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ON THE EFFECT OF DISTRIBUTED ATTACHED MASSES ON EXPONENTIAL ROTATING DISKS

Shubhankar Bhowmick[1] and Kashinath Saha[2]

Department of Mechanical Engineering, Jadavpur University, Kolkata, INDIA
[1]shubh.ju@gmail.com, [2]kashinathsaha@gmail.com

ABSTRACT

Based on principle of Minimum Potential Energy and von-Mises yield criterion, limit angular speed of high speed rotating disk carrying externally distributed attached masses is obtained and the stress and displacement behaviour is investigated. The effect of geometry and distributed loading parameters on the stress and deformation states is reported.

Keywords: von-Mises stress, Limit Speed, Distributed Attached Masses, Variational Method.

1. INTRODUCTION

High speed rotating disks find widespread application in industry, some common examples being high speed impellers, gears and flywheels. The most primitive form of a rotating disk is a thin cylindrical object and its state of stress is a function of its angular speed. At a particular speed the developed stresses exceed the yield stress of the material. This particular speed is termed as limit angular speed of the rotating disk. It is well understood that stresses in a rotating uniform disk are maximum at its root. This finding initiated the idea of radially varying the thickness of a disk without hampering its axisymmetric nature, so that more material is accumulated at the root of the disk. This leads to higher range of operating speeds apart from reducing the rotary inertia of the disk.

Analysis of state of stress in such disks up to the point of yielding has been extensively carried out till date by many researchers. In [1], investigation of elastic stress behaviour for isotropic material possessing non-linear stress-strain relationship is performed using dynamic relaxation technique. Exponential variation of thickness was discussed in [2] assuming Tresca's criterion and its associated flow rule along with linear strain hardening material behaviour, although solution satisfying all boundary and continuity conditions was not obtained. Eraslan and Orcan [3], studied a similar problem and presented an analytical solution that satisfy all boundary and continuity conditions. The analytical method was further applied in [4] to obtain solution for elastic deformation of solid and annular disks having parabolically varying thickness subjected to different boundary conditions. Bhowmick et al [5], has provided an efficient numerical scheme for the estimation of limit angular speed of disks with and without attached masses by application of variational principle involving Galerkin's principle and assuming a series solution. The proposed methodology in [5] has been further extended to study the post-elastic behaviour and elastic behaviour in [6] and [7] for uniform and non-uniform disks and the results reported were found to be in good agreement with those available in literature.

Substantial amount of research have been carried out over the past few decades on the mechanical design of high-speed rotating disk applications. However, the effect of external loading of such applications on the limit speed has not been considered by any researchers. In the present study, the problem is formulated by using a variational method and analysis is carried out for exponential disk geometry and limit speed is obtained for each case. The results are presented in dimensionless form and the curves documented may be readily used as design monographs. The obtained results have provided a substantial insight into our understanding and may facilitate analysis in new application areas such as centrifugal impellers.

2. MATHEMATICAL MODELLING

A high speed rotor carrying blades, symmetrically distributed at any radius, is a classic example of externally loaded rotating disk (refer Fig. 1). It is quite obvious that the blades attached with the central disk imparts additional centrifugal load on it. These blades, modeled as attached masses, are distributed radially at various points within a certain portion of the disk. Here it is important to note that for a single inlet impeller, the blades will also introduce bending moment on the disk. However, in the present study, such complication is avoided and hence the result is readily applicable for double inlet impellers only.

The effect of attached mass is taken into consideration through a factor $\boldsymbol{\alpha}$, which indicates the ratio of the attached mass to the mass of the total disk. The mathematical modeling of the problem has been given in detail in [7] for the case of concentrated attached mass. In the present paper this attached mass is modeled as distributed over a span defined by s_m about the mean

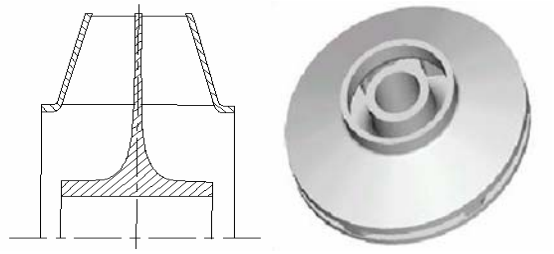


Fig 1: Typical double inlet impeller

radius r_m. This attached mass will contribute to the centrifugal force, thus appearing in the right hand side external load expression (refer eq.(1)) of the governing equation given below as

$$\frac{E}{1-\mu^2}\sum_{i=1}^{n}\sum_{j=1}^{n}c_i\int_0^1\left\{\frac{\phi_i\phi_j}{\xi}+\mu(\phi_i'\phi_j+\phi_i\phi_j')+\xi\phi_i'\phi_j'\right\}hd\xi$$

$$=\rho\omega^2b^3\sum_{i=1}^{n}\int_0^1\left\{\xi^2\phi_i\right\}hd\xi+\rho\lambda h_b\omega^2\int_0^1(\lambda\xi_1+r_{ml})^2\phi_i d\xi_1 \quad (1)$$

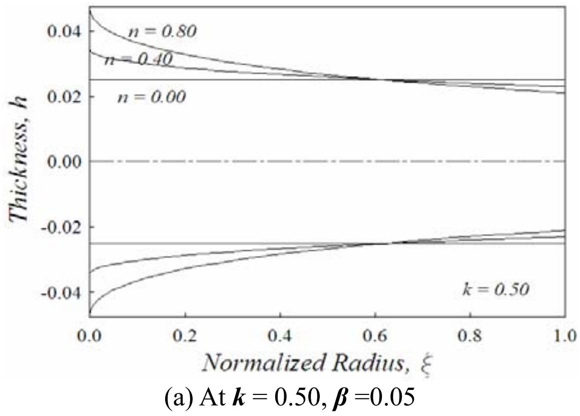
The above equation can be expressed in matrix form as, $[K]\{c\}=\{R\}$ and solution of unknown coefficients is obtained from $\{c\} = [K]^{-1}\{R\}$, implemented numerically by using IMSL routines. The present study is based on von Mises' yield criteria and assuming plane stress condition the yield condition is considered as

$$\sigma_r{}^2-\sigma_r\sigma_\theta+\sigma_\theta{}^2\geq\sigma_0{}^2\,. \quad (2)$$

Further the disk thickness is considered to vary exponentially according to the function

$$h(\xi)=h_0\,exp\left\{-n(\xi)^k\right\}. \quad (3)$$

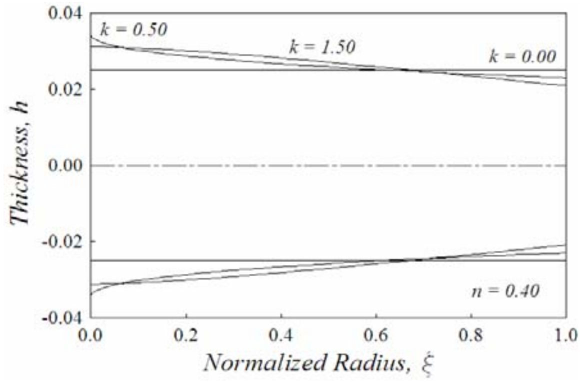
where n and k are the geometry parameters affecting the thickness variation as shown in Fig. 2.



(a) At k = 0.50, β =0.05

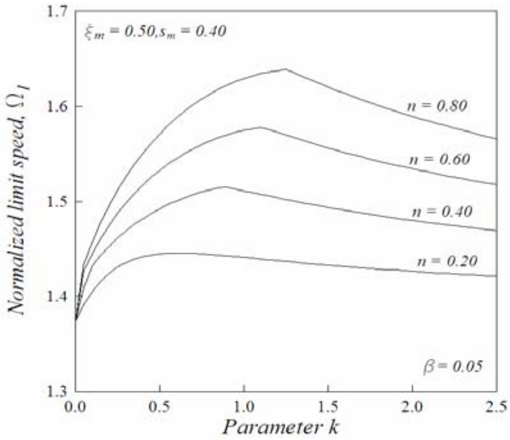
3. RESULTS AND DISCUSSION

The numerical values of the different system parameters, considered in the present work are, E = 207 GPa, μ = 0.3, ρ = 7850 kg/m^3 and σ_0 = 350 MPa. The dimensionless angular speed, $\omega b\sqrt{\rho/\sigma_0}$ corresponding to the onset of yielding is defined as Ω_l and is considered as an important design parameter. The variation of disk thickness for different values of geometry parameters n, k and β is plotted in Fig.2 (a-b). Here β is the thickness parameter (h_c/b) that governs the thickness of the disks. The analysis is carried out for various exponential disk geometries equal to that of a uniform thickness disk having b = 1.0 and thickness h_c = 0.05. In Fig. 2(a), it is observed that for a given value of parameter k, increase in n results in increase in root thickness of the disk while keeping the nature of the profile same. The dependence of nature of profile on the other geometry parameter k is indicated in Fig. 2(b). The present study is carried out on variable thickness disks having mass equal to that of a uniform disk whose thickness is β times the outer radius of the disk. In the present formulation, β is assumed to be 0.05, thereby justifying plane stress formulation.

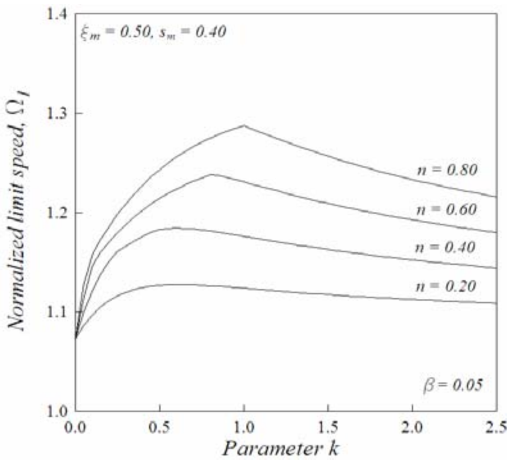


(b) At n = 0.40, β =0.05

Fig 2: Thickness variation of exponential disks

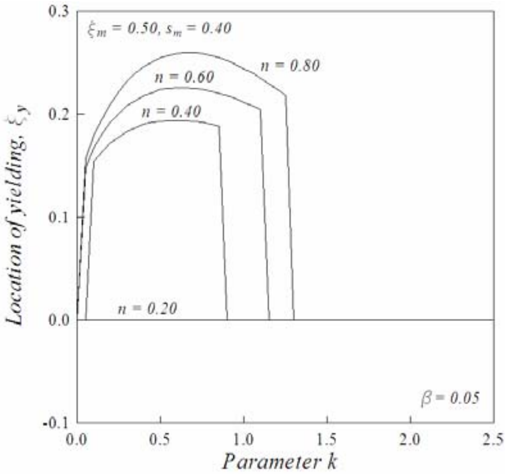


(a) at α = 0.25

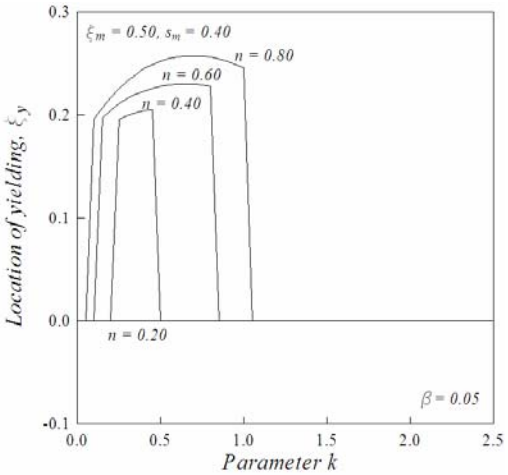


(b) at α = 1.00

Fig 3: Variation of limit speed with geometry parameters



(a) at $\alpha = 0.25$



(b) at $\alpha = 1.00$

Fig 4: Variation of location of yielding at limit speed .

The investigation of disk behaviour under external loading is carried out by considering the loading to be distributed over a span of s_m = 0.40. The effect of geometry parameters on variation of limit angular speed and location of yielding is investigated for α = 0.25 and 1.00 distributed at a mean radius ξ_m = 0.50.

The variation of limit angular speed with geometry parameters for the respective values of loading parameters is plotted in Fig. 3. It is observed that as ***n*** increases, limit angular speed also continues to increase for a given value of parameter ***k***. The variation of limit speed is observed to be similar to that of disk without attached mass [5, 7] but the values are understandably reduced. The variation of limit speed with parameter ***k*** for each given value of ***n*** shown in Fig. 3 hints at the existence of an optimum value of ***k*** at which maximum limit angular speed is attained. To gain a better insight into the phenomenon, location of yielding is plotted against parameter ***k*** in Fig. 4. Under external loading, the disk is never observed to yield at the root. This observation leads to the conclusion that external loading in form of attached masses causes the location of yielding to shift away from the root.

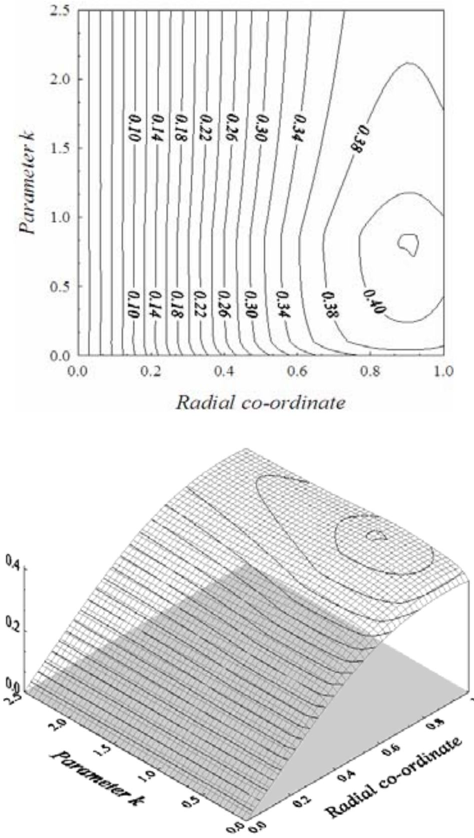


Fig 5: Contour and 3-D plot of normalized displacement ($\bar{u} = uE / b\sigma_0$) field for varying disk geometries at n = 0.40, α = 0.25, s_m =0.4

As was observed in case of disks without attached masses in [5, 7] the variation of geometry caused smooth and gradual shifting of yield location; it can now be concluded safely that in addition to geometry variation external loading causes shift in yield location. However, the nature of shifting of location of yielding caused by external loading is abrupt as observed in Fig. 4. Further study reveals that at a particular value of ***k*** the location of yielding suddenly falls back to the position where an externally loaded uniform thickness disk (***k*** = 0) yields.

It is observed that limit angular speed attains a maximum value corresponding to this value of parameter ***k*** and thus can be identified as *optimum value* of geometry parameter ***k***. The radial variation of displacement field with parameter ***k*** at α = 0.25 and ξ_m = 0.50 is plotted in figs. 5 and 6 for ***n*** = 0.40 and ***n*** = 0.80 respectively. This is followed by plots depicting von-Mises stress distribution for the same values of loading and geometry parameters in figs 7 and 8 respectively. The study is further carried out for α = 1.00 and ξ_m = 0.50 for similar geometry parameters. Similar plots of variation of displacement field and von-Mises stress distribution against the geometry parameters is shown in figs 9-12. These figures provide a good insight into the displacement and stress behaviour for different combinations of geometry and loading parameter. In Fig. 13 the variation of limit speed is plotted for α varying from 0.0 to 1.0 at any location within 0.2 to 0.8 along the disk radius. The variation of limit speed is observed to be more predictable than that of lumped attached mass reported in [7].

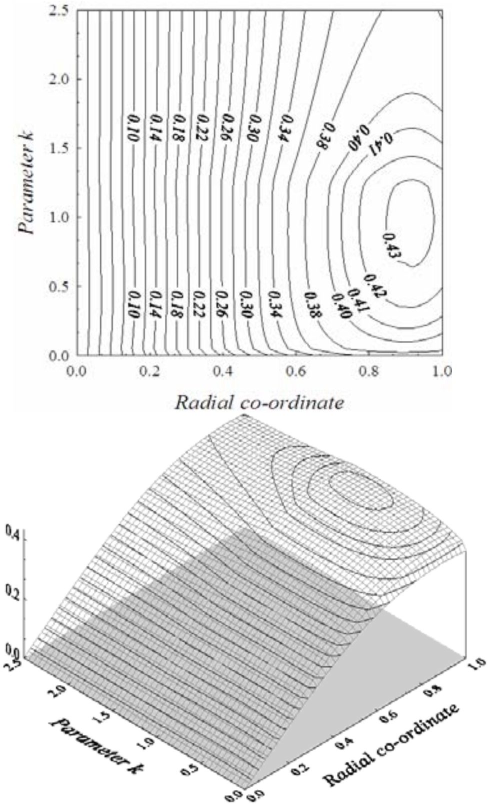


Fig 6: Contour and 3-D plot of normalized displacement ($\overline{u} = uE / b\sigma_0$) field for varying disk geometries at n = 0.80, α = 0.25, s_m = 0.4

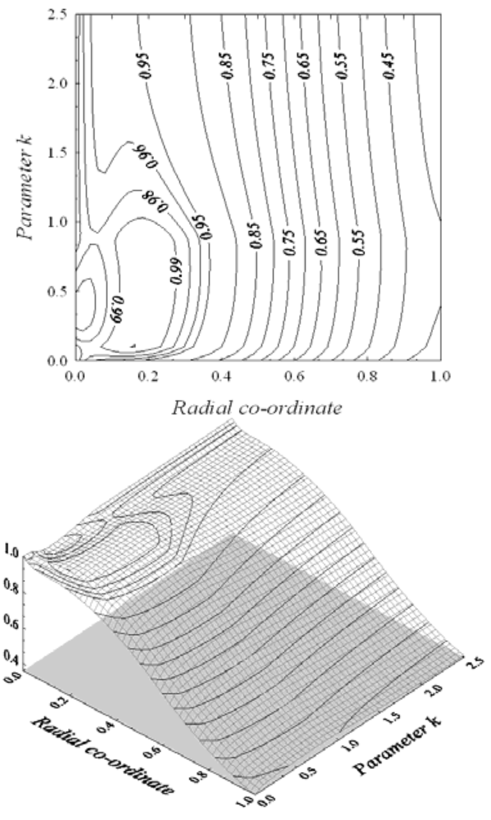


Fig 7: Contour and 3-D plot of normalized von-Mises stress ($\overline{\sigma} = \sigma_v / \sigma_0$) for varying disk geometries at n = 0.40, α = 0.25, s_m = 0.4

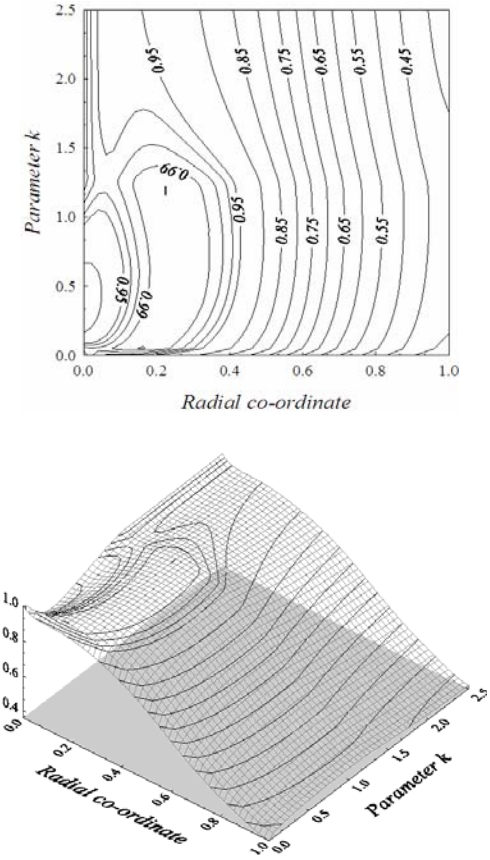


Fig 8: Contour and 3-D plot of normalized von-Mises stress ($\overline{\sigma} = \sigma_v / \sigma_0$) for varying disk geometries at n = 0.80, α = 0.25, s_m = 0.4

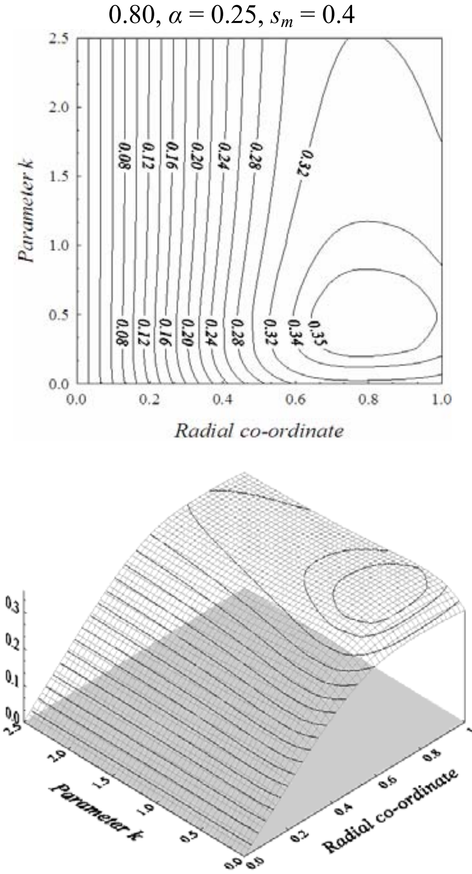


Fig 9: Contour and 3-D plot of normalized displacement ($\overline{u} = uE / b\sigma_0$) field for varying disk geometries at n =

0.40, α = 1.00, s_m = 0.4

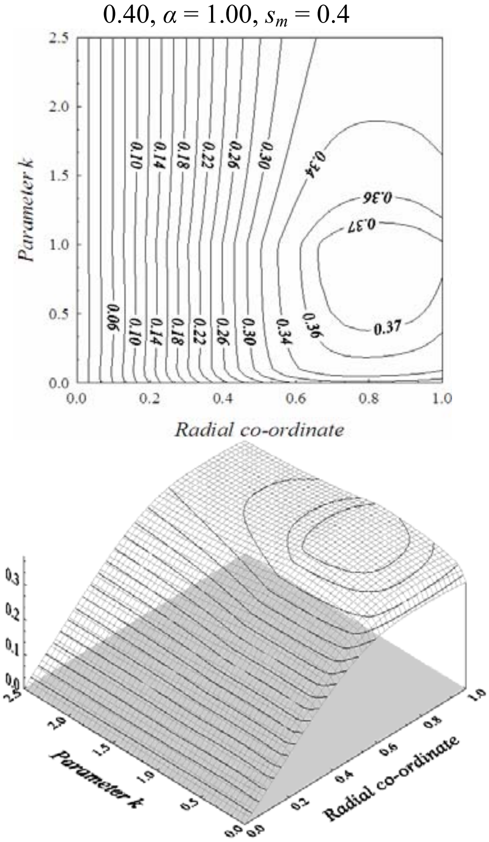


Fig 10: Contour and 3-D plot of normalized displacement ($\overline{u} = uE / b\sigma_0$) field for varying disk geometries at n = 0.80, α = 1.00, s_m = 0.4

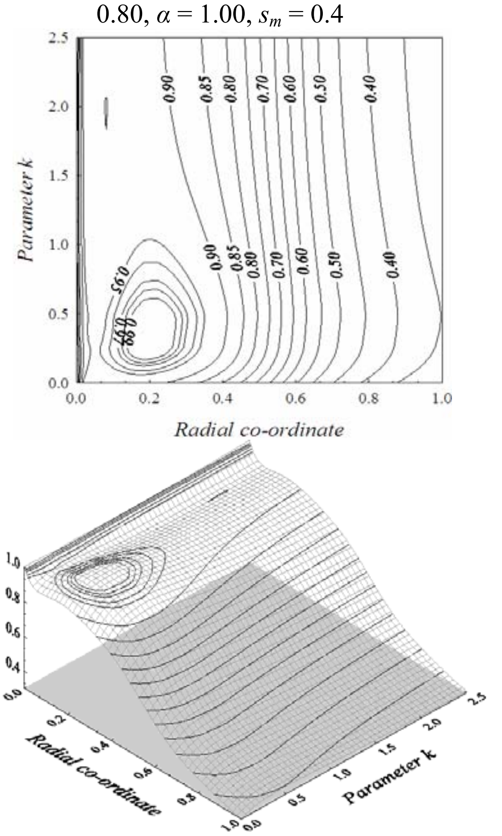


Fig 11: Contour and 3-D plot of normalized von-Mises stress ($\overline{\sigma} = \sigma_v / \sigma_0$) for varying disk geometries at n = 0.40, α = 1.00, s_m = 0.4

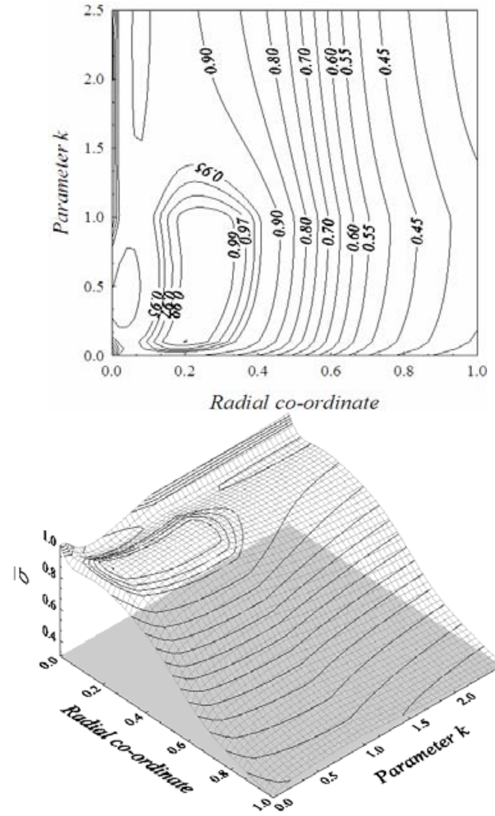


Fig 12: Contour and 3-D plot of normalized von-Mises stress ($\overline{\sigma} = \sigma_v / \sigma_0$) for varying disk geometries at n = 0.80, α = 1.00, s_m = 0.

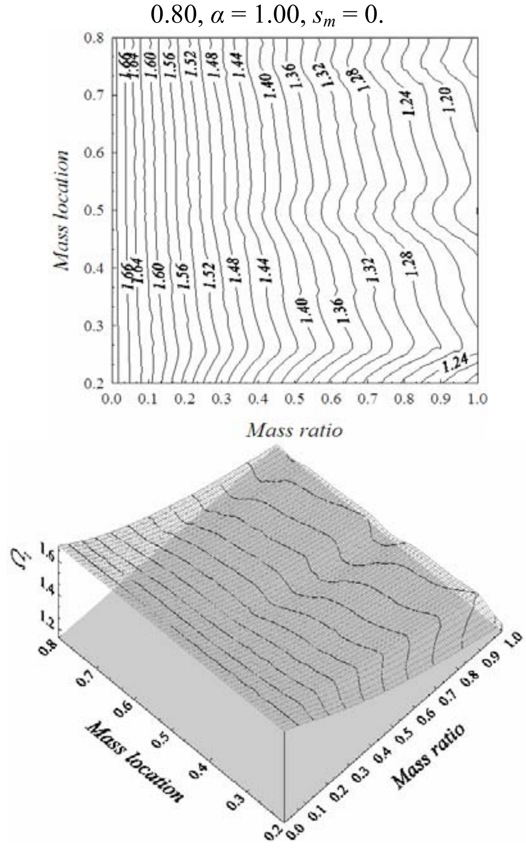


Fig 13: Contour and 3-D plot of variation of limit speed with loading parameters at n = 0.80, k = 0.9, s_m = 0.4

4. CONCLUSION

The application of variational principle provides an advantage over other methods in terms of simplicity and ease with which various complicating effects such as addition of attached masses at various radial locations, rigid inclusions, etc. are incorporated. The present work employs variational principle to study the stress distribution and estimate the limit angular speed of rotating disks with exponentially varying thickness carrying distributed attached masses. The effect of geometry and loading parameters is studied individually with an objective of maximizing the limit angular speed. The results are reported in dimensionless form and the curves documented may be readily used as design monographs. The generated results have provided a substantial insight to the modelling and analysis of centrifugal impellers. However, post-elastic analysis is needed for more realistic understanding of the system behaviour.

5. REFERENCES

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6. ACKNOWLEDGEMENT

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7. NOMENCLATURE

Symbol	Meaning
b	Outer radius of the disk.
c_i	The vector of unknown coefficients.
E	Modulus of Elasticity of the disk material
h_0	Thickness at the root of the disk.
h	Thickness at any radius r.
h_c	Thickness of the equivalent uniform disk.
n, k	Geometry parameters of the disk.
r, θ, z	Radial, tangential and axial directions.
r_m	Attached mass location.
r_{ml}, r_{mu}	Inner and outer radii of mass distribution span.
r_y	Radial location of yielding.
s_m	Span of distributed attached mass, $(r_{ml} - r_{mu})$.
u	Displacement field of the disk.
$\bar{u}$	Normalized displacement field, $uE/b\sigma_0$
U	Strain energy of the disk.
V	Potential energies of the disk due to rotation.
α	Attached mass ratio.
β	Thickness ratio, h_c / b.
ρ, μ	Density and Poisson's ratio
ω, Ω	Angular speed and dimensionless angular speed of the disk respectively.
ω_l	Limit angular speed
Ω_1	Dimensionless limit speed, $\omega_1 b\sqrt{\rho/\sigma_0}$.
$\varepsilon_r, \varepsilon_\theta$	Strains in radial and tangential direction.
σ_r, σ_θ	Radial and tangential stresses.
σ_0	Yield stress of the disk material.
σ_v	von-Mises stress, $\sqrt{\sigma_r^2 - \sigma_r\sigma_\theta + \sigma_\theta^2}$
$\bar{\sigma}$	Normalized von-Mises stress of the disk, σ_v/σ_0
ϕ_i	Set of orthogonal polynomials (used as coordinate functions)
ξ	Normalized radial co-ordinate, (r/b).
ξ_l	Normalized location, $((r - r_{ml})/ s_m)$.
ξ_y	Normalized location of yielding (r_y/b).