

## OPTIMIZATION OF FRP LAMINATED COMPOSITES USING FEM AND PARALLEL GENETIC ALGORITHM

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### ABSTRACT

In the present work, Island Model Parallel Genetic Algorithm (IMPGA) in conjunction with 3D Finite Element Method (FEM), has been used for optimization of hybrid Fiber Reinforced Plastic (FRP) laminated plates subjected to transverse impact loading. Fiber orientation, material and thickness in each lamina as well as number of lamina in the laminate have been used as design variables and impact induced delamination and matrix cracking have been used as failure criteria. Multi-objective approach has been used to achieve the optimum design of a laminate for combined normalized weighted cost and weight minimization. It has been observed that IMPGA-FEM module leads to far superior results compared to serial GA(SGA)-FEM both in terms of convergence as well as computational time.

**Keywords:** Island Model Parallel Genetic Algorithm, Fiber Reinforced Plastic Composites

### 1. INTRODUCTION

An important issue in the design of laminated FRP structures is the optimal selection of number of plies, fiber angle of each ply, material of each ply, thickness of each ply and thus the laminate thickness. By appropriate selection of stacking sequence and material of each lamina, it is possible to impart directional strength in FRP laminates allowing the designer to achieve a reduced cost and weight of the component while ensuring the required safety. From the manufacturing constraints, ply angle and ply thickness are to be selected from a set of discrete values and the optimum design of laminated FRP structure becomes a discrete optimization problem. Optimal selection of stacking sequence or optimum design of laminated FRP composites has been an important area of research and many research works have been reported in this direction till date. Some of them are heuristic approaches towards composite material selection and design [1, 2] based on expert systems and knowledge of experienced designers and are problem specific and non-unique. Schmit and Farshi [3] studied stacking sequence design by treating ply thicknesses as continuous design variables, which were finally rounded off. Some of the important design issues that have been successfully discussed in tailoring are thickness balance of laminated composites, stacking sequence design [3, 4, 5], and maximizing buckling load [6, 7]. Todoroki and Haftka [8] presented a repair strategy using GA for stacking sequence optimization of laminated composites leading to a balance laminate. Qu et. al. [9] used deterministic and reliability based designs of composite laminates for cryogenic environments. Savic et. al. [10] optimized composite I-sections using a global search

algorithm namely improving hit and run, which allowed design variables to be either discrete or continuous with user defined discretization. Genetic Algorithms (GAs) are non-deterministic, have the ability to work in a discrete search space and in the recent time have been successfully applied to the problem of composite design optimization [11, 12, 13, 14]. Venkataraman [15] reviewed major works on the optimization of composite panels in recent years. Few works in the optimization of hybrid laminates are also reported in literature [5, 16]. Multi-objective optimization of hybrid composite laminates using SGA and FEM has also been reported for static and dynamic loading [17, 18]. Though GAs have demonstrated the potential to overcome many of the problems associated with gradient-based methods and are most effective when the design space is large, high computational time and storage requirement often forces to work with a reduced design space and in such cases parallel GA could be advantageously used. Island model GA [19, 20] is one, which ideally suits to parallel computing environment leading to Island Model Parallel Genetic Algorithm (IMPGA). Applicability of parallel GA in structural optimization of composite laminates has not been studied in details though preliminary attempt has been made by the present authors [21], showing the superiority of IMPGA in optimization of composite laminates over SGA.

Therefore, in the present work, IMPGA has been implemented in a distributed memory platform for optimization of symmetric and balanced laminates in order to study its effectiveness by varying the number of processors as well as the population size in each processor. A multi-objective design problem for

combined normalized weighted cost and weight optimization of a transversely impacted Graphite/Epoxy (T300/5208)-Aramid/Epoxy (Kevlar 49) hybrid composite plate has been carried out.

## 2. CONTACT IMPACT ANALYSIS

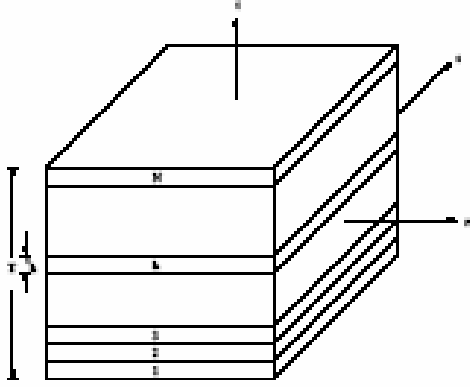


Fig 1: 3D layered solid element

Layered 3D eight-nodded solid elements as shown in Fig.1 with three translational degrees of freedom at each node have been used for finite element modeling of the laminated plate.

The shape functions of the element are:

$$N_i = \frac{1}{8}(1 + rr_i)(1 + ss_i)(1 + tt_i) \quad (1)$$

$i = 1, 2, 3, \dots, 8$

To adequately simulate the flexural response, extra shape functions have been introduced.

Considering a laminate made up of  $N$  layers with a total thickness of  $T$ , the stiffness matrix is given by [22]

$$[K] = \frac{2}{T} \int_{-1}^1 \int_{-1}^1 \sum_{k=1}^N \frac{t_k - t_{k-1}}{2} \int_{-1}^1 [B]^T [C] [B] J |drdsdt \quad (2)$$

where,  $t_k$  is the thickness of the  $k^{th}$  layer.

The impactor is modeled as an elastic body with a spherical nose as shown in Fig.2. During loading and unloading, the contact force distribution is determined using the Hertzian contact law. Thus, the contact force  $f$  can be related to the indentation depth  $\alpha$ , the distance between the center of the impactor's nose and mid surface of the plate [22] as

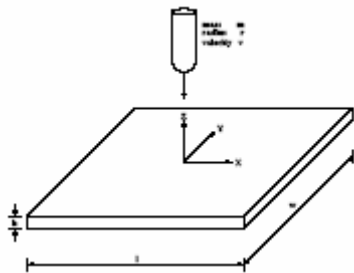


Fig 2: Description of the problem

$$f = K' \alpha^{1.5} \quad \text{during loading and} \quad (3)$$

$$f = f_m \left[ \frac{\alpha - \alpha_0}{\alpha_m - \alpha_0} \right]^{2.5} \quad \text{during unloading} \quad (4)$$

where  $K'$  is the modified constant of the Hertz contact theory,  $f_m$  is the maximum contact force just before unloading,  $\alpha_m$  is the maximum indentation corresponding to  $f_m$  and  $\alpha_0$  is the permanent indentation during this loading/unloading process calculated in terms of critical indentation  $\alpha_{cr}$  following [22].

## 3. FAILURE ANALYSIS

### 3.1 Critical matrix cracking

In order to assess the possible matrix cracking in the laminated plate due to transverse impact, matrix cracking criterion proposed by Choi et al [23] has been used in the present work. The criterion is expressed as:

$$\left( \frac{\bar{\sigma}_{yy}^n}{Y^n} \right)^2 + \left( \frac{\bar{\sigma}_{yz}^n}{S_i^n} \right)^2 = e_M^2 \quad (5)$$

$$e_M \geq 1 \text{ failure}$$

$$e_M < 1 \text{ no failure}$$

$$Y^n = Y_t^n \text{ if } \bar{\sigma}_{yy} \geq 0$$

$$Y^n = Y_c^n \text{ if } \bar{\sigma}_{yy} < 0 \quad (6)$$

### 3.2 Delamination at interface

In order to assess delamination initiation at the interface of the laminate, the criterion proposed by Choi et al [23] for impact induced delamination has been used in the present work. The criterion is:

$$D_a \left[ \left( \frac{\bar{\sigma}_{yz}^n}{S_i^n} \right)^2 + \left( \frac{\bar{\sigma}_{xz}^{n+1}}{S_i^{n+1}} \right)^2 + \left( \frac{\bar{\sigma}_{yy}^{n+1}}{Y^{n+1}} \right)^2 \right] = e_D^2 \quad (7)$$

$$e_D \geq 1 \text{ failure}$$

$$e_D < 1 \text{ no failure}$$

$$Y^{n+1} = Y_t^{n+1} \text{ if } \bar{\sigma}_{yy} \geq 0$$

$$Y^{n+1} = Y_c^{n+1} \text{ if } \bar{\sigma}_{yy} < 0 \quad (8)$$

and  $D_a$  is an empirical constant determined from experiment, which has been taken as 1.8 [23].

## 4. OPTIMUM DESIGN OF LAMINATE S

In the present work Island Model Parallel GA has been used where the overall population of chromosomes is partitioned into a number of sub-populations. Each sub-population evolves independently for optimizing the same objective function. Some logical topology for how the populations are interconnected is defined and

periodically each sub-population replaces its chosen chromosome, as per migration strategy, with the best of its neighbor's. Three types of optimization problems have been considered viz. cost minimization, weight minimization and combined cost weight minimization of Graphite/Epoxy(T300/5208)-Aramid/Epoxy(Kevlar 49) hybrid composite plate while subjected to transverse impact. The design variables are, ply angle, ply material and ply thickness of each ply along with the total number of plies in the laminate and hence the thickness of laminate.

The objective functions for cost minimization is:

$$\text{Minimize } f_{\text{cost}}(t, \rho_{\text{ply}}, C_{\text{ply}}) = (L \times B) \sum_{i=1}^N t_i \rho_i C_i \quad (9)$$

The objective functions for weight minimization is:

$$\text{Minimize } f_{\text{wt}}(t, \rho_{\text{ply}}) = (L \times B) \sum_{i=1}^N t_i \rho_i \quad (10)$$

where  $t$ ,  $\rho$ ,  $C$ ,  $L$  and  $B$  represent ply thickness, ply material density, cost of the material per unit, length and breadth of the plate respectively.

The objective function for combined cost and weight optimization has been obtained through a process of normalization as:

$$\text{Minimize } f_{\text{combined}}(t, \rho_{\text{ply}}, C_{\text{ply}}) = (1-\alpha) \frac{f_{\text{wt}}}{W_{\text{min}}} + \alpha \frac{f_{\text{cost}}}{C_{\text{min}}} \quad \text{where } 0 \leq \alpha \leq 1.0 \quad (11)$$

Here  $C_{\text{min}}$  and  $W_{\text{min}}$  represent the cost and weight corresponding to the laminates with minimum cost (Eq.9) and minimum weight (Eq.10) respectively and  $\alpha$  is the weight assigned to normalized units,  $0 < \alpha < 1$ .

The combined effect of critical matrix cracking (Eq.5) and delamination at interface (Eq.7) has been taken as the failure criterion. Whichever occurs first is taken as the cause of failure. The failure criterion is: Failure Index

$$F.I = \max\{e_D, e_M\} \text{ fails if either } e_D \text{ or } e_M \geq 1 \quad (12)$$

## 5. NUMERICAL RESULTS AND DISCUSSIONS

In the present work, a computer code has been developed in 'C', which has two distinct modules viz. the FEM module and the GA module (serial or IMPGA). The code has been run on a parallel platform, PARAM Padma, which is having one Teraflop peak computing power and having Power4 RISC processors.

Orientation gene - [E/E/0°/-15°/75°/45°],  
 Material gene - [E/E/m<sub>2</sub>/m<sub>2</sub>/m<sub>2</sub>/m<sub>2</sub>],  
 Thickness gene - [E/E/0.10/0.45/0.15/0.25].

	E	E	E
	0°	m <sub>2</sub>	0.10
	0°	m <sub>2</sub>	0.10
	-15°	m <sub>1</sub>	0.45
	15°	m <sub>1</sub>	0.45
	75°	m <sub>2</sub>	0.15
	-75°	m <sub>2</sub>	0.15
	45°	m <sub>2</sub>	0.25
	-45°	m <sub>2</sub>	0.25

Mid - Plane

Fig 3: Three gene strings for hybrid laminate. "E" is the gene corresponding to a null layer

Table 1: Material properties

Property	Gr/Epoxy	Kev/Epoxy
$E_{xx}$	181 GPa	76 GPa
$E_{yy}$	10.3 GPa	5.5 GPa
$G_{xy}$	7.17 GPa	2.3 GPa
$\nu_{xy}$	0.28	0.34
$\nu_{yz}$	0.28	0.34
$\rho$	1600 kg/m <sup>3</sup>	1360 kg/m <sup>3</sup>
Cost	6. units/kg	2. units/kg
$X_t$	1500 MPa	1400 MPa
$X_c$	1500 MPa	235 MPa
$Y_t$	40 MPa	12 MPa
$Y_c$	246 MPa	53 MPa
$S_t$	68 MPa	34 MPa
$V_f$	0.7	0.7

A square laminated plate (0.0762m × 0.0762m) with arbitrary ply orientations and thicknesses, clamped along all the edges, subjected to transverse impact (of an aluminum spherical impactor with diameter 0.0127 m) at the center has been considered for the analysis. The symmetric laminate may have any number of plies and each ply may be made of either Aramid/Epoxy or Graphite/Epoxy (refer Table 1 for properties). The ply orientation of each ply could be between -90° and 90° with increments of 15°. The ply thickness of each ply can vary between 0.1mm to 0.5mm with increments of 0.1mm.

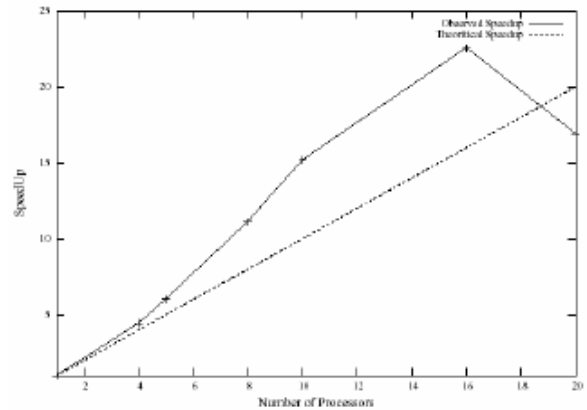


Fig. 4: Actual speedup with theoretical speedup

In the present work, the length of the gene string is kept fixed throughout the optimization process, however, having empty plies makes it possible to change the laminate thickness during the optimization process. To accommodate two or more materials, three strings of genes have been introduced, namely ply orientation, ply material and ply thickness. Figure 3 represents triple gene strings for hybrid laminates with provision for thickness alterations. The initial population starts with laminates having randomly chosen number of plies and

corresponding to each ply, ply thickness, ply material and fiber orientation are also chosen at random. Single point crossover is the main genetic operator while mutation induces random changes in the genes and prevents the search from getting stuck in a local optimum. Gene swap is used to swap the positions of two genes in the chromosome. In addition to the existing GA operators, in the island model parallel GA implementation, migration

Table 2: Optimum laminates obtained by 8 processors IMPGA and SGA for weight minimization (Population size=80 and impactor velocity=9 m/s)

No. of Proc.	Optimum Laminate Configuration
1	[ 0.0 <sub>2K</sub>   0.000500/ 0.0 <sub>2K</sub>   0.000200/ 0.0 <sub>2GR</sub>    0.000200/ 0.0 <sub>2K</sub>   0.000450/ ± 15.0 <sub>2K</sub>    0.000100 ] <sub>s</sub> Weight =4.6916x10 <sup>-2</sup> kg, Thickness=0.0058 m
8	[0 <sub>2K</sub>   0.000100/ μ 45.0 <sub>2K</sub>   0.000100/ ± 60.0 <sub>K</sub>   0.000100 ] <sub>s</sub> Weight =1.263481x10 <sup>-2</sup> kg, Thickness0.001600 m

In a laminate[01<sub>m1</sub>||t<sub>1</sub>/ 02<sub>m2</sub>||t<sub>2</sub>] θ<sub>1</sub> and θ<sub>2</sub> are ply angles t<sub>1</sub> and t<sub>2</sub> ply thicknesses and m1 and m2 are ply materials

routine has been used to facilitate the exchange of chromosomes among the subpopulations at different processors. Figure 4 shows the CPU time required in the present study for solving weight optimization problem using FEM–SGA module and FEM-IMPGA module and it could be observed that the actual time reduction outperforms the theoretical time reduction. It should be noted here that in the case of simple parallel GA, the maximum speed-up possible is *n* (in the case of negligible communication overhead). This again justifies the use of IMPGA rather than the use of simple parallel GA.

To show the performance of the parallel algorithm with increasing number of processors, for a fixed population size of 80, convergence pattern for weight optimization has been studied and the results obtained are presented in Table 2, which shows the comparison of the laminates obtained using 8 processor IMPGA on parallel platform and the SGA. It could also be observed from the Table 2 that the optimum laminate obtained from 8 processor IMPGA is much lighter and thinner compared to that obtained from SGA. But beyond 10 processors, when the same optimization has been carried out with 16 and 20 processors, the optimal laminates have been observed to be inferior though they are still lighter and thinner compared to that obtained from SGA. This increase in weight/thickness beyond 10 processors in the present case shows that at each node, IMPGA requires a minimum size of subpopulation (critical sub-population size) below which the performance deteriorates. Figure 5 shows the convergence of weight minimization problem with increasing number of processors in the integrated FEM-IMPGA module. Table 3 shows the optimum laminates obtained for weight minimization, cost minimization and combined

cost-weight minimization problems using the integrated FEM and IMPGA module with 8 processors.

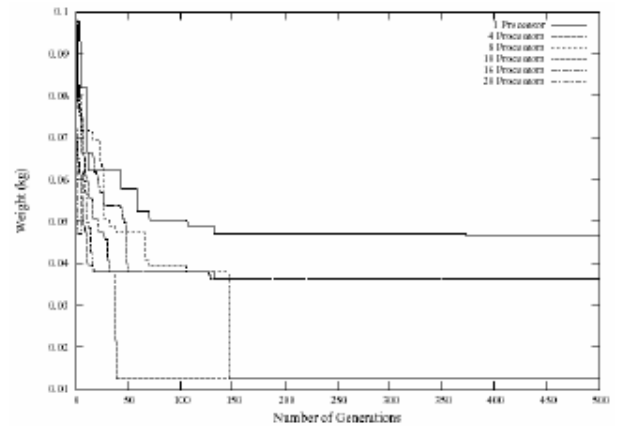


Fig 5: Convergence of weight minimization problem (Population size=80 and impactor velocity=9 m/s)

Table 3: Optimum laminates obtained FEM-IMPGA with 8 processors population size = 80 and impact velocity=12 m/s

Opti. Mode	Laminate Thickness	Optimum Laminate Configuration
Cost	0.006400 m	[0.0 <sub>2K</sub>   0.000450/ μ 15.0 <sub>K</sub>   0.000300/ μ 30.0 <sub>K</sub>   0.000500 / μ 45.0 <sub>K</sub>   0.000350 ] <sub>s</sub> cost=1.114391x10 <sup>-1</sup> units
Weight	0.005600 m	[0.0 <sub>2K</sub>   0.000500/0.0 <sub>2GR</sub>   0.000400/ ± 30.0 <sub>K</sub>   0.000350/ ± 45.0 <sub>K</sub>   0.000150 ] <sub>s</sub> weight=4.645152x10 <sup>-2</sup> kg
Combined	0.006400 m	[0.0 <sub>2K</sub>   0.000450/ μ 15.0 <sub>K</sub>   0.000200/ μ 15.0 <sub>K</sub>   0.000100 / μ 30.0 <sub>K</sub>   0.000500/ μ 45.0 <sub>K</sub>   0.000350 ] <sub>s</sub> cost=1.114391x10 <sup>-1</sup> units weight=5.053925x10 <sup>-2</sup> kg

## 6. CONCLUSIONS

An integrated module comprising of IMPGA and FEM has been developed for optimization of symmetric and balanced laminated hybrid composites. The present integrated module gives faster convergence along with lighter and less costly balanced symmetric laminate as compared with those obtained from integrated FEM and SGA module. Further, it has been observed that the speed up obtained from the present module is better than the possible theoretical speed up. A probabilistic migration strategy has been found to yield better convergence compared to the deterministic one. It has also been observed that for efficient working of IMPGA, a minimum size of sub population on each processor is a necessary requirement. In summarizing, the IMPGA with an improved migration strategy in searching optimal laminates outperforms the SGA in terms of reduced computational time and better-converged solution.

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