THE STUDY OF THE BACKLASH EFFECTS ON GEOMETRY FACTOR OF SPUR GEARS BY THE FINITE ELEMENT METHOD BY USING ANSYS

Alireza Dastan

Mechanical engineer from Bahonar University of Kerman-Iran
Tel: +98 917 732 7737 , Fax: +98 731 333 8594
E-mail:A.R.Dastan@Gmail.com

ABSTRACT
In this paper, the concept of backlash in spur gears and the method of its creation will be explained. I am going to state the concept of the Highest Point of Single Tooth Contact (HPSTC) which is the most critical loading point in gears. Then necessary equations will be derived to determine the accurate position of HPSTC and loading angle at this point will be derived. Stress analysis by the finite element method (FEM) by using ANSYS software is the basic idea of this paper. At the end, some charts based on non-dimensional variables are provided in order to state the effects of backlash on tooth stress by using the mentioned method.

Keywords: backlash, highest point of single tooth contact, geometry factor

1. INTRODUCTION
According to the increasing usage of power transmission systems in industry, especially in transportation industry, researchers are certain to focus on this subject. Since gear power transmission system provides proper working conditions, high life time of machinery and low power loss, scientists are more attracted to this issue.

A lot of studies and research on some sorts of gears and the standard of their characteristics have been done by Mabie and his colleagues [1]. Also Sheigly has done some studies on gear stresses [2]. And as we know American Gear Manufacturing Association (AGMA) is a well-known organization which has done some wide research on many kinds of gears. The standards offered by this organization are international acceptable criteria. There are various methods for stress analyzing and deflection in gear tooth. Among these methods we can refer to Lewis improved equation [2], classic elasticity theorem based on complex variables, Cornell method and finite element method.

The finite element method is the most efficient and available method for determining gear stress and tooth deflection. Nowadays due to the progress in computer technology, designers and researchers have attended to this method.

In this paper, I attempt to offer a general approach for stress analysis of some kinds of standard and non-standard gears based on the finite element method and using some available software. By using this approach we can also analyze the spur gears with backlash.

2. BACKLASH
When the sum of the thickness of two teeth out of mating gears on operating pitch circle would be less than circular pitch, it is said those two gears are engaged with backlash, meaning there is a gap between teeth of two mating gears(see Fig. 1). Being designed with standard features and being installed in standard center distance, the gears work ideally and there will be no backlash.

The basic purpose of designing backlash is to prevent locking the gears, as well as to prevent coming into contact in both side of one tooth simultaneously. A little amount of backlash is designable to make necessary gap for lubrication and partially expansion of gears. Also, the amount of backlash should not be increased exceedingly, because the backlash causes high manufacturing costs.

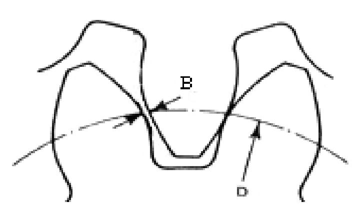

Fig 1: Two gears have been engaged withbacklash

In general, there are two ways for making backlash in mating gears:
1. Reducing tooth thickness.
2. Installing two mating gears in a distance which would be more than the standard center distance.

Both methods can happen by designing or unintentionally. It means that either errors in gear generation process and installing or designing conditions considered by designer in advance can be the reasons of making backlash.

If designer wants to design backlash by reducing tooth thickness it is common to consider half the amount of backlash for each gear. So that, the sum of the decreased thickness amount in each gear would be equal to backlash amount. While the gear generation process for making backlash, cutter tool should shave gear blank a little more than the standard cutting depth. Though, there are some exceptions to this rule. For instance in pinions having fewer teeth, it is reasonable to consider the whole amount of backlash for bigger gear in order not to weaken the pinion teeth.

If the backlash is made based on increasing center distance, the amount of backlash will be calculable according to the changes in center distance. It has been done by Mabie [1]:

$$B = \frac{C'}{C}[\pi m - (t_1 + t_2) + 2C(inv\phi' - inv\phi)] \quad (1)$$

$$\cos\phi' = \frac{C}{C'}\cos\phi \quad (2)$$

Involute function is defined as follows:

$$inv\phi = \tan\phi - \phi \quad (3)$$

3. HIGHEST POINT OF SINGLE TOOTH CONTACT

Lewis equation for calculating tooth bending stress in spur gear [2] is given by:

$$\sigma = \frac{W_t}{FmY} \quad (4)$$

Equation (4) has been derived according to the load acting on tooth tip and ignoring the effects of stress concentration.

In fact, if the gear has been cut accurately enough, the other pair of teeth will come into contact in the situation of load acting on tooth tip. Therefore a portion of the load will be carried by these teeth. So the whole load does not apply on one tooth. Thus, the critical stress happens when just one pair of teeth carries the load. This position is called highest point of single tooth contact (HPSTC).

Highest point of single tooth contact is a point on involute portion of tooth having the following conditions:

Firstly, just one tooth of each gear is in contact with each other. Secondly, it has the farthest distance to the center of the gear (the largest radius). It means that the next teeth of gears are going to be in contact at the same time.

When two gears are meshed in HPSTC, symmetry axis of teeth does not cover the line of centers of the two gears. And load acting angle based on tooth symmetry axis will not be the pressure angle (ϕ) any more. By considering the tooth symmetry axis as one of the coordinate axis, the load is acted on tooth with the angle called ϕ_l.

According to AGMA standards, HPSTC is the critical point for the load acting and geometry factor J is defined based on load acting to this point [2].

In contrast with Lewis equation (Eq. (4)), in AGMA equation (Eq. (5)) it has been considered the effects of stress concentration as well as the load sharing ratio. And tooth bending stress is calculated according to the load acting on HPSTC based on the angle ϕ_l :

$$\sigma = \frac{W}{FmJ} \quad (5)$$

Where W is the total load and J is geometry factor.

Therefore it is necessary to define the exact position of HPSTC and load angle ϕ_l at this point, accurately.

In Fig. 2, A is the beginning contact point and B is the end of contact point. E_1 and E_2 are interference points. P is the pitch point and H is the HPSTC. Angle ϕ_l is the angle between load vector (pressure line direction) and the perpendicular line to teeth symmetry axis.

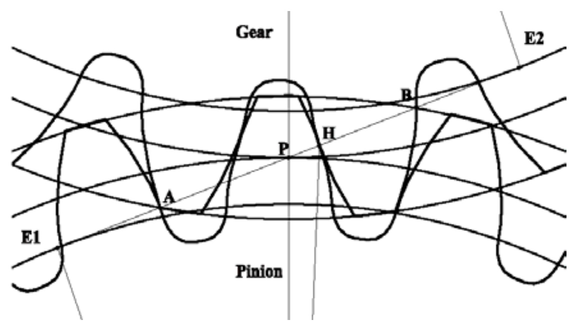

Fig 2: Two gears are meshed at HPSTC

As known:

$$AH = P_b \quad (6)$$

$$AB = Z = \sqrt{Ro_1^2 - Rb_1^2} + \sqrt{Ro_2^2 - Rb_2^2} - C\sin\phi \quad (7)$$

According to Fig. 2:

$$E_1H = E_1B - BH \quad (8)$$

$$E_1B = \sqrt{(O_1B)^2 - (O_1E_1)^2} = \sqrt{Ro_1^2 - Rb_1^2} \quad (9)$$

$$BH = AB - AH = Z - P_b \quad (10)$$

$$O_1H = \sqrt{(O_1E_1)^2 + (E_1H)^2} \quad (11)$$

The result of Eqs. (8),(9) and (10) will be:

$$E_1H = \sqrt{Ro_1^2 - Rb_1^2} - (Z - P_b) \quad (12)$$

By combining Eqs. (11) and (12) we get:

$$O_1H = \sqrt{Rb_1^2 + ((P_b - Z) + \sqrt{Ro_1^2 + Rb_2^2})^2} \tag{13}$$

We can rewrite Eq. (13) as the following form:

$$R_{H(1)} = O_1H \tag{14}$$

$$= \sqrt{Ro_1^2 + P_b(1-m_P)[P_b(1-m_p) + 2\sqrt{Ro_1^2 - Rb_1^2}]}$$

Equation (14) calculates the radius of HPSTC for gear 1. In this equation by changing subscript 1 to 2, we can calculate HPSTC radius for gear 2.

In Fig. 3, those two gears meshed in HPSTC have been shown. Line O_1O_2 is the line of centers, line O_1M is the symmetry line of tooth as well as the vertical axis of the coordinate system. Angle ϕ_H is the involute pressure angle in point H.

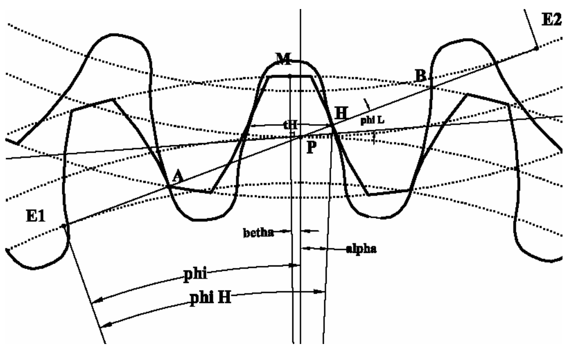

Fig 3: Two gears are meshed at HPSTC. H is HPSTC. ϕ_l and the perpendicular line to symmetry axis of tooth are shown.

As shown in Fig. 3:

$$\hat{\phi}_H = \cos^{-1} \frac{R_b}{R_H} \tag{15}$$

Recalling Eq. (16) from reference [1], in order to calculate tooth thickness we have:

$$t_B = 2R_B[\frac{t_A}{2R_A} + inv\,\phi_A - inv\,\phi_B] \tag{16}$$

In Eq. (16) by naming pitch point as point A and HPSTC as point B, we get:

$$t_H = 2R_H[\frac{t}{2Rp} + inv\phi - inv\phi_H] \tag{17}$$

$$t = \frac{\pi Rp}{N} \tag{18}$$

Where, N is teeth number of gear.
By inserting Eq. (17) into Eq. (18):

$$t_H = 2R_H[\frac{\pi}{2N} + inv\,\phi - inv\,\phi_H] \tag{19}$$

According to Fig. 3 and Eq. (19):

$$M\hat{O}_1H = \hat{\alpha} + \hat{\beta} = \frac{t_H}{2R_H} = inv\phi - inv\phi_H + \frac{\pi}{2N} \tag{20}$$

$$\hat{\alpha} = \hat{\phi}_H - \hat{\phi} \tag{21}$$

$$\hat{\beta} = M\hat{O}_1H - \hat{\alpha} \tag{22}$$

$$\hat{\phi}_l = \hat{\phi} - \hat{\beta} \tag{23}$$

By combining these four equations:

$$\phi_l = \phi_H + inv\phi_H - inv\phi - \frac{\pi}{2N} \tag{24}$$

Or

$$\phi_l = \phi - (\tan\phi - \tan\phi_H + \frac{\pi}{2N}) \tag{25}$$

Note that in Eqs. (24) and (25), angle ϕ_l would be calculated in terms of radian.

4. MODELING AND STRESS ANALYSIS

Analytic methods to solve gear problems need many simplifications and initial assumptions and due to the simplifications some errors may happen in the calculation. And the taken results may be contrary to what occurs in real situation.

Researchers have been interested in numerical methods since computers have been powerful. In this way they could prevent the errors which might happen in analytic hand calculations. This caused an increase in the gear analysis by using computer because as usual there are no limiting assumptions in numerical solutions.

The finite element method is a very efficient approach for stress analysis of elastic objects having complicated geometry. Nowadays, there is some various software installed on PC in the market. So they can model and draw the complicated geometry of mechanical models. And they can operate stress analysis by means of finite element method. Some of the commonest software for modeling and stress analysis are SolidWorks and ANSYS, respectively. We can also use GearTrax on SolidWorks in order to sketch the 3-D and 2-D models of some sort of standard and non-standard gears.

In this paper the 2-D sketch of three teeth of gears as the one offered by Shigley [2], is modeled in SolidWorks. And it is saved by ACIS(.sat) format, then imported in ANSYS for stress analysis process.

Since the stress distribution in spur gears along the face width is uniform, we analyze the 2-D model of gear based on the plane stress with thickness assumption instead of the 3-D model. I considered the face width of 10 millimeters for the whole of paper. It seems the elements of Slolid42 and Solid82 available in ANSYS, are proper for this purpose. As Solid82 has 8 nodes and curved edge lines, for meshing of gear model it is preferable to Solid42. Figure 4 shows the Solid82 element. In this paper in the whole problems I consider steel as the material of gears. It means that Elasticity module is equal to 200 GPa (E=200 GPa) and Poisson's ratio is equal to 0.3 (υ=0.3).

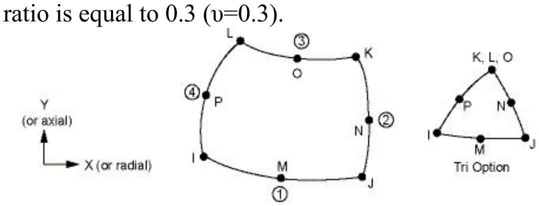

Fig 4: Solid82 element

As mentioned, the load with angle given in Eqs. (24) or (25), is applied on HPSTC which is calculable through Eq. (14).
Figure 5 shows a model of the gear by applying boundary conditions (load and supports).

The maximum Von Mises stress obtained from ANSYS is the criterion in this work. The software shows the maximum stress at the load acting point, because the whole load is applied on one node at this point. And this value is not valid. To obtain the real maximum stress happened in tooth fillet, after finishing solving, we should either omit the zone around the load acting point or select the zone around the tooth fillet.

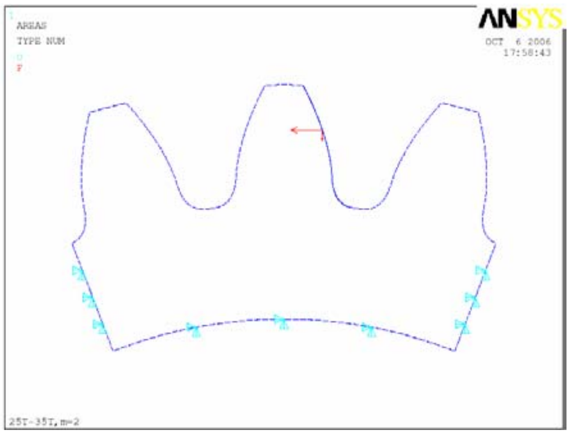

Fig 5: The model of a gear in ANSYS
by applying boundary conditions.

Figure 6 shows the Von Mises stress distribution in pinion of 25 teeth, 2 mm module and 20˚ pressure angle which meshed with a gear of 35 teeth.

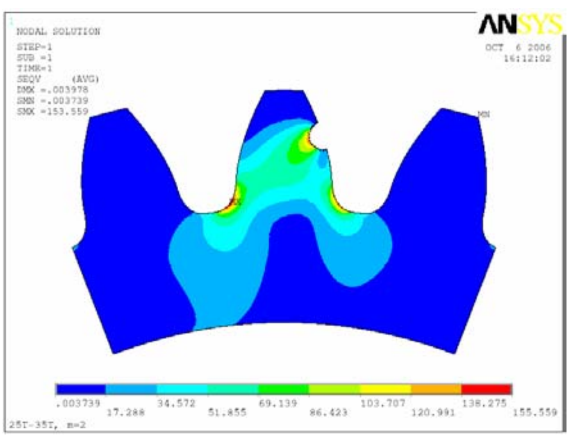

Fig 6: Von Mises stress distribution in
a pinion of 25 teeth, no backlash and 2mm module
engaged with a 35 teeth gear.

Based on the amount of stress obtained through the finite element method and by recalling Eq. (5), we can define an especial equation to calculate gear geometry factor:

$$J = \frac{W}{mF\sigma_{\max}} \tag{26}$$

Since in mating gear having the same material, the pinion is always weaker; I analyzed just the pinions and ignored the gear analysis.

In table 1, I have explained the characteristics of some gears which have been studied in this method and their geometry factor calculated by Eq. (26). In the last column, geometry factor calculated by AGMA [2] is written to be compared.

Table 1- Stress analysis results for some gear sets

No	φ (deg)	m (mm)	N_P	N_G	Load (KN)	HPSTC radius (mm)	φ_l (deg)	stress (Mpa)	J (FEM)	J (AGMA)
1	20	1.5	18	18	1	13.891	19.70	227.87	0.2925	0.325
2	20	2	25	35	1	25.331	18.66	153.52	0.3257	0.3669
3	20	1.5	36	36	1	27.24	19.04	186.48	0.3575	0.4073
4	20	6	35	80	1	105.54	18.33	47	0.3546	0.4233
5	20	20	80	80	10	801.77	19.27	133.8	0.3737	0.4832

5. THE EFFECTS OF BACKLASH ON THE GEOMETRY FACTOR

As mentioned, we can produce backlash in two ways; reducing tooth thickness and increasing the center distance. Both of them can happen by designing or unintentionally. To state the permitted amount of backlash in spur gears based on diametral pitch and center distance in terms of inches, AGMA has composed some standards which are proper reference to design coarse pitch gears.
In table 2, the amount of permitted backlash in terms of millimeter is stated based on module of two meshed gears and their center distance.

Table 2- Permitted amount of backlash based on AGMA standard

Center distanc (mm)	Module (mm)				
	50.8-12.76	12.7-7.278	7.258-4.24	4.233-2.542	2.54-1.27
to 127	_	_	_	_	0.127-0.381
127-254	_	_	_	0.254-0.508	0.254-0.508
254-508	_	_	0.508-0.762	0.381-0.635	0.254-0.508
508-762	_	0.762-1.016	0.635-0.762	0.508-0.762	_
762-1016	1.016-1.524	0.889-1.143	0.762-1.016	0.635-0.762	_
1016-1270	1.27-1.778	1.016-1.397	0.889-1.143	0.762-1.016	_
1270-2032	1.524-2.032	1.143-1.651	1.016-1.397	_	_
2032-2540	1.778-2.286	1.27-2.032	_	_	_
2540-3048	2.032-2.794	_	_	_	_

Since the gear geometry and operating conditions change by making backlash, the position of HPSTC, load acting angle at this point and the tooth stress will change too. For two methods to make backlash we should examine the validity of Eqs. (14) and (25):
1. Making backlash by reducing tooth thickness:

By reviewing of Eq. (14) it is revealed that in this case none of the effective variables have changed and the equation can be used again.
But load acting angle ϕ_l changes because tooth thickness has changed. In Eq. (17), the amount of Eq. (18) should be replaced with the tooth thickness on pitch circle will be stated in the following equation:

$$t = \frac{\pi R_P}{N} - \frac{B}{2} \quad (27)$$

Where B is the final amount of backlash divided up into equal parts on each gear. Therefore Eq. (25) based on Eq. (27) changes as the following form:

$$\phi_l = \phi - (\tan\phi - \tan\phi_H + \frac{\pi}{2N} - \frac{B}{4R_P}) \quad (28)$$

2. Making backlash by increasing the center distance:

For this type of backlash making according to Mabie calculations [1] and Eq. (14) we will have:

$$R_H = \sqrt{R_{o1}'^2 + P_b(1-m_p)[P_b(1-m_p) + \sqrt{R_{o1}'^2 - R_{b1}^2}]} \quad (29)$$

Two gears engaged with backlash on HPSTC are shown in Fig. 7. The backlash has been made by increasing center distance.

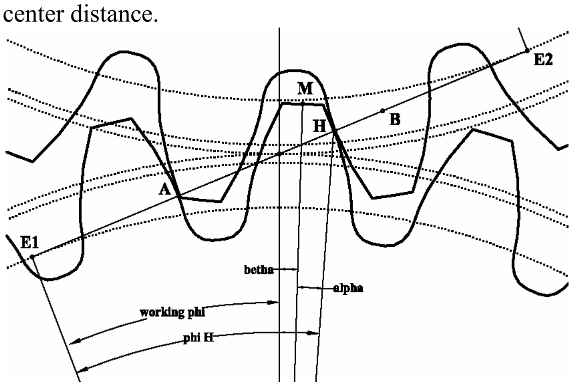

Fig 7: Two gears are engaged at HPSTC with backlash

To calculate load acting angle in this case, Eq. (17) is rewritten as the following form:

$$t_H = 2R_H[\frac{t'}{2R'} + inv\phi' - inv\phi_H] \quad (30)$$

Where t′ is stated as:

$$t' = 2R'[\frac{t}{2R_P} + inv\phi - inv\phi'] \quad (31)$$

R_H is gotten from Eq. (29) and ϕ_H is obtained from Eqs. (29) and (15).

According to Eq. (30) and Fig. 7:

$$M\hat{O}_1H = \alpha - \beta = \frac{t_H}{2R_H} = inv\phi - inv\phi_H + \frac{\pi}{2N} \quad (32)$$

$$\beta = M\hat{O}_1H - \alpha = M\hat{O}_1H - (\phi_H - \phi') \quad (33)$$

$$\phi_l = \phi' - \beta \quad (34)$$

By combining last 3 equations:

$$\phi_l = \phi - (\tan\phi - \tan\phi_H + \frac{\pi}{2N}) \quad (35)$$

Comparing Eqs. (25) and (35), it seems that there is no difference between them. But note that the amount of angle ϕ_H is different in no backlash case in comparison to the state having backlash.

Providing design charts in order to determine the effects of backlash on geometry factor of mating gears is the final purpose in this paper. In this work, it is considered only spur gears with pressure angle of 20° whose backlash is produced by reducing tooth thickness. According to previous experience and research, I chose the spur gears with tooth number of 18, 25, 35, 50 and 80, as a pinion for analysis. Also, the gears with module of 0.5, 1, 2, 2.5, 3, 4, 6, 8, 12 and 20 millimeter are chosen based on the standard modules offered in reference [1]. The proper interval of backlash is derived for each pair of gears with mentioned module from table 2. This interval has been divided into 5 or 7 suitable parts and each part is applied to its own gear set.

For instance, in the case of N_p=50, N_G=80 and module of 4 mm and referring to table 2, the proper backlash interval of 0.381 to 0.635 mm is suggested for this set. Therefore to get reliable statistic data we analyze the pinions of 50 teeth having module of 4 mm and backlash of 0.0, 0.39, 0.47, 0.55 and 0.63 mm. So, 362 gears in various modules, different numbers of teeth and the backlash amount proportional to mentioned AGMA standards were modeled and analyzed. In order to make the proper design charts and using them easily, I defined another non-dimensional variable in the form of B/m, in addition to considering non-dimensional geometry factor. In this fraction B is backlash and m is module of the gear. In fact, the non-dimensional variable B/m, shows the ratio of backlash to the amount of circular pitch (P=πm)

Figures 8 to 12 show the amount of geometry factor of pinion, based on non-dimensional variable B/m. Note that all of gear sets have pressure angle of 20° and teeth number of gear meshed with the pinion is written on the charts.

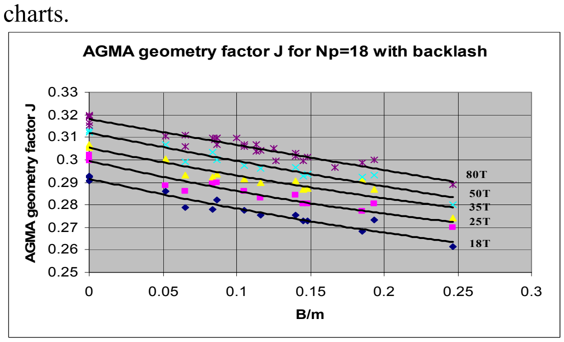

Fig 8: Pinion geometry factor with teeth number of 18

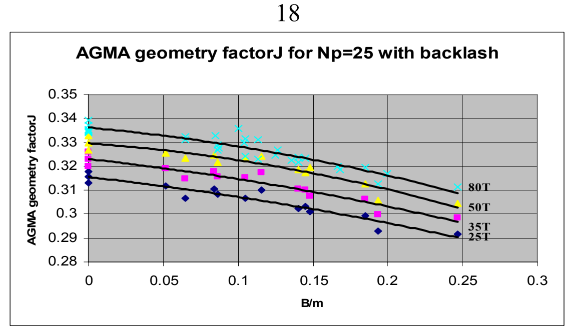

Fig 9: Pinion geometry factor with teeth number of 25

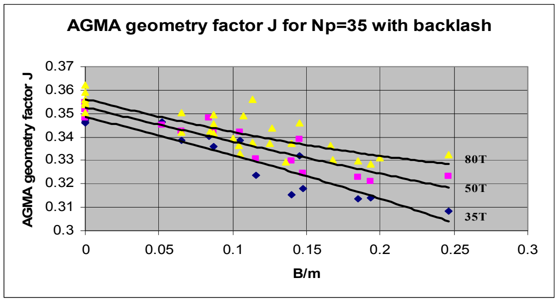

Fig 10: Pinion geometry factor with teeth number of 35

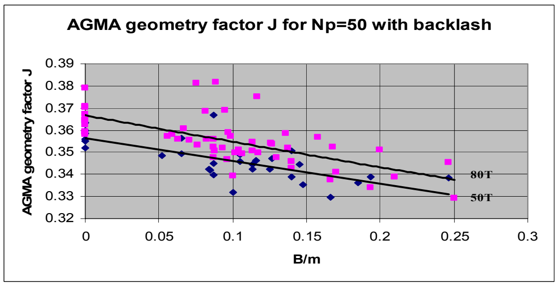

Fig 11: Pinion geometry factor with teeth number of 50

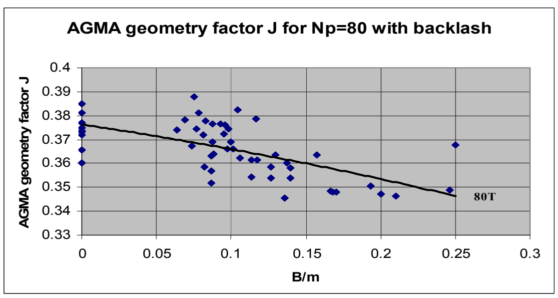

Fig 12: Pinion geometry factor with teeth number of 80

I advise noting the table 2 when using design charts 8 to 12. Meaning at first, it is necessary to refer to table 2 for a gear set having determined teeth number, module and backlash. If the backlash amount satisfies the offered interval, using the charts will be acceptable. Otherwise they are not recommended. However their inaccuracy has not been proved.

6. DISCUSSION AND CONCLUSION

In this work backlash was studied as a phenomenon which can occur in gear systems. Also, highest point of single tooth contact in gear sets was introduced. This position is the critical load acting point because the whole transferred load is applied to one tooth and it has the maximum distance to the tooth fillet. So the bending moment is the maximum. This point is the criterion for the maximum stress in the whole gear design. So, it was necessary to define the exact position of this point and the circumstance of load acting. It was preceded to this by analytic method and the results were expressed.

In this paper, I offered some practical and reliable approach, by combining the previous analytic methods and new numerical methods done by using software. In this way we can study both standard and non-standard gears. And the results are expressed like the former analytic ones. So we can use them easily in future. It was done for the gears whose backlash was made because of manufacturing errors as well as designer need. And the results were illustrated in some charts. As we observed the charts, the gear geometry factor will be decreased by increasing the ratio of backlash to module. It means the stress established in the tooth fillet will be increased.

7. REFERENCES

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8. NOMENCLATURE

Symbol	Meaning	Unit
B	backlash	(mm)
C	standard center distance	(mm)
C′	operating center distance	(mm)
m	module	(mm)
t	tooth thickness on standard pitch circle	(mm)
ϕ	cutting tool pressure angle	(deg)
ϕ'	operating pressure angle	(deg)
σ	tooth bending stress	(MPa)
W_t	transferred tangent load	(N)
F	face width	(mm)
Y	Lewis form factor	dimensionless
J	geometry factor	dimensionless
P_b	base pitch	(mm)
R_b	base circle radius	(mm)
R_O	addendum circle radius	(mm)
Z	length of action	(mm)
m_p	contact ratio	dimensionless
N	teeth number of gear	dimensionless
σ_{max}	the taken stress from ANSYS analysis	(MPa)
W	total load	(N)
R'_O	operating addendum circle radius	(mm)
t′	tooth thickness on operating pitch circle	(mm)
R′	operating pitch circle radius	(mm)
R_P	standard pitch circle radius	(mm)