of the boundary layer flow of an electrically conducting fluid up a vertical flat plate in the presence of a strong magnetic field is also very common because of its application in nuclear engineering in connection with the cooling of reactors. Gebhart [1] has shown that the viscous dissipation effect plays an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotative speeds and also in strong gravitational field processes on large scales (on large planets) and in geological processes. With this understanding, Takhar and Soundalgekar [2] studied the effects of viscous and Joule heating on the problem posed by Sparrow and Cess[3], using the series expansion method of Gebhart [1]. Hossain [4] have studied the effect of viscous and Joule heating on the flow of an electrically conducting and viscous incompressible fluid past a semi infinite plate of which temperature varies linearly with the distance from the leading edge and in the presence of uniform transverse magnetic field. He has solved the equations numerically governing the flow applying the finite difference method along with Newton's linearization approximation. Here the viscous dissipation effect on the skin friction and the rate of heat transfer in the entire region from up stream to down stream of 0a viscous incompressible and electrically conducting fluid from a vertical flat plate in presence of transverse magnetic field will be investigated.

2. GOVERNING EQUATIONS OF THE FLOW

The steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a side of a vertical flat plate of thickness 'b' in the presence of a uniformly distributed transverse magnetic field is considered. The flow configuration and the coordinates system are shown in Fig. 1

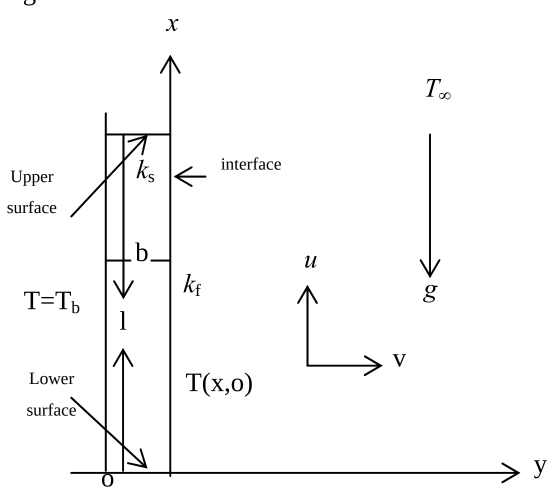

Fig 1. Physical model and coordinate systems.

The equations governing the flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)-\frac{\sigma H_0{}^2 u}{\rho} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa_f}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

The appropriate boundary conditions to be satisfied by the above equations are

$$u=0, v=0 \text{ at } y=0 \qquad (3a)$$

$$u\to 0, T\to T_\infty \text{ as } y\to\infty \qquad (3b)$$

The temperature and the heat flux are required continuous at the interface for the coupled conditions and at the interface we must have

$$\frac{k_s}{k_f}\frac{\partial T_s}{\partial y}=\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (4)$$

where k_s and k_f are the thermal conductivity of the solid and the fluid respectively. The temperature T_s in the solid as given by Pozzi and Lupo [5] is

$$T_s=T(x,0)-\left\{T_b-T(x,0)\right\}\frac{y}{b} \qquad (5)$$

where $T(x,0)$ is the unknown temperature at the interface to be determined from the solutions of the equations.

We observe that the equations (2) - (3) together with the boundary conditions (3) - (5) are non linear partial differential equations. In the following the solution methods of these equations are discussed.

3. TRANSFORMATION OF THE GOVERNING EQUATIONS

Equations (2) – (3) may now be nondimensionalized by using the following dimensionless dependent and independent variables:

$$\bar{x}=\frac{x}{L}, \bar{y}=\frac{y}{L}d^{\frac{1}{4}}, u=\frac{\nu}{L}d^{\frac{1}{2}}\bar{u}, \quad v=\frac{\nu}{L}d^{\frac{1}{2}}\bar{v}, \frac{T-T_\infty}{T_b-T_\infty}=\theta, L=\frac{\nu^{2/3}}{g^{1/3}}, \quad d=\beta\left(T_b-T_\infty\right) \qquad (6)$$

Introducing (6) in equations (2) and (3) we get the following dimensionless equations.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta \qquad (7)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+N\left(\frac{\partial u}{\partial y}\right)^2 \qquad (8)$$

where $M=\frac{\sigma H_0{}^2 L^2}{\mu d^{1/2}}$, dimensionless magnetic parameter, $P_r=\frac{\mu C_p}{\kappa}$, Prandtl number and $N=\frac{\nu^2 d}{L^2 c_p\left(T_b-T_\infty\right)}$, dimensionless viscous dissipation parameter.

The corresponding boundary conditions (3) - (5) take the following form:

$$u=v=0,\ \theta-1=p\frac{\partial\theta}{\partial y} \text{ at } y=0 \qquad (9a)$$

$$u\to 0,\ v\to 0 \text{ as } y\to\infty \qquad (9b)$$

where p is the conjugate conduction parameter given by $p=\left(\frac{\kappa_f}{\kappa_s}\right)\left(\frac{b}{L}\right)d^{1/4}$

Here the coupling parameter 'p' governs the described problem. The order of magnitude of ‘p’ depends actually on $\frac{b}{L}$ and $\frac{\kappa_f}{\kappa_s}$, $d^{1/4}$ being the order of unity. The term $\frac{b}{L}$ attains values much greater than 1 because of L being small. In case of air, $\frac{\kappa_f}{\kappa_s}$ becomes very small when the vertical plate is highly conductive i.e. $\kappa_s >> 1$ and for materials, $O\left(\frac{\kappa_f}{\kappa_s}\right)=0.1$ such as glass. Therefore in different cases 'p' is different but not always a small number. In the present investigation we have considered $p=1$ which is accepted for $\frac{b}{L}$ of $O\left(\frac{\kappa_f}{\kappa_s}\right)$.

To solve the equations (7) – (8) subject to the boundary conditions (9), the following transformations are introduced for the flow region starting from up stream to down stream.

$$\psi=x^{4/5}(1+x)^{-1/20}f(\eta,x), \quad \eta=yx^{-1/5}(1+x)^{-1/20}, \quad \theta=x^{1/5}(1+x)^{-1/5}h(\eta,x) \qquad (10)$$

Here η is the dimensionless similarity variable and ψ is the stream function which satisfies the equation of continuity and $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$ and $h(\eta,x)$ is the dimensionless temperature.

Substituting (10) into equations (7) and (8) we get the following transformed non-dimensional equations.

$$f'''+\frac{16+15x}{20(1+x)}ff''-\frac{6+5x}{10(1+x)}f'^2-Mx^{2/5}(1+x)^{1/10}f'$$

$$+h=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \quad (11)$$

$$\frac{1}{P_r}h''+\frac{16+15x}{20(1+x)}fh'-\frac{1}{5(1+x)}f'h + Nxf''^2=x\left(f'\frac{\partial h}{\partial x}-h'\frac{\partial f}{\partial x}\right) \quad (12)$$

In the above equations the primes denote differentiation with respect to η.

The boundary conditions (9) then take the following form

$$\begin{aligned} &f(x,0)=f'(x,0)=0,\\ &h'(x,0)=-(1+x)^{1/4}+x^{1/5}(1+x)^{1/20}h(x,0) \quad (13)\\ &f'(x,\infty)=0,\, h'(x,\infty)=0 \end{aligned}$$

4. METHOD OF SOLUTION

To get the solutions of the parabolic differential equations (11) and (12) along with the boundary condition (13), we shall employ a most practical and accurate solution technique, known as implicit finite difference method together with Keller- box elimination technique. To apply the method, we first discretize the equations (11) and (12) along with the boundary condition (13) by the simple implicit finite difference scheme, similar to that used by Keller and Cebeci [7] to a system of non linear algebraic equations. These non linear systems of algebraic equations are then linearized by means of Newton's quasi-linearization scheme. The system of linear algebraic equations are solved by using Keller box method which has been most efficiently by Cebeci and Bradshw[8] and very recently by Hossain[4].

5. RESULTS AND DISCUSSION

Here we have investigated the problem of the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid with viscous dissipation along a side of a vertical flat plate of thickness 'b' in the presence of a uniformly distributed transverse magnetic field. Solutions are obtained for the fluid having Prandtl number Pr = (0.05, 0.73, 1.0) and for a wide range of the values of the viscous dissipation parameter N = (0.01, 0.1, 0.3, 0.5, 0.6, 0.8, 1.0) and the magnetic parameter M = (0.1, 0.2, 0.5, 0.8, 1.0). If we know the values of the functions f (η, x), h (η,x) and their derivatives for different values of the Prandtl number Pr, the magnetic parameter M and the viscous dissipation parameter N, we may calculate the numerical values of the surface temperature θ (0, x) and the skin friction coefficient f'' (0, x) at the surface that are important from the physical point of view.

Numerical values of the skin friction coefficient f'' (0, x) and the surface temperature θ (0, x) are depicted graphically in Fig. 2 and Fig. 3 respectively against the axial distance x in the interval [0, 30] for different values of the viscous dissipation parameter N (= 0.3, 0.6, 0.8, 1.0) for the fluid having Prandtl number Pr = 0.73 and the magnetic parameter M=1.0. From Fig. 2, it can be observed that increase in the value of the viscous dissipation parameter N leads to increase the value of the shear stress coefficient f'' (0,x) which is usually expected. Again Fig. 3 shows that the increase of the viscous dissipation Parameter N leads to increase the surface temperature θ (0, x). In Fig. 4 and Fig. 5, the shear stress coefficient f'' (0, x) and the surface temperature θ (0, x) are shown graphically for different values of the Prandtl number Pr (= 0.05, 0.73, 1.0) when value of the magnetic parameter M is 0.1 and the viscous dissipation parameter N = 0.01. The values of the Prandtl number Pr are taken to be 0.05 that corresponds physically the sodium, 0.73 that corresponds the air and 1.0 corresponding to electrolyte solutions such as salt water. From Fig. 4, it is shown that the shear stress coefficient f'' (0, x) decreases with the increase of the Prandtl number Pr (= 0.05 , 0.73 , 1.0) and from the Fig. 5, the same result is observed on the surface temperature distribution due to increase of the value of the Prandtl number. Similar results are observed from Fig. 6 and Fig. 7 for skin friction f'' (0,x) and surface temperature distribution θ(0, x) respectively for different values of the magnetic M when Prandtl number Pr = 0.73 and viscous dissipation parameter N = 0.5. Fig. 8 and Fig. 9 deal with the effect of the viscous dissipation parameter N (= 0.1, 0.5, 0.8, 1.0) for Pr = 0.73 and for the magnetic parameter M = 1.0 on the velocity profile f'(η ,x) and the temperature profile θ (η, x) . From Fig. 8, it is revealed that the velocity profile f'(η,x) increases slowly with the increase of the viscous dissipation parameter N which indicates that viscous dissipation accelerates the fluid motion. Small increment is shown from Fig. 9 on the temperature profile θ(η,x) for increasing values of N. Fig. 10 depicts the velocity profile for different values of the Prandtl number Pr (= 0.05, 0.73, 1.0) while the magnetic parameter M is 0.1 and the viscous dissipation parameter N = 0.01. Corresponding distribution of the temperature profile θ (η, x) is shown in Fig. 11. From Fig. 10, it can be seen that if the Prandtl number increases, the velocity of the fluid decreases. On the other hand, from Fig. 11 it is observed that the temperature profile decreases within the boundary layer due to increase of the Prandtl number Pr. From Fig. 12 we observe that the velocity profiles decrease with the increase of the magnetic parameter M when the values of the Prandtl number Pr and the dissipation parameter N are respectively 0.73 and 0.5. Opposite result is shown in Fig. 13 for temperature distribution for the same values of the magnetic parameter M.

6. CONCLUSIONS

The effects of viscous dissipation and magnetic parameters N and M respectively for small Prandtl number Pr (= 0.05, 0.73, 1.0) on the magneto hydrodynamic (MHD) natural convection boundary layer flow with viscous dissipation from a vertical flat plate has been studied introducing a suitable transformations. The transformed non similar boundary layer equations governing the flow together with the boundary conditions based on conduction and convection were solved numerically using the implicit finite difference method together with Keller box scheme. The coupled effect of natural convection and conduction requires that the temperature and the heat flux be continuous at the interface.

From the present investigation, the following conclusions may be drawn:

1. The skin friction coefficient and the velocity distribution increase for increasing value of the viscous dissipation parameter *N*.
2. Increased value of the viscous dissipation parameter *N* leads to increase the surface temperature distribution as well as the temperature distribution.
3. The skin friction coefficient, the surface temperature distribution, the temperature distribution over the whole boundary layer and the velocity distribution decrease with the increase of the Prandtl number *Pr*.
4. The skin friction coefficient, the surface temperature distribution and the velocity profile decrease and the temperature profile increases for the increased values of the magnetic parameter M.

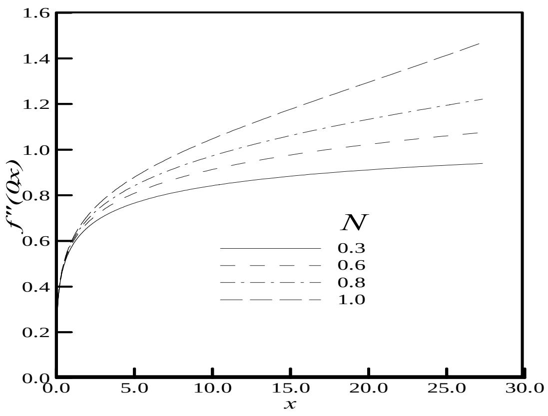

Fig 2. Skin frictions for different values of *N* when number *Pr* = 0.73 and *M* = 1.0

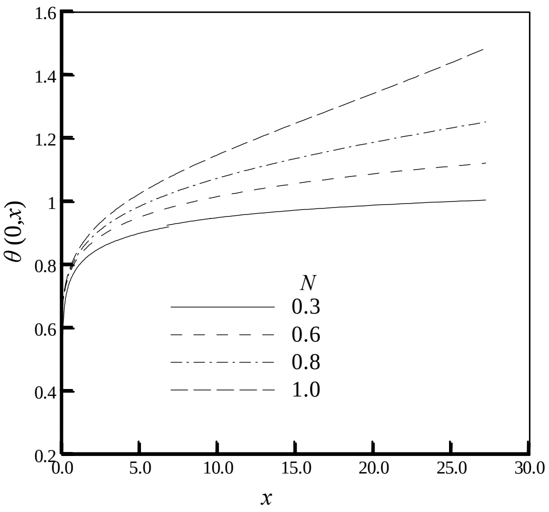

Fig 3. Surface temperature distribution for different values of *N* when *Pr* =0.73 and *M* =1.0

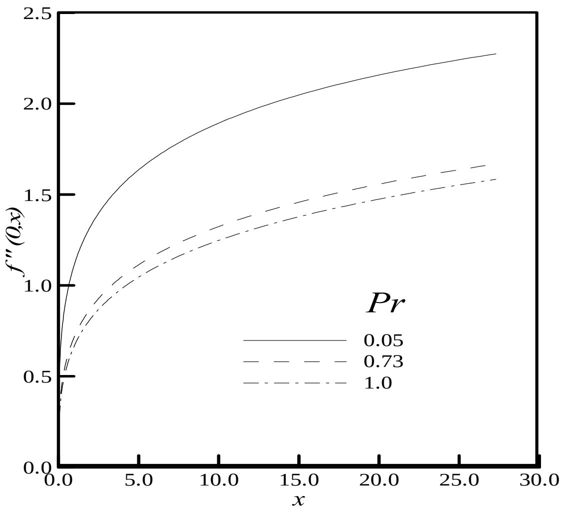

Fig 4. Skin frictions for different values of *Pr* when *M* = 0.1 and *N* = 0.01

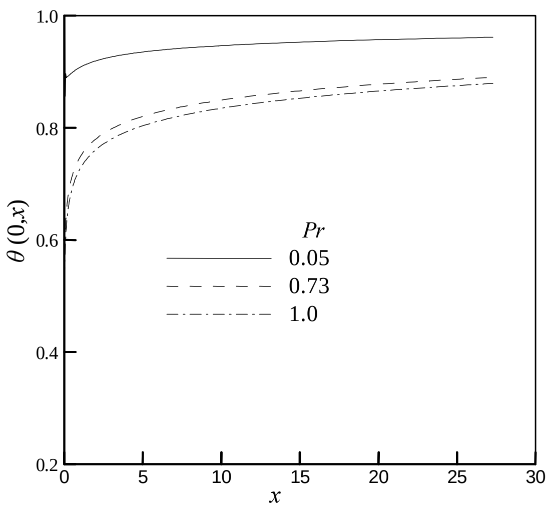

Fig 5. Surface temperature distribution for different values of *Pr* when *M* =0.1and *N* = 0.01

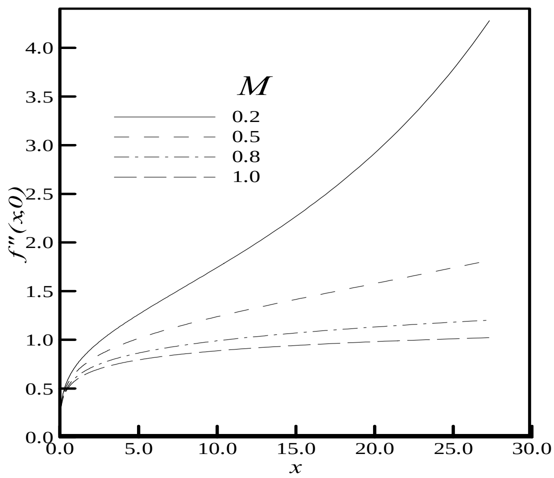

Fig 6. Skin frictions for different values of *M* when *Pr* = 0.73 and *N* = 0.5

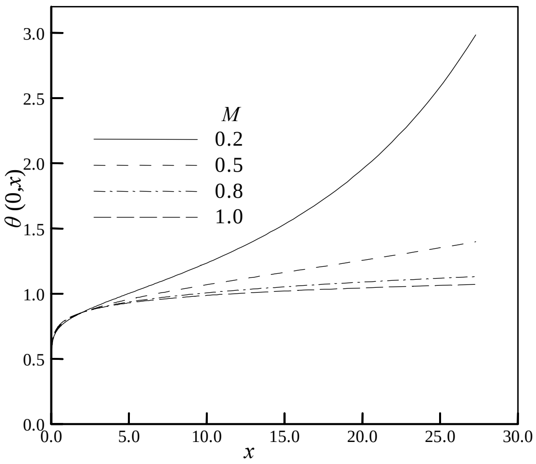

Fig 7. Surface temperature distribution for different values of M when Pr =0.73 and N =0.5

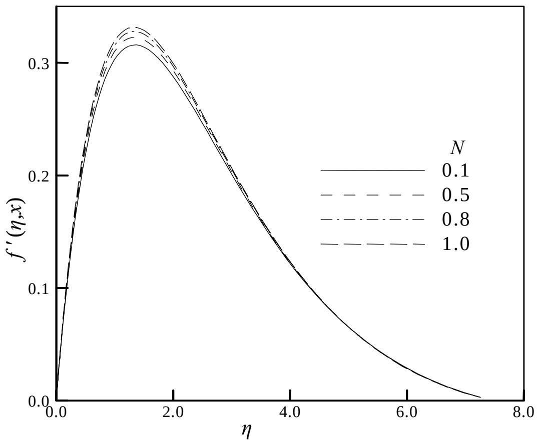

Fig 8. Velocity profile for different values of N when Pr = 0.73 and M = 1.0

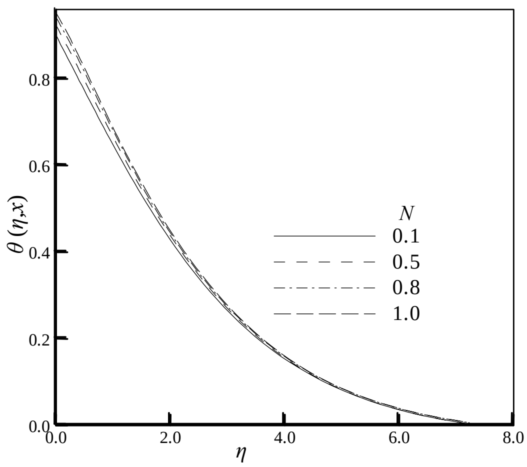

Fig 9. Temperature profile for different values of N when Pr = 0.73 and M = 1.0

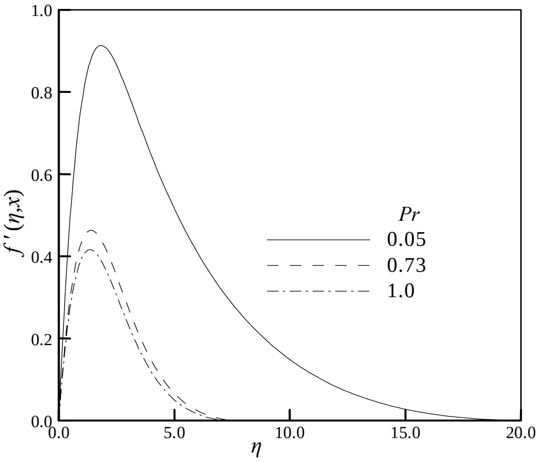

Fig 10. Velocity profile for different values of Pr when M = 0.1 and N = 0.01

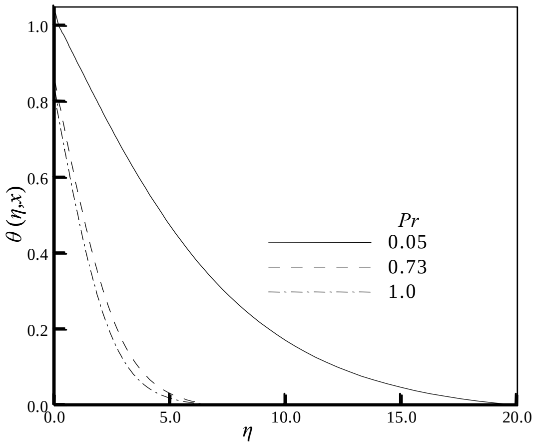

Fig 11. Temperature profile for different values of Pr when M = 0.1 and N = 0.01

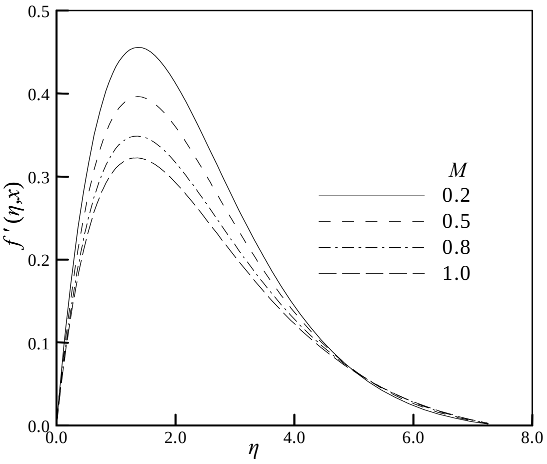

Fig 12. Velocity profile for different values of M when Pr = 0.73 and N = 0.5

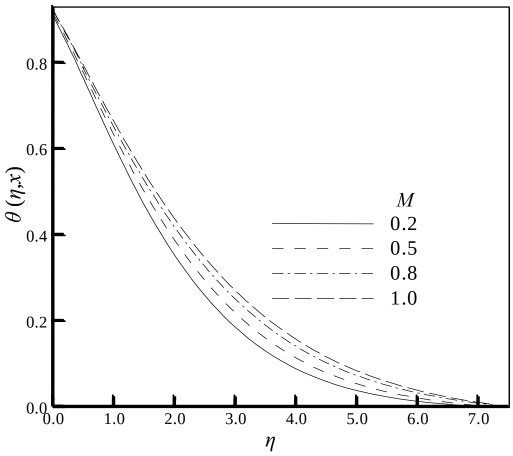

Fig 13. Temperature profile for different values of M when Pr = 0.73 and N = 0.5

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8. NOMENCLATURE

symbol	Meaning	Unit
b	Plate thickness	(cm)
C_p	Specific heat	(J/KgK)
D	$(T_b - T_\infty) / T_\infty$	(-)
f	Dimensionless stream function	(-)
g	Acceleration due to gravity	(cm/s^2)
h	Dimensionless temperature	(-)
H_0	Applied magnetic field	(-)
L	Reference length, $\nu^{2/3} / g^{1/3}$	(cm)
l	Length of the plate	(cm)
M	Magnetic parameter	(-)
N	Viscous dissipation parameter	(-)
p	Coupling parameter	(-)
Pr	Prandtl number	(-)
T	Temperature of the flow fluid	(K)
T_b	Temperature at outside of the plate	(K)
T_s	Solid temperature	(K)
T_∞	Temperature of the ambient fluid	(K)
u	Velocity component in the x-direction	(cm/s)
v	Velocity component in the y-direction	(cm/s)
x	Stream wise co-ordinate	(cm)
y	Transverse co-ordinate	(cm)

Greek symbols

β	Co-efficient of thermal expansion	(-)
ψ	Stream function	(-)
η	Dimensionless similarity variable	(-)
ρ	Density of the fluid inside the boundary layer	(kg/m^3)
ν	Kinematic viscosity	(m^2/s)
μ	Viscosity of the fluid	(N.s/m^2)
θ	Dimensionless temperature	(-)
σ	Electrical conductivity	(W/m^2.K^{-4})
K_f	Thermal conductivity of the ambient fluid	(kW/mK)
K_s	Thermal conductivity of the ambient solid	(kW/mK)