FEATURE RECOGNITION OF ROTATIONAL PARTS USING BIQUADRATIC BEZIER PATCHES

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ABSTRACT
For the automation of the process planning function a complete representation of the part is very useful. Features recognition is considered as the bridge between computer aided design (CAD) and computer aided process planning (CAPP). The purpose of feature recognition is to convert the geometric CAD data of a part into a set of features suitable to support manufacturing activities such as in process planning system. This paper proposes a method for features recognition by using biquadratic Bezier patch, which is similar to syntactic pattern recognition approach. In this paper a part is represented by the some specific coordinates according to given table and these coordinates are used only for identifies the different faces of the part with the help of pattern primitives. Now these coordinates are implemented in biquadratic Bezier patch equation for the purpose of feature recognition of the rotational parts by matching from predefined features. This method of feature recognition is limited to the turning features such as all types of grooves, chamfer, concave & convex surfaces, tapers, etc.

Keywords: Form features; Feature recognition; Surface representation; Part representation Bezier patch

1. INTRODUCTION

For manufacturing a part process planning is necessary which involves detailed description of the part for the purpose of process planning activity including selection of processes, sequencing of the processes, selection of machine tools and cutting tools, decision about cutting parameters, choice of jigs and fixtures and calculation of machining times and costs. Computer aided process planning has a very important role for the integration of computer aided design and computer aided manufacturing, and makes a purpose full relation between CAD and CAM. Part description of the CAD model in basic form of geometry and topology that can not be used for directly for process planning because it requires information in the form of features and can be extracted from this CAD model. It is a problem for process planning that how the feature can be manufactured and this information can be used in process planning. In this paper a algorithm to recognize features by using biquadratic Bezier patch which has got similarities with syntactic pattern recognition approach. The application of this technique to recognize turning features is presented. Turning features are selected in this study because they form the majority type of features in mechanical parts and products.

Process planning requires part form feature information not geometric information. Hence extracting promising approach to solve this problem [1]. the required information from the CAD data is a problem that must be solved. Feature recognition is the most

Oral et al. [2] define computer aided optimum operation and tool sequences that are to be used in Generative Process Planning System developed for rotational parts. The recognition of features such as U-groove, V-groove, slot, tapers, convex and concave form an important subset of feature recognition [3]. Kim et al. [12] deals with the development of a computer-aided process planning system interfaced with CAD for turning operation. The method developed by Joshi and Chang is a good example of a graph based approach to feature recognition. In this the boundary representation of the part is translated into a graph where its nodes represent faces and its arcs represent edges [13]. Features are considered to be machined volumes and are described in a hierarchical taxonomy that is designed to be useful across a range of machined components [6]. Feature recognition is a sub-discipline of solid modeling that focuses on the design and implementation of algorithms for detecting manufacturing information from solid models produce by CAD system [4]. For the purposes of feature recognition, segmentation or shape analysis it is often important to compute curvature information. Razdan and Bae [7] present a novel method, which uses biquadratic Bezier patches as a surface fitting technique. Based on the CIM strategy, CAPP system allows the user to develop an integrated structure that deals with the flow of information between CAD, CAPP, MRP and CAM activities within the company [10]. Razdan and Bae [5] proposed a hybrid method, which takes advantage of both methods, and create regions with complete feature loops. Satisfactory results have been achieved for both CAD parts as well as other laser s canned objects such as bones and ceramic.

2. PART REPRESENTATION

An object oriented part modular represents a part as an assembly of the form features. The part modular has the capability to represent and store geometrical and technological information of each form feature. The form features class hierarchy classifying the form features of a part as primary and secondary features. The primary features give the overall shape to the part. The secondary features reside on the primary features and add details to the part [11].

3. SURFACE REPRESENTATION

Surfaces can be described mathematically in 3D space by non-parametric or parametric equations. There are several methods to fit non- parametric surfaces to a given set of data points. The equation of the surface or surface patch is given by

$P = [x\ y\ z]^T = [x\ y\ f(x, y)]^T$

Where P is the position vector of a point on the surface (figure 4.1). The natural form of the function f (x, y) for a surface to pass through all the given data points is a polynomial that is

$$Z = f(x,y) = \sum_{i=0}^{m}\sum_{j=0}^{n} a_{ij} x^i y^j \qquad (1)$$

Where the surface is described by an x y grid of size (m+1) * (n+1) points. The parametric representation of a surface means a continuous vector valued function P (u, v) of two variables or parameters u and v, where the variables are allowed to range over some connected region of the u v plane and as they do so. P (u, v) as sums every position on the surface.

The function P (u, v) at certain u and v values is the point on the surface at these values to described the parametric equation of a 3D curved surface in space is

$$P(u, v) = [x\ y\ z]^T = [x(u, v)\ y(u, v)\ z(u, v)]^T \qquad (2)$$

$$u_{min} \leq u \leq u_{max},\ v_{min} \leq v \leq v_{max}$$

This equation gives the coordinates of a point on the surface as the components of its position vector. It uniquely maps the parametric space (E^2 in u and v values) to the Cartesian space (E^3 in x, y and z) [8].

This equation suggests that a general 3-d surface can be modeled by dividing it into an assembly of topological patches. The interval is (0, 1) for both u and v. A patch is considered the basic mathematical element to model a composite surface. Some surfaces may consist of one patch only while others may be a few patches. The topology of a patch may be rectangular or triangular. Triangular patches add more flexibility in surface modeling because they do not require ordered rectangular array of data points to create the surface as the rectangular patches do.

4. BEZIER CURVES AND SURFACES

To obtain a more free form design for aesthetic surfaces that satisfy some requirements, the modeling techniques need to provide more flexibility for changing the shape. This can be achieved by the use of Bezier curves named after P. Bezier, the designer of the French car company Renault, who invented the procedure in the 1960's. Bezier curve uses the vertices as control points. The curve will pass through the first and last point with all other points acting as control points. Figure 1 shown the control polygon and the generated smooth curve [9].

The flexibility of the process can be seen by changing the position of the individual control points in space, thereby altering the control polygon. The flexibility of the curve becomes more with more control points. The process can be extended for the surfaces as well. Bezier select Bernstein polynomials as the basis functions for the curves. They are specified as

$$B_{i,n}(u) = C(n,i)u^i(1-u)^{n-i}$$

Where $C(n,i) = \dfrac{n!}{i!(n-i)!}$

Based on these basis functions, the equation for the Bezier curve is given by

$$P(u) = \sum_{i=0}^{n} b_i B_{i,n}(u) \qquad u \in (0,1)$$

For n=3, we get

$$B_{0,3} = (1-u)^3$$

$$B_{1,3} = 3u(1-u)^2$$

$$B_{2,3} = 3u^2(1-u)$$

$$B_{3,3} = u^3$$

$$P(u) = (1-u)^3 b_0 + 3u(1-u)^2 b_1 + 3u^2(1-u)b_2 + u^3 b_3$$

The Bezier surface is the direct extension of the Bezier curve. Points on a Bezier surface can be specified as an extension of the Bezier curve.

$$P(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n} b_{ij} B_{i,m}(u) B_{j,n}(v)$$

$$u, v \in (0,1)$$

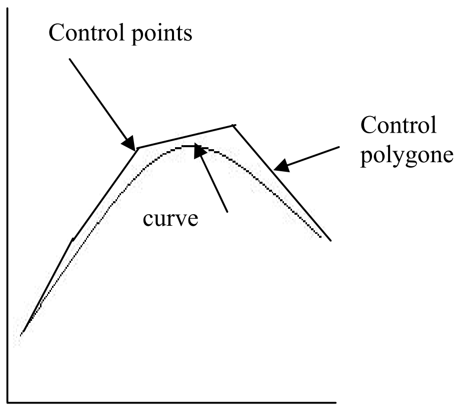


Fig 1: Bezier curve and the associated control polygon

Where b_{ij} represent the rectangular array of control points (m+1) $\times$ (n+1) defining the vertices of the characteristic polyhedron of the Bezier patch for a 4×4 points. The matrix form of this equation for a 4×4-control points is

$$P(u,v)=\left[(1-u)^3 3u(1-u)^2 3u^2(1-u)u^3\right]b\begin{bmatrix}(1-v)^3\\3v(1-v)^2\\3v^2(1-v)\\v^3\end{bmatrix}$$

The matrix b contains the points that define the characteristic polyhedron

$$b=\begin{bmatrix}b_{01} & b_{02} & b_{03} & b_{04}\\ b_{11} & b_{12} & b_{13} & b_{14}\\ b_{21} & b_{22} & b_{23} & b_{24}\\ b_{31} & b_{32} & b_{33} & b_{34}\end{bmatrix}$$

5. SYNTACTIC PATTERN-RECOGNITION APPROACH

The basic of the syntactic pattern recognition approach is the decomposition of pattern into sub patterns or primitives such a decomposition of a chromosome structures in terms of primitives and to detect and encode these primitives in the form of string. Feature recognition is a design interface for process planning which is an automatic transfer of part description data from CAD system to process planning system.

Oral et al. [2] have developed a syntactic pattern recognition technique for the extraction of manufacturing details from geometric models. In this system, a part is represented as a syntactic pattern made up of geometric primitives in 2D and sixteen pattern primitives were defined. . Turning surfaces can be defined an elements such as diameter, taper, face, arc, chamfer, and grooving with the aim of pattern primitives.

6. FEATURE DEFINITION

A machining feature can be considered as a portion of part havening a manufacturing attribute and can be created with certain machining operations. Rotational features are those features that can be created by machining operations on lathe or a turning machine. Prismatic features can be created by machining operations on a milling machine or a three-axis machining center. With the special characteristics of the shapes and operations, rotational features are defined based on the 2D profile pattern, and the prismatic features are defined in terms of the 3D boundary faces

7. FEATURE RECOGNITION PROCEDURE

The part feature recognition system that is developed has got similarities with syntactic pattern recognition approach. In syntactic pattern recognition approach 16 pattern primitives are used to formalize the pattern recognition process. For example, a diameter can be represented by either the pattern primitives A or C; a face can be represented by B or D.

7.1 Pattern Primitives

The basic of this method is the decomposition of patterns into sub patterns or primitives, that is the decomposition of complex patterns and sub patterns is carried out until the simplest sub patterns, known as pattern primitives, are obtained. Pattern primitives will have simple description and can easily be recognized. To design part feature recognition system, twenty pattern primitives have been defined as shown in figure 2. Each pattern primitive is represented by a specific coordinate and this coordinate only for identifies the particular direction of primitive. These coordinates are shown in table 1. It has different shapes of line and arc segments with a start point, a terminal point, a center point, and a direction. A code in terms of coordinate is assigned to pattern primitives to identify them. With the help of pattern primitives, turning surfaces such as diameter, taper, face, and arc can be defined specifically.

7.2 Pre- defined Features

Each predefined feature consists of three surfaces and has a unique pattern string where pattern string is defined as a group of characters. According to the direction some predefined features are shown in table 2. The part feature recognition system depends on the number of predefined features included and the system is sophisticated when it is used for a large number of predefined features.

In this method of feature recognition a part is represented by coordinates to formalize the pattern recognition process. In this all coordinates are varies from 0 to 1 only [u, v , (0, 1)]. Bezier surface equation is a parametric equation of characteristic polyhedron and the parameters are u and v. Point on a Bezier surface are given by a simple expression of general equation for points on a Bezier curve[9].

$$P(u,v)=\sum_{i=0}^{m}\sum_{j=0}^{n}b_{ij}B_{i,m}(u)B_{j,n}(v)$$
$$u,v\in(0,1) \qquad (3)$$

For features recognition a biquadratic Bezier patch has been used. The equation of a biquadratic Bezier patch is

$$P(u,v)=\sum_{i=o}^{2}\sum_{j=0}^{2}b_{ij}B_{i,2}(u)B_{j,2}(v)$$

$$u, v \in (0,1) \tag{4}$$

Where b_{ij} are Bezier control points, $B_{i,2}(u)$ and $B_{j,2}(v)$ are Bernstein basis functions in the u and v parametric direction.

$$B_{i,m}(u) = C(m,i)u^i(1-u)^{m-i}$$

$$C(m,i) = \frac{m!}{i!(m-i)!}$$

$$B_{0,2} = (1-u)^2$$

$$B_{1,2} = 2u(1-u)$$

$$B_{2,2} = u^2$$

The equation (4) can be expanded to give

$$P(u,v) = \sum_{i=0}^{2} B_{i,2}(u)\,[b_{i0}B_{0,2}(v) + b_{i1}B_{1,2}(v) + b_{i2}B_{2,2}(v)]$$
$$= B_{0,2}(u)\,[b_{00}B_{0,2}(v) + b_{01}B_{1,2}(v) + b_{02}B_{2,2}(v)]$$
$$+ B_{1,2}(u)\,[b_{10}B_{0,2}(v) + b_{11}B_{1,2}(v) + b_{12}B_{2,2}(v)]$$
$$+ B_{2,2}(u)\,[b_{20}B_{0,2}(v) + b_{21}B_{1,2}(v) + b_{22}B_{2,2}(v)] \tag{6}$$

$$P(u,v) = [\, b_{00}B_{0,2}(u)B_{0,2}(v) + b_{01}B_{0,2}(u)B_{1,2}(v) + b_{02}B_{0,2}(u)B_{2,2}(v) + b_{10}B_{1,2}(u)B_{0,2}(v) + b_{11}B_{1,2}(u)B_{1,2}(v) + b_{12}B_{1,2}(u)B_{2,2}(v + b_{20}B_{2,2}(u)B_{0,2}(v) + b_{21}B_{2,2}(u)B_{1,2}(v) + b_{22}B_{2,2}(u)B_{2,2}(v)] \tag{7}$$

The equation can be written in a matrix form as

$$P(u,v) = \left[B_{0,2}(u)B_{0,2}(v) \quad B_{0,2}(u)B_{1,2}(v) \quad - - - - - - \quad B_{2,2}(u)B_{2,2}(v)\right]\begin{bmatrix} b_{00} \\ b_{01} \\ b_{02} \\ b_{10} \\ b_{11} \\ b_{12} \\ b_{20} \\ b_{21} \\ b_{22} \end{bmatrix}$$

Now setting all the above values in matrix form according to equation AB=P, where A is computed from the Bernstein basis function contribution of each point, B is single column matrix of Bezier control points b_{ij} (figure 3) and P is the single column matrix of n points to be fitted (p_n) for the part [7].

$$\begin{bmatrix} B_{0,2}(u_0)B_{0,2}(v_0) & - - - - - - - - & B_{2,2}(u_0)B_{2,2}(v_0) \\ - & - - - - - - - - & \\ - & - - - - - - - - & \\ - & - - - - - - - - & \\ - & - - - - - - - - & \\ - & - - - - - - - - & \\ - & - - - - - - - - & \\ - & - - - - - - - - & \\ B_{0,2}(u_n)B_{0,2}(v_n) & - - - - - - - - & B_{2,2}(u_n)B_{2,2}(v_n) \end{bmatrix}\begin{bmatrix} b_{00} \\ b_{01} \\ b_{02} \\ b_{10} \\ b_{11} \\ b_{12} \\ b_{20} \\ b_{21} \\ b_{22} \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_n \end{bmatrix}$$

7.3 Pre-requisites of the model

The following assumptions are made for development of model:

1. Each pattern primitive has given a unique code and its coordinate according to given table.
2. Each pre-defined feature has a unique string and is a set of three coordinates.
3. The length of pattern primitive is not considered.
4. The part feature recognition system is applicable only for two-dimensional profile of any rotational object.
5. Always read the strings from left hand sides.
6. At any condition remaining pattern string will be $p_2p_4p_6p_0$ [(.5, .5)(1, 0)(.6, -.6)(0, 0)], which show shape of the raw material.

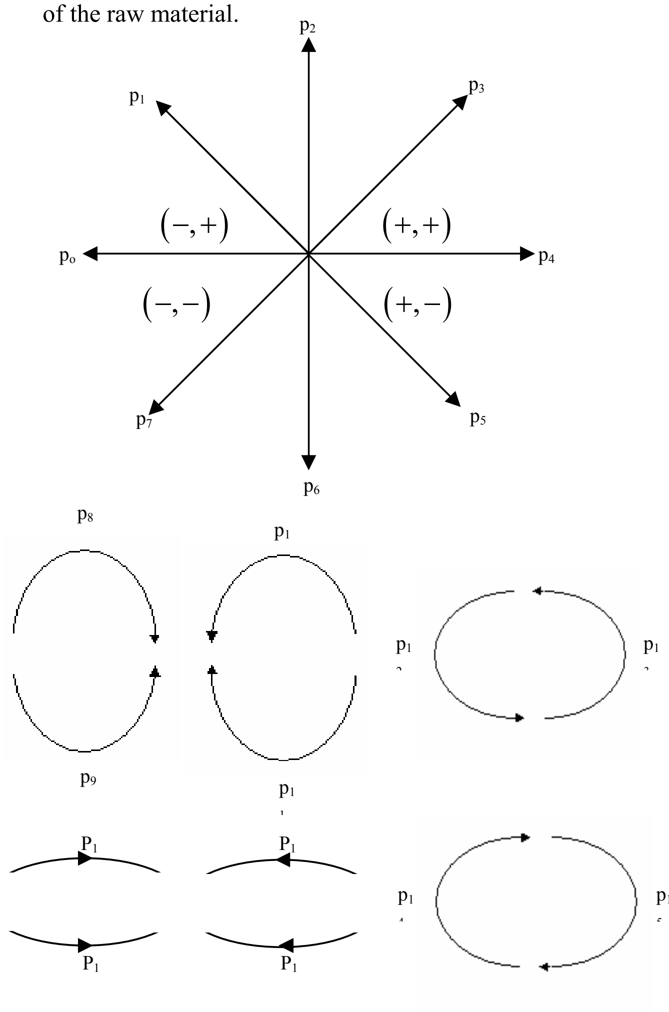


Fig 2 : Pattern Primitives

Table 1:

Sr. No.	Pattern Primitive	Coordinate
1	p_o	(0, 0)
2	p_1	(0, .5)
3	p_2	(.5, .5)
4	p_3	(1, .5)
5	p_4	(1, 0)
6	p_5	(1, -.3)
7	p_6	(.6 -.6)
8	p_7	(0, -.3)
9	p_8	(1, .4)
10	p_9	(.4, 1)
11	p_{10}	(1, .7)
12	p_{11}	(.7, 1)
13	p_{12}	(1, .8)
14	p_{13}	(.8, 1)
15	p_{14}	(1, .6)
16	p_{15}	(.6, 1)
17	p_{16}	(.3, .5)
18	p_{17}	(.5, .3)
19	p_{18}	(.2, .4)
20	p_{19}	(.4, .2)

7.4 Model Development

The model is follows a fixed procedural step. It has been developed in C environment. The specific steps of procedure for recognizing features from the two-dimensional profile of any rotational object are described in the successive paragraphs of this paper.

The algorithm is divided in six phases. The first phase considers the number of turning features and turning surfaces present in 2-D profile of a rotational object.

In the second phase, a part is represented by 2-D profile information is expressed as a pattern string with the help of primitives and its coordinates.

In the third phase, an input file is opened for pattern string and taking the print out of the input file makes subsequently a manual checking. The file can be subsequently modified in case if there is any error

In the fourth phase, the input file created in phase three is utilized to calculating Bernstein basis function for each surface and used in biquadratic Bezier patch equation, and getting the results in the form of single column matrix of coordinates.

In the fifth phase, the results are utilized to match the coordinates with the coordinate set of pre-defined feature.

In the sixth phase, an output file is opened to tore the number of turning surface present and feature strings that are matched with pre-defined features. The remaining pattern string shows the shape of raw material.

7.5 Algorithm

Step 1:

a. Create the profile view of the part.
b. Assign a unique identification number corresponding to each turning surfaces. Specific rules are outlined for the identification of turning surfaces:
    (i) The first surface is left end of the part.
    (ii) Labeling will begin at surface 1 and proceed in a clockwise manner.

Step 2: Represent the part as a pattern string by using pattern primitives and its corresponding coordinates. All coordinates are varies from 0 to 1 $\left[u, v \in (0,1)\right]$.

Step 3: Calculating Bernstein basis function for each surface and setting these in matrix form equation of single column matrix of a biquadratic Bezier patch.

Step 4: Examine the result of single column matrix of coordinates if:

a. A match is found, extract the feature and fill the void. Go to step 6.
b. No match is found. Go to step 5.

Step 5: Create a new feature, if possible and store in the table of pre-defined feature.

Step 6: Retrieve the features in chronological order. Stop.

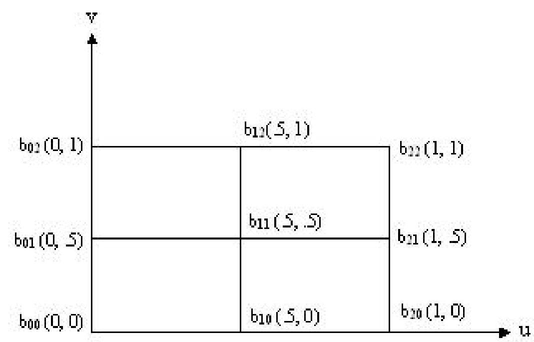


Fig 3: Parametric space (Source: Ref. 9

8. ILLUSTRATIVE EXAMPLE

A rotational part is considered and figure 4 shows a biquadratic Bezier patch fitted to surface of the rotational part.

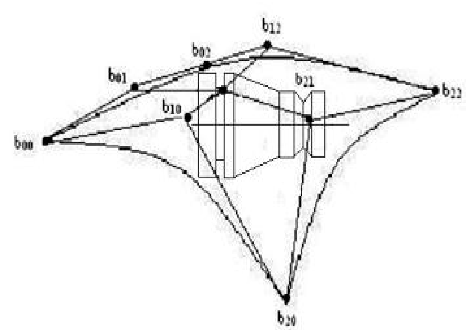


Fig 4: A biquadratic Bezier Patch fitted to surface of the rotational part rotational part

Step 1
The part considered in the figure 5, creating a two–dimensional profile of rotational part having two grooves one rectangular and other is v-groove. Part is designed by revolving the two-dimensional profile about x-axis. In this figure total number of turning surfaces are 13

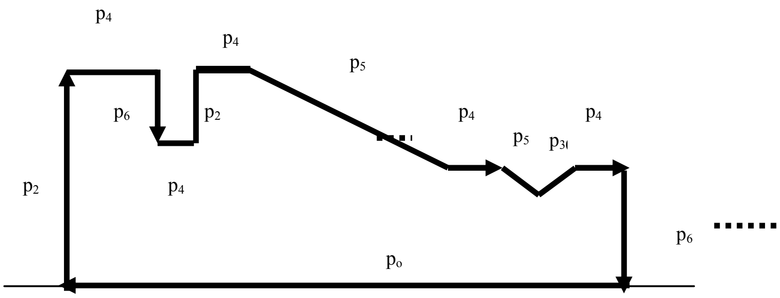


Fig 5.: Two-dimensional profile view of the part represent by pattern primitives

Step 2
figure 5 represents the part as a pattern strings and its coordinates by using pattern primitives according to table 1. All coordinates are varies from 0 to 1 [u, v ,(0,1)].
Pattern String: p_2 (.5, .5) p_4 (1, 0) p_6 (.6, -.6) p_4 (1, 0) p_2 (.5, .5) p_4 (1, 0) p_5 (1, -.3) p_4 (1, 0) p_5 (1, -.3) p_3 (1, .5) p_4 (1, 0) p_6 (.6, -.6) p_0 (0, 0)

Step 3
Now calculating Bernstein basis function for each surface according to equation (5) and setting the results in matrix form according to equation (9). We get the following result in the form of single column matrix of coordinates.

For $p_2(.5,.5)$

$$B_{0,2}(u)B_{0,2}(v) = .0625$$
$$B_{0,2}(u)B_{1,2}(v) = .125$$
$$B_{0,2}(u)B_{2,2}(v) = .0625$$
$$B_{1,2}(u)B_{0,2}(v) = .125$$
$$B_{1,2}(u)B_{1,2}(v) = .25$$
$$B_{1,2}(u)B_{2,2}(v) = .125$$
$$B_{2,2}(u)B_{0,2}(v) = .0625$$
$$B_{2,2}(u)B_{1,2}(v) = .125$$
$$B_{2,2}(u)B_{2,2}(v) = .0625$$

Step 4
Read three coordinates at a time start from upper side and match with predefined features from table 2. For example read coordinates (.5, .5)(1, 0)(.6, -.6) [$p_2p_4p_6$] and matched with pre-defined features. We can see that this string is not matched with pre-defined features; skip the first coordinate and read next three coordinate set. Repeat the procedure, till the lowermost coordinate is evaluated and update the pattern string. We get the following results in the form of features. In this part total number of extracted features are three (figure 6). Remaining pattern strings give the shape of the blank. For example $p_2\, p_4\, p_6\, p_0$ [(.5, .5) (1, 0) (.6, -.6) (0, 0)].

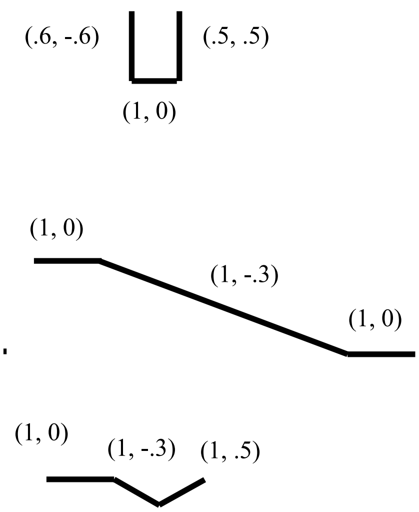


Fig 6:. Extracted features

9. RESULT AND DISCUSSION

This method of feature recognition by biquadratic Bezier patch performs better than others and has some similarities from syntactic pattern recognition approach. From above example we can see that every part is requiring coordinates which are easily selected from the table: 1. Feature recognition by this method is easy and simple from earlier methods of feature recognition.

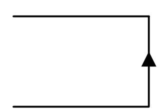
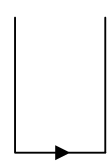
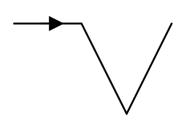
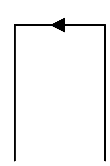
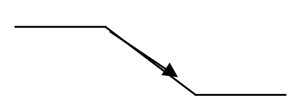
The methodology developed in this thesis has been used for two-dimensional profile view of different rotational parts covering various types of features. Subsequently, a few examples are used to illustrate the validation of the methodology. We can see that the methodology developed here can give approximately same result as compared to syntactic pattern recognition approach.

10. CONCLUSION

The important advantages of recognizing features using coordinates information of the rotational parts are its ability to easily recognize turning features. Further research is required to be conducted for compound features. The feature information can be used to determine tool approach directions for machining and development of machining sequences. It has been observed that new features can be identified and added to pre defined feature table. The outcome of the analysis will result in creating the blank, represented by a set of primitives. The remaining pattern string represents the shape of the raw material. The procedure developed by the author are based on certain assumptions such as every turning face is represented by a specific coordinate.

$$
\begin{bmatrix}
.0625 & .125 & .0625 & .125 & .25 & .125 & .0625 & .125 & .0625 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
.4096 & -.3072 & .0576 & 1.23 & -.9216 & .1728 & .9216 & -.6912 & .1296 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
.0625 & .125 & .0625 & .125 & .25 & .125 & .0625 & .125 & .0625 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.69 & -.78 & .09 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.69 & -.78 & .09 \\
0 & 0 & 0 & 0 & 0 & 0 & .25 & .5 & .25 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
.4096 & -.3072 & .0576 & 1.23 & -.9216 & .1728 & .9216 & -.6912 & .1296 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\ 0 & .5 \\ 0 & 1 \\ .5 & 0 \\ .5 & .5 \\ .5 & 1 \\ 1 & 0 \\ 1 & .5 \\ 1 & 1
\end{bmatrix}
=
\begin{bmatrix}
P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \\ P_{11} \\ P_{12}
\end{bmatrix}
\begin{bmatrix}
P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \\ P_{11} \\ P_{12}
\end{bmatrix}
=
\begin{bmatrix}
.5 & .5 \\ 1 & 0 \\ .6 & -.6 \\ 1 & 0 \\ .5 & .5 \\ 1 & 0 \\ 1 & -.3 \\ 1 & 0 \\ 1 & -.3 \\ 1 & .5 \\ 1 & 0 \\ .6 & -.6 \\ 0 & 0
\end{bmatrix}
$$

Table 2:

Sr. No.	Predefined Features	String	Coordinates
1		$p_4p_2p_0$	(1, 0)(.5, .5)(0, 0)
2		$p_6p_4p_2$	(.6, -.6)(1, 0)(.5, .5)
3		$p_4p_5p_3$	(1, 0)(1, -.3)(1, .5)
4		$p_2p_0p_6$	(.5, .5)(0, 0)(.6, -.6)
5		$p_4p_5p_4$	(1, 0)(1, -.3)(1, 0)

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