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DESIGN, DYNAMICS SIMULATION AND CONTROLLABILITY OF SNAKEBOARD

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ABSTRACT
This paper considers the modeling and control of a commercially available variant of the skateboard, known as snakeboard. For this purpose, a mechanism is designed in order to model the body of a human on the snakeboard, its movements on the board and the effect of each part on the neighbouring parts. For the designed mechanism, the generalized coordinates are assigned and the equations of motion are derived with respect to assigned set of generalized coordinates by means of Lagrange method. Analysis and simulations are performed for this set of equations. In the final step, using both sliding mode control method and neural network controller, controllability analysis is performed and the results of trajectory tracking along two different paths (a straight line and a sinusoidal trajectory) are compared as to determine the performance of each proposed controllers.

Keywords: Snakeboard, Dynamics Simulation, Sliding Mode Control

1. INTRODUCTION
Snakeboard is a commercially available variant of the skateboard (figure 1).The snakeboard is consist of a pair of footpads and sets of wheels. Each of these footpads is hinged to a coupler link so they are free to rotate about the coupler. The snakeboard allows the rider to propel him/herself forward without having to make touch to the ground. To do so the rider fixes his/her feet to the footpads. To propel the vehicle the rider should harmonically rotate the upper part of his/her body (figure 2).

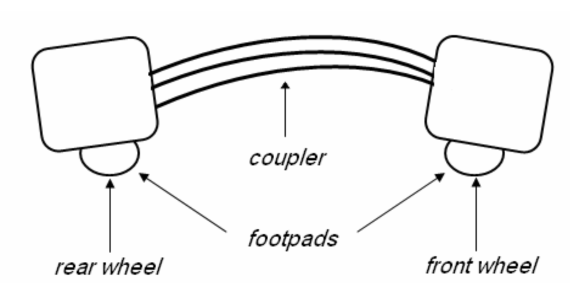


Fig 1: The schematic of Snakeboard [1]

The motion of snakeboard is roughly accomplished by coupling a conservation of angular momentum effect with the nonholonomic constraints defined by the condition that the wheels roll without slipping. Lewis, Ostrowski, Murray, and Burdick [1] as an interesting mechanical control system with nonholonomic constraints, firstly studied the snakeboard.

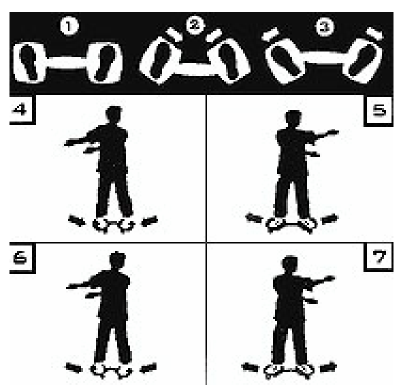


Fig 2: How to ride a snakeboard

In this paper, a mechanism is designed in order to model the body of human on the snakeboard. For the designed mechanism, the generalized coordinates are assigned and the equations of motion are derived with respect to assigned set of generalized coordinates by means of Lagrange method. A controllability analysis is performed by means of two methods, sliding mode control method and neural networks as the controller of the system.

2. MECHANISM DESIGN
In order to model the human body on the snakeboard, a mechanism is designed, and the movement of each part of the mechanism on the board and the effect of separate

part on the neighbouring are studied. For modeling of the human body on the board, the designed mechanism should include three parts. The first part is for modeling of the human legs on the board. The second part is designed for modeling the motion of the human waist. A third part should be designed to model the hands of the rider. A pictorial model of the designed mechanism is shown in figure 3. In figure 3, parts 1 and 2 represent the footpads. Part C in the figure shows the extension of the human feet on the board. These parts rotate with the footpads. Part 3 in figure 3 is the resemblance of the waist and part 4 is for modeling the motion of the human hands. In human body, the rotation of the waist causes a rotation in each leg. In the designed mechanism, this effect is considered by a gear contact between the parts for modeling of the waist and the legs (point B in figure 3).

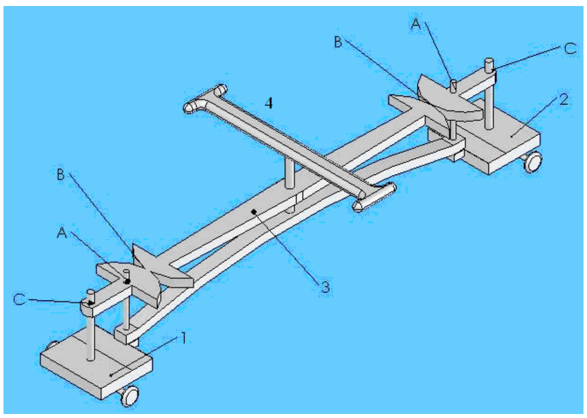


Fig 3: The snakeboard and the designed mechanism for modeling the human body

3. DERIVING EQUATIONS OF MOTION

For the designed mechanism, the generalized coordinate is shown in figure 4. In figure 4, (x, y, θ) is the set of coordinates which locate the center of the coupler, γ is the angle between the waist and the coupler, ψ is the angle between the waist and the coupler and n is the gear ratio for the contact between the waist and the legs and φ_b and φ_f are the angles between the each set of wheels and the respective footpad. Thus for this mechanism, the set of generalized coordinates is described as $\{q^n\} = \{x, y, \theta, \gamma, \psi, \varphi_b, \varphi_f\}$, [3].

The wheels of the snakeboard are assumed to roll without lateral sliding. This condition is modeled by constraints which may be shown to be nonholonomic (Eq.s 1 and 2).

$$g_1 = \dot{x}\sin(n\gamma + \theta - \varphi_b) - \dot{y}\cos(n\gamma + \theta - \varphi_b) + cn\dot{\gamma}\cos(\varphi_b) + l\dot{\theta}\cos(n\gamma - \varphi_b) = 0 \tag{1}$$

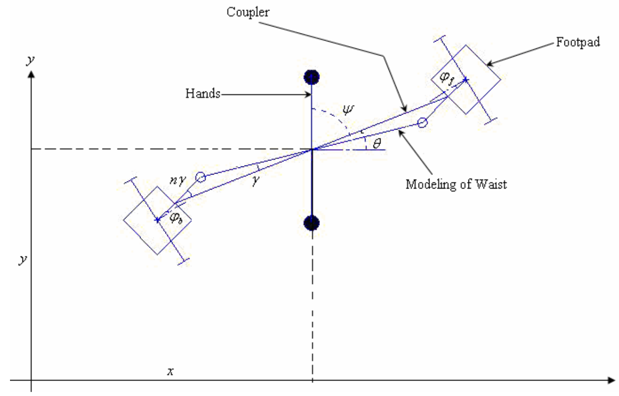


Fig 4: A Simplified Model of the Designed Mechanism with Generalized Coordinate

$$g_2 = \dot{x}\sin(n\gamma + \theta + \varphi_f) - \dot{y}\cos(n\gamma + \theta + \varphi_f) - cn\dot{\gamma}\cos(\varphi_f) - l\dot{\theta}\cos(n\gamma + \varphi_f) = 0 \tag{2}$$

For this set of generalized coordinates, the kinetic energy is calculated in equation 3.

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}J_{wr}(\dot{\theta} + \dot{\gamma})^2 + J_{pad}(\dot{\theta} + n\dot{\gamma})^2 + \frac{1}{2}J_w(\dot{\theta} + n\dot{\gamma} + \dot{\varphi}_b)^2 + \frac{1}{2}J_w(\dot{\theta} + n\dot{\gamma} + \dot{\varphi}_f)^2 + \frac{1}{2}J_h(\dot{\theta} + \dot{\psi})^2 \tag{3}$$

Where m is the total mass of the board, J is the inertia of the coupler, J_{wr} is the inertia of the waist, J_{pad} is the inertia of the footpads, J_w is the inertia of the wheels, J_h is the inertia of the hands. The equations of motion are derived by Lagrange equation of motion (Eq. 4) and are as follows:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}^n}\right) - \frac{\partial T}{\partial q^n} - Q_n + \sum_r \lambda^r \frac{\partial g_r}{\partial q^n} = 0 \tag{4}$$

$$m\ddot{x} + \lambda_1\sin(n\gamma + \theta - \varphi_b) + \lambda_2\sin(n\gamma + \theta + \varphi_f) = 0 \tag{5}$$

$$m\ddot{y} - \lambda_1\cos(n\gamma + \theta - \varphi_b) - \lambda_2\cos(n\gamma + \theta + \varphi_f) = 0 \tag{6}$$

$$\left(J+2J_{pad}+2J_w+2J_{wr}+2J_h\right)\ddot{\theta}$$
$$+J_w\left(n\ddot{\gamma}+\ddot{\varphi}_b\right)+J_w\left(n\ddot{\gamma}+\ddot{\varphi}_f\right)+$$
$$J_h\ddot{\psi}+J_{wr}\ddot{\gamma}+2nJ_{pad}\ddot{\gamma}$$
$$+\lambda_1 l\cos\left(n\gamma-\varphi_b\right)-\lambda_2 l\cos\left(n\gamma+\varphi_f\right)=0 \tag{7}$$

$$J_h\ddot{\psi}+J_h\ddot{\theta}=\tau_1 \tag{8}$$

$$J_w\ddot{\varphi}_b+J_w\left(\ddot{\theta}+n\ddot{\gamma}\right)=\tau_2 \tag{9}$$

$$J_w\ddot{\varphi}_f+J_w\left(\ddot{\theta}+n\ddot{\gamma}\right)=\tau_3 \tag{10}$$

$$J_{wr}\left(\ddot{\gamma}+\ddot{\theta}\right)+2J_{pad}\left(n\ddot{\gamma}+\ddot{\theta}\right)$$
$$+J_w\left(2n\ddot{\gamma}+2\ddot{\theta}+\ddot{\varphi}_b+\ddot{\varphi}_f\right)$$
$$+\lambda_1 cn\cos\left(\varphi_b\right)-\lambda_2 cn\cos\left(\varphi_f\right)=\tau_4 \tag{11}$$

Where $\left(\tau_1,\tau_2,\tau_3,\tau_4\right)$ are the input torques in $\left(\psi,\varphi_b,\varphi_f,\gamma\right)$ directions, respectively. In these set of equations, l is half of the coupler length and c is the distance between points A and C in figure 3.

4. DYNAMICS SIMULATION

For the equations of motion, dynamics simulation is carried out for a system by the following specifications:

$m=6 \quad Kg$,

$J=J_h=0.01 \quad Kg.m^2$,

$J_{pad}=J_w=0.001 \quad Kg.m^2$ [1]

$J_{wr}=0.002 \quad Kg.m^2$.

The simulations are performed for $\left(\tau_1,\tau_2,\tau_3,\tau_4\right)$ as the inputs. The output of the simulation is the set $\left(x,y\right)$. In the actual snakeboard, the rider turns his feet in opposite directions. Thus for modeling the motion, it is considered that $\varphi_b=-\varphi_f$. There are two parameters which can be changed in the simulations. One of these parameters is the angular velocity of the input torques with respect to each other and the other parameter is the amplitude of the torques [1,2]. The results of simulation for four different set of inputs are shown in figures 5 through 8.

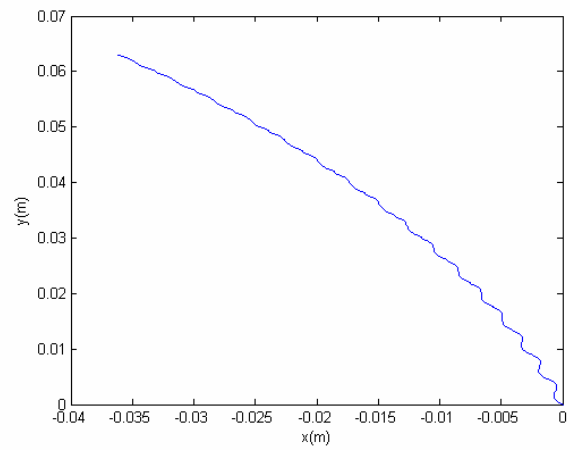


Fig 5: The result of dynamic simulation for $\tau_1=\tau_2=-\tau_3=0.06\sin\left(t\right)$ and $\tau_4=\sin\left(t\right)$

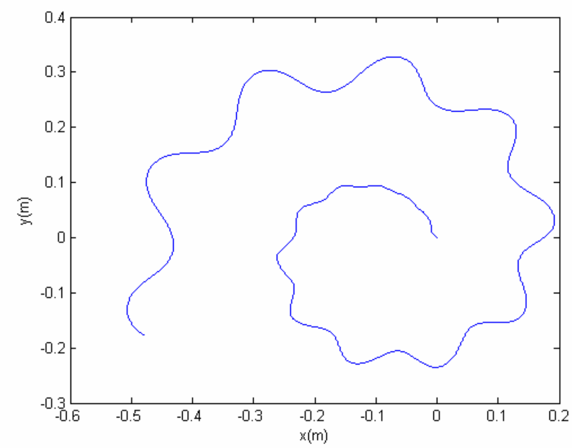


Fig 6: The result of dynamic simulation for $\tau_1=\tau_2=-\tau_3=0.06\sin\left(t\right)$ and $\tau_4=\sin\left(t\right)$

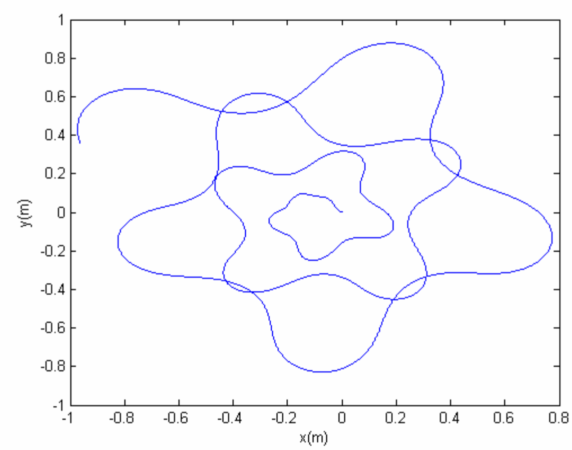


Fig 7: The result of dynamic simulation for $\tau_1=\tau_2=-\tau_3=0.06\sin\left(2t\right)$ and $\tau_4=0.8\sin\left(t\right)$

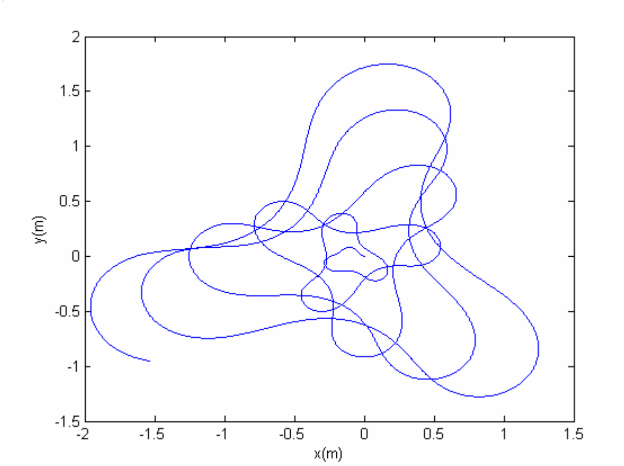


Fig 8: The result of dynamic simulation for $\tau_1 = \tau_2 = -\tau_3 = 0.06\sin(2t)$ and $\tau_4 = 0.6\sin(t)$

There is the change of two parameters that affect the results of simulations. First, the change in the amplitude of the torques and second, the change in the angular velocity of the input torques. As a result of dynamics simulation, it can be seen that the changes in the amplitude of the torques, result in change of the general amplitude of the motion (figures 5 and 6). In addition, an increase in the angular velocity of the torques, make the snakeboard to pass through a spiral path (figure 7 and 8).

5. CONTROL OF THE SNAKEBOARD

In order to control of the snakeboard, two methods for controlling of nonlinear systems are applied to the snakeboard. One of these methods is Sliding Mode Control and the other one is using Neural Network as the controller of the system.

Since $\tau_2 = -\tau_3$, the controller output is the set of torques (τ_1, τ_2, τ_4). Additionally, the desired output of the control system is the set variables (x, y, θ).

5.1 Sliding Mode Control

For controlling the system by sliding mode method, the system should be reduced to a square system. In a square system, the number of desired control variables is the same as number of control inputs. By the means of equations of motion, three equations are derived to be used in the controller [4].

$$\ddot{x} = \frac{1}{m}\sin(n\gamma - \varphi_b + \theta) \times \left(-\frac{J_1}{J_2}\tau_1 - \frac{J_1}{J_2}\tau_2 + \frac{J_1}{J_2}a_2(t) - a_1(t) + \tau_4\right) \tag{12}$$

$$\ddot{y} = -\frac{1}{m}\cos(n\gamma - \varphi_b + \theta) \times \left(-\frac{J_1}{J_2}\tau_1 - \frac{J_1}{J_2}\tau_2 + \frac{J_1}{J_2}a_2(t) - a_1(t) + \tau_4\right) \tag{13}$$

$$\ddot{\theta} = \frac{1}{J_2}(\tau_1 + \tau_2 - a_2(t)) \tag{14}$$

where a_1 and a_2 are functions of time with limited bounds which are perceived from the results of dynamics simulation.

$$0 \le a_1(t) \le 0.14 \tag{15}$$

$$0 \le a_2(t) \le 0.17 \tag{16}$$

For tracking a straight line, a sliding mode controller is designed with the following specifications: $k_x = 450$, $k_y = 480$, $k_\theta = 550$, $\lambda_x = 0.4$, $\lambda_y = 0.3$, $\lambda_\theta = 0.2$, $\varphi_x = \varphi_y = \varphi_\theta = 0.1$. For this controller the results of trajectory tracking are shown in figures 9 through 12.

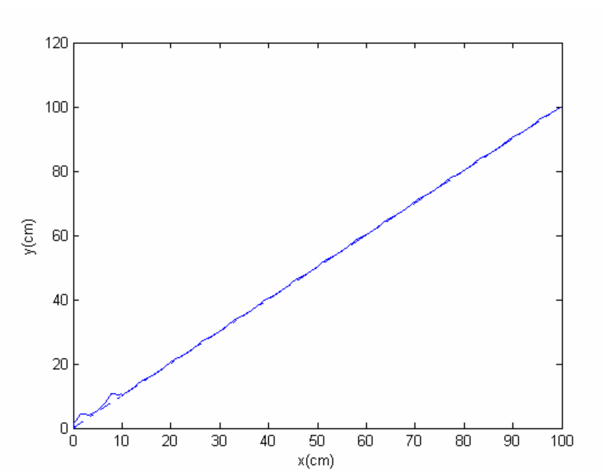


Fig 9: Result of trajectory tracking (The dashed line is the target and the bold line is the sliding mode system response for $y = x$ path)

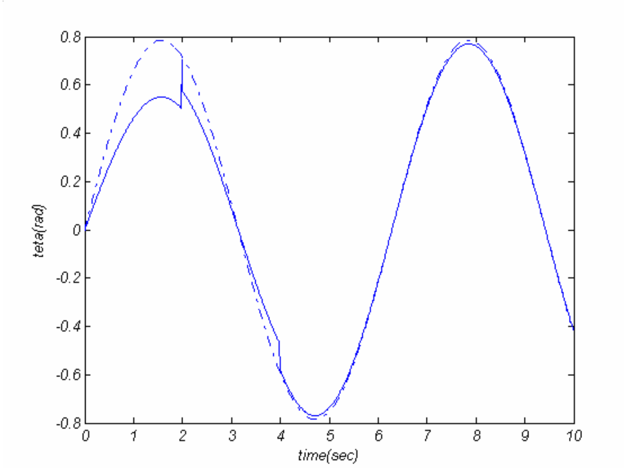
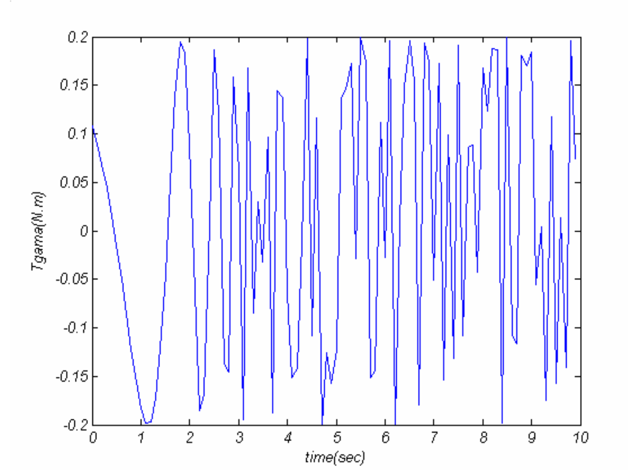


Fig 10: Result of trajectory tracking for θ (The dashed line is the target and the bold line is the sliding mode system response for $y = x$ path)



(a)

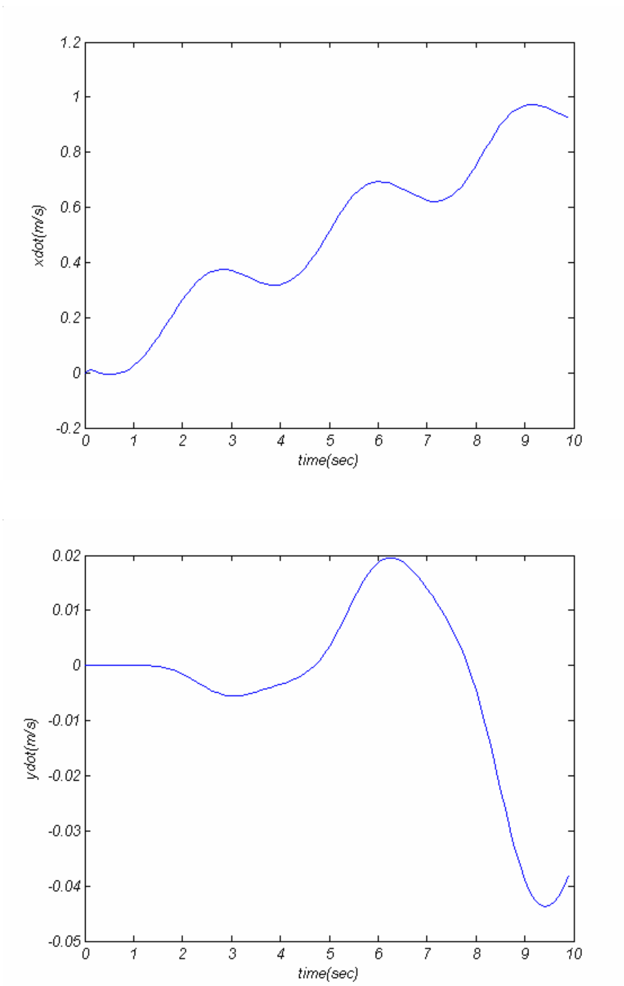
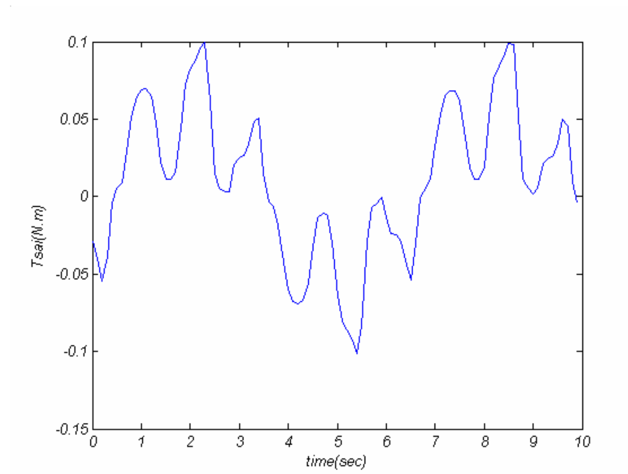
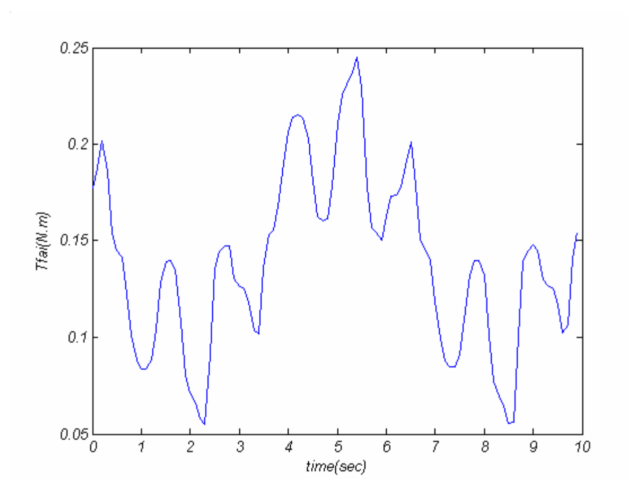


Fig 11: Velocities along x and y directions for sliding mode controller for $y = x$ path



(b)



(c)

Fig 12: The control torques,(a) waist torque, (b) hands torque and (c) wheels torque, for sliding mode controller for $y = x$ path

For tracking a sinusoidal path a sliding mode controller is designed with the specifications as follows: $k_x = 300$ ، $k_y = 350$ ، $k_\theta = 270$ ، $\lambda_x = 0.4$ ، $\lambda_y = 0.3$ ، $\lambda_\theta = 0.2$ ، $\varphi_x = \varphi_y = \varphi_\theta = 0.1$. The path tracking result is depicted in figure 13.

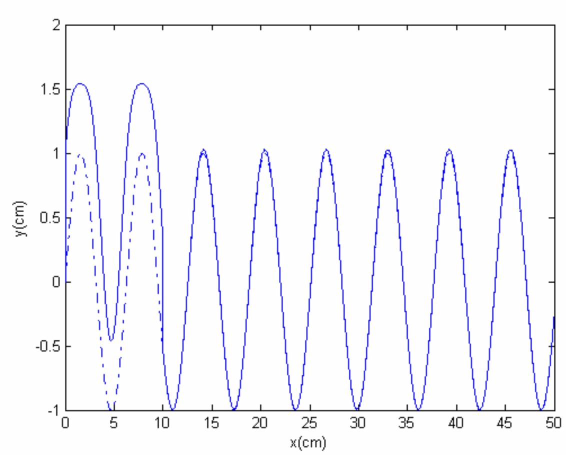


Fig 13: Result of trajectory tracking (The dashed line is the target and the bold line is the sliding mode system response for $y = \sin(x)$ path)

5.2 Using Neural Networks as the Controller of the System

Here again the problem of trajectory tracking of the system is discussed but this time using a neural network as the controller of the system is in use. The results are shown in figures 14 and 15.

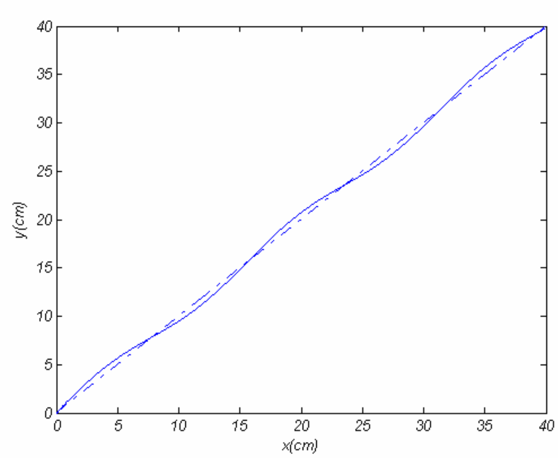


Fig 14: Result of trajectory tracking by a neural network as the controller (The dashed line is the target and the bold line is the system response)

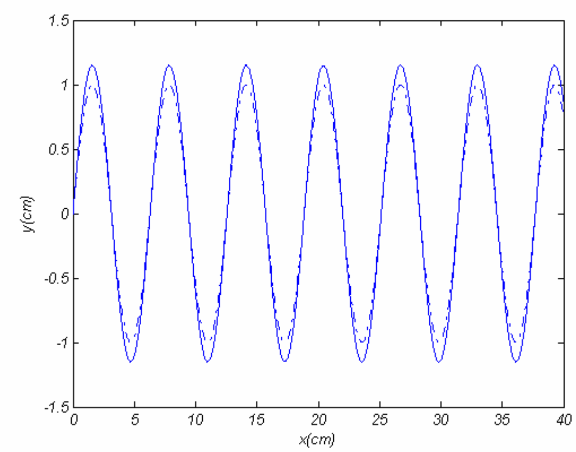


Fig 15: Result of trajectory tracking by a neural network as the controller (The dashed line is the target and the bold line is the system response)

It is seen in both situations (tracking a straight line and a sinusoidal path) that the speed of the neural network controller is more than the speed of the sliding mode controller. But in both situations the accuracy of the sliding mode controller is much higher than that of accuracy of the neural network controller.

6. CONCLUSION

In this paper, a mechanism is designed to simulate the motion of the human body on a snakeboard. For this mechanism and the board the equations of motion are derived by Lagrange's method. For these equations dynamics simulation is carried out. It is seen that the motion of the snake board, is a snake-like motion. Finally, two methods of nonlinear control, sliding mode and neural networks are applied to the system and the results of trajectory tracking are compared.

7. REFERENCES

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