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THEORETICAL STRESS CONCENTRATION FACTORS FOR SHORT FLAT TENSION BARS WITH OPPOSITE U-SHAPED NOTCHES

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ABSTRACT
This paper presents solutions for distribution of stresses and displacements in plate having symmetrical opposite U-shaped notches under the application of uniform tension. Solutions especially at the notch region, which are observed as the most critical zone, are determined by finite difference numerical technique for different dimensions of short flat bar. Analyzing the solution obtained, stress concentration factors are determined. Effect of notch radius and the thickness ratio of the selected bar on the stress concentration factors have also been studied. The rationality and the reliability of the solution are checked by comparing the results obtained with the results of finite element method available in the literature. The stress concentration factor (K_t) is presented graphically as a function of notch radius ratio (r/h) and height ratio (H/h) for the plate under uniform tension scheme.

Keywords: Finite difference technique, Stress analysis, Circular notch.

1. INTRODUCTION
At present design and analyses of loaded members specially when subjected to fatigue as well as brittle conditions aim mainly at two factors: greater reliability of such design and achieving better cost effectiveness. This work presents an analysis on the theoretical stress concentration factor (TSCF), which is based on the recent observations to establish the dependence on the parameters, rather than the traditional ones for the particular geometry studied herein.

To perform both design and analyses of loaded parts especially in case of mechanical or structured members subjected to fatigue conditions, even in the cases of loaded members where the full account of stress distribution is unavailable due to geometric discontinuities, the concept of TSCF is frequently used [Ref. 1, 2]. The designer while performing designs and analyses in the context of the field of fatigue or for the members having brittle material behavior, extracts the value of the stated TSCFs from the experimentally determined charts described in the well known texts for most of the part and uses them together with the other ideas to complete his design.

The theoretical stress concentration factor, K_t is normally defined as the ratio of the maximum localized stress, σ_{max} and the nominal stress, σ_{nom} [Ref. 3]. Therefore, it is described as

$$K_t = \sigma_{max} / \sigma_{nom} \quad (1)$$

The existing values of the TSCF for the type of the member and loading relevant to this work were originally determined experimentally and have been reproduced in numerous mechanical design texts and handbooks (See for example reference [4]). In all these references the influence of the length of the members in the direction of the applied load was not considered and no specific values for the lengths for which the results are applicable are given. The established values are valid only for the relatively large (infinite) values of the ratio L/H (Fig 1).

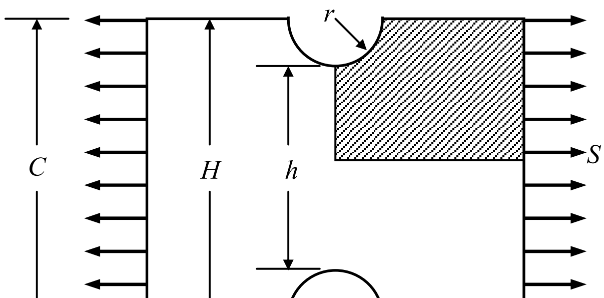

Fig 1: Plate indicating significant geometric parameters

In this work the in-plane TSCF for uniform thickness tension plates with opposite U shaped notches for practical ranges of the notch radius are presented in the standard graphical form, where the length of the member in the direction of the applied load is considered as a fixed parameter. The results reported herein are valid for the lengthwise centre of the U notches to be in the lengthwise middle point of the members. The analyses were performed for the unit value of L/H ratio and for four values of the H/h ratio {1.15, 1.2, 1.5 and 2.0}. In addition the study extends existing results to smaller ratios of r/h {0.1, 0.15, 0.2, and 0.3}.

Because of the symmetry of the geometry and the loading condition of the member under study, only one quarter of the plate was modeled using Finite difference method (FDM) as indicated in the shaded area in Fig 1. In this work the values of the maximum localized stress, σ_{max} are obtained by numerical method in finite difference scheme with displacement potential function formulation for a constant value of the nominal stress, σ_{nom}.

The displacement potential function formulation used here for solution of two-dimensional elastic problems was first introduced by Uddin [5], later Idris et.al [6-7] used it for obtaining analytical solutions of a number of mixed boundary-value elastic problems and Ahmed [8-11] extended it's use where he obtained finite difference solutions of a number of mixed boundary value problems of simple rectangular bodies. Later, Akanda developed a new numerical scheme [12-14] by which solved irregular shaped elastic bodies under mixed mode of loading. The utility and the reliability of this numerical scheme is verified and established by solving a number of geometrically complicated problems [15-20]. The present study focuses the solution of the problems of short flat tension bars with opposite U-shaped notches in regard to the estimation of theoretical stress concentration factors using this Akanda's developed numerical scheme. Solutions especially at the notch region, which are observed as the most critical zone, are looked into. Analyzing the solution obtained, stress concentration factors are determined. Effect of notch radius and the thickness ratio of the selected bar on the stress concentration factors have also been studied. The rationality and the reliability of the solution is checked by comparing the results obtained with the results of finite element method [21] available in the literature. The stress concentration factor (K_t) is presented graphically as a function of notch radius ratio (r/h) and height ratio (H/h) for the plate under uniform tension.

2. PROCEDURE FOR DETERMINATION OF THE THEORETICAL STRESS CONCENTRATION FACTOR

Owing to the geometric symmetry of the member under study and the type of the applied boundary conditions, only one quarter of the plate was used as delimited by the shaded area indicated in the Fig 1**.** The boundary conditions for every model used in the calculations are indicated in Fig 2, i.e. the lower boundary was entirely restricted in the vertical direction but at the same time allowed to move freely in the horizontal direction. On the other hand, the left boundary was entirely restricted in the horizontal direction but at the same time allowed to move freely in the vertical direction.

The maximum stress was obtained at the edge of the notches, as shown in Fig 2. The stress distribution in the plate was obtained by Finite Difference [12-14] with the use of well-known displacement potential function formulation [5]. From this distribution, the maximum value of stress in the plate is found.

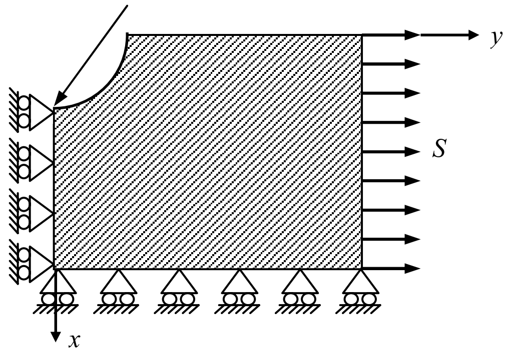

Fig 2: Quarter plate showing the applied boundary conditions

A constant nominal stress, S, was applied on the plate as shown in Fig 1, which was used to determine the theoretical stress concentration factor by using Eq. 1. The denominator of Eq. 1, i.e. the nominal stress was calculated using the expression

$$\sigma_{nom} = SC/h \qquad (2)$$

The geometry under consideration is discritized by rectangular meshes as shown in Fig 3. The parameters of interest, especially the stresses and displacements are obtained at the rectangular nodal points. The details description of the numerical scheme can be seen in references [12-14], therefore, these descriptions have been omitted here.

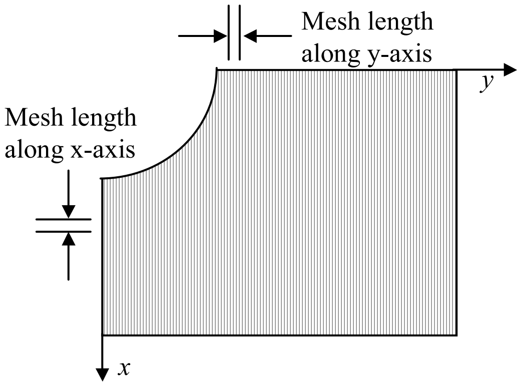

Fig 3: Typical finite difference mesh used in the calculation

3. ANALYSIS OF RESULTS

The solution of the displacements and the stress components u, v, σ_x, σ_y and σ_{xy} for each rectangular nodal point within the elastic field has been obtained. For a case of L/H = 1.0, C/H = 1.0, H/h = 2.0 and r/h = 0.2, the variations of the values of u, v, σ_x, σ_y and σ_{xy} with the change of the non-dimensional ratio $2x/H$ were plotted for some selected sections. The selected sections are shown in Fig 4.

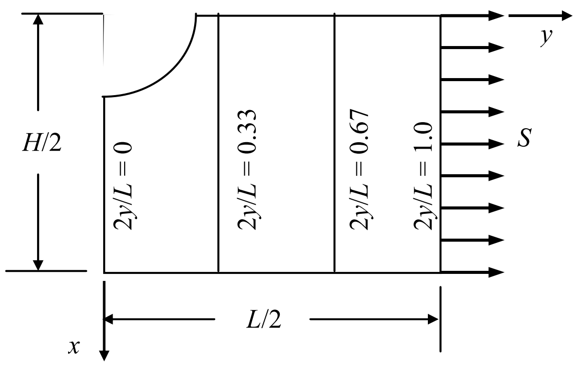

Fig 4: Selected sections for studying the distribution of stresses and displacements.

3.1 Distribution of *u*

The distribution of the displacement component (along *x*-axis) *u* for some selected sections are shown in Fig 5. It is observed that the values of the displacement component are anti symmetric with respect to the horizontal centerline. For a vertical section the value of the displacement is maximum at the upper surface of the plate and it gradually decreases along the vertical direction. No displacement was found at the section $2x/H = 1$.

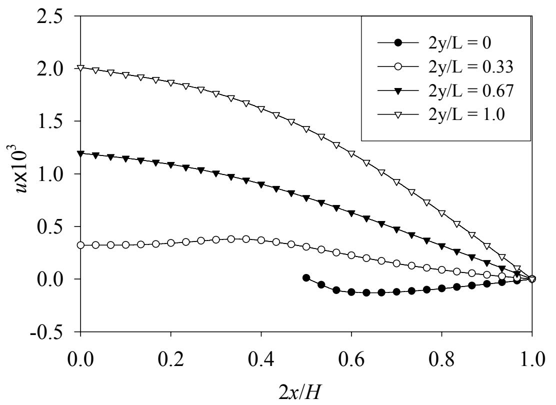

Fig 5: Distribution of *u*

3.2 Distribution of *v*

The distribution of the displacement component (along *y* axis) *v* for some selected sections are shown in Fig 6. It is observed that the value of *v* for a constant $2x/H$ is maximum at the end of the plate and decreases gradually along the middle section of the plate.

3.3 Distribution of σ_x

The distribution of the stress component along *x*-axis for some selected sections is shown in Fig 7. For a constant value of $2x/h$ the maximum or minimum value of σ_x is obtained at the mid plane along length of the plate. The value of σ_x decreases and eventually becomes negative as approached towards the end along the direction of the applied force.

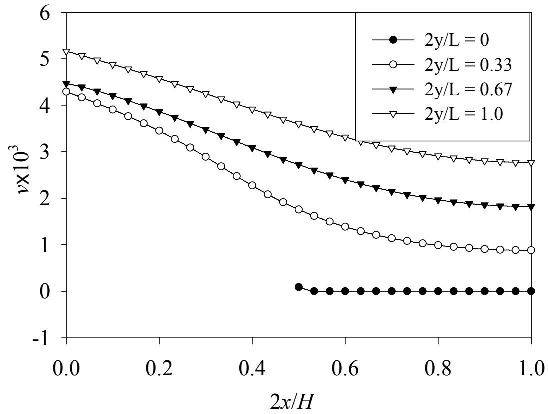

Fig 6: Distribution of *v*

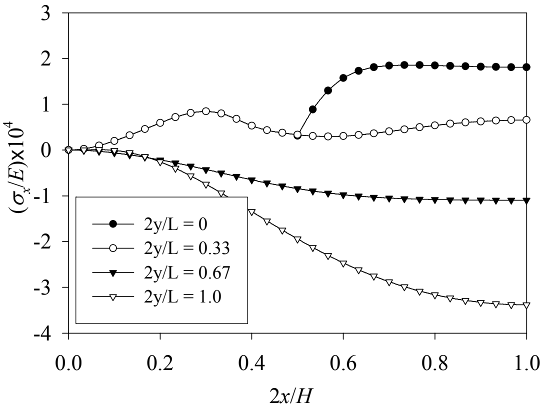

Fig 7: Distribution of σ_x

3.4 Distribution of σ_y

The distribution of the stress component along *y*-axis for the selected sections is shown in Fig 8. For a constant value of $2x/H$ the maximum value of σ_y is at mid plane along the thickness of the plate where the U-shaped notch root is present. The value of σ_y is constant at the outermost end, which satisfies the applied boundary conditions. The value of σ_y across the notch is less than the constant uniform applied stress and the value of σ_y across the middle part of the plate is higher than the applied stress. This indicates that the resultant force at any section is same as the applied one.

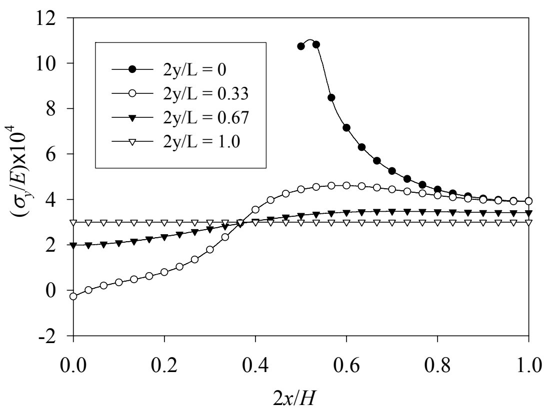

Fig 8: Distribution of σ_y

3.5 Distribution of σ_{xy}

The distribution of the stress component along *xy* plane for some selected sections is shown in Fig 9 and the results are found reasonable.

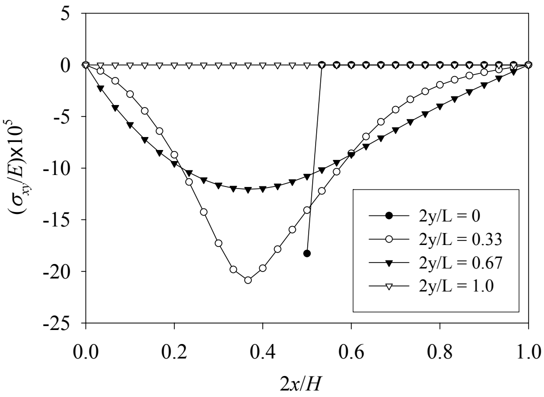

Fig 9: Distribution of σ_{xy}

3.6 Effect of Notch Radius and Thickness on Stress Concentration Factor

In this work four new curves for the TSCF for the geometry treated herein were determined for the short member regime, corresponding to each value of the H/h ratio of the set {1.15,1.2,1.5 & 2.0}. The H/h values selected correspond to each of the existing experimentally determined curves for the geometry treated in this work.

For the body under uniform tension the variation of stress concentration factor K_t as a function of notch radius and thickness is presented in Fig 10 to Fig 13.

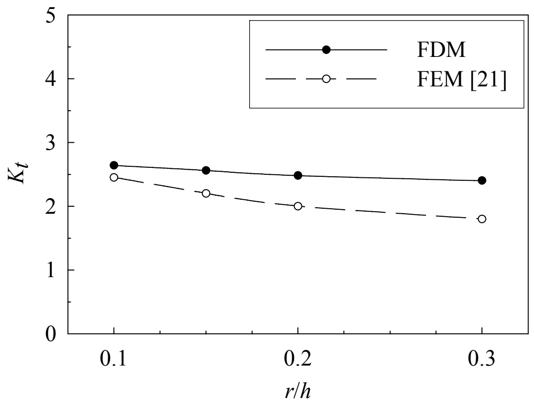

Fig 10: Comparison of TSCF for *H*/*h* = 1.15

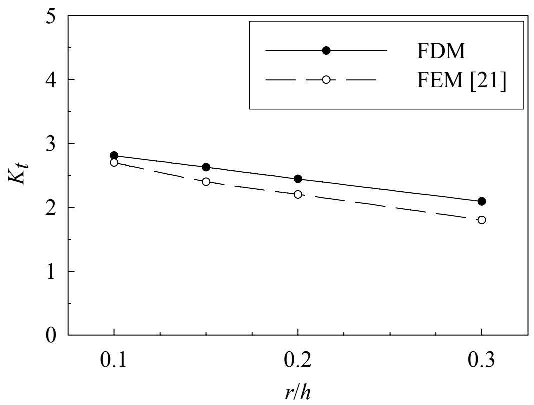

Fig 11: Comparison of TSCF for *H*/*h* = 1.2

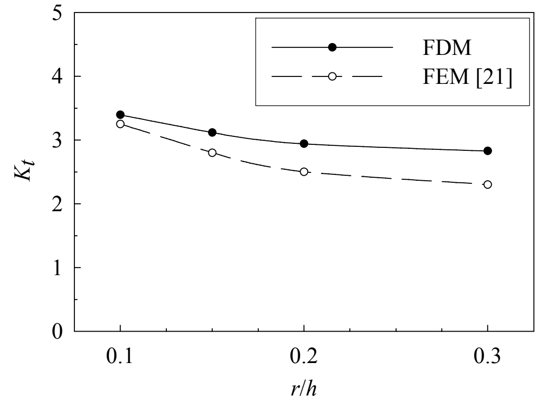

Fig 12: Comparison of TSCF for *H*/*h* = 1.5

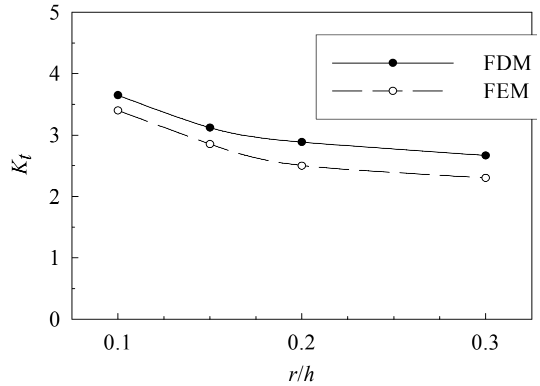

Fig 13: Comparison of TSCF for *H*/*h* = 2.0

In all these figures for a constant value of *L*/*H*, *C*/*H* & *H*/*h*, the value of K_t decreases either linearly or parabolically with increasing notch radius. Besides for a constant value of r/h, the value of K_t increases as the value of *H*/*h* increases. So for short flat tension bars with opposite U-shaped notches, the thickness ratio has significant effect on the stress concentration. These behaviors are compared with the values of K_t for similar cases determined by Finite Element Method and same behaviors were found. In every case the value of K_t determined by FDM is greater than the values determined by FEM [Ref. 21]. Here it should be noted that the result obtained in our study is the direct solution, whereas the result obtained in FEM [Ref. 21] is the extrapolated value, as the FEM cannot give solution at the boundary directly. Therefore, the present solution will be the more realistic one.

3.7 Effect of the Degree of Load Concentration on Stress Concentration Factor

The effect of the degree of load concentration is studied with the aid of the parameter *C*/*H* (a parameter that assumes the value of zero for totally concentrated loads and assumes a value of one for uniformly distributed loads). For a constant value of *L*/*H* = 1.0 the distribution curves of K_t are shown in Fig 14 to Fig 17 with respect to *C*/*H* keeping either *r*/*h* or *H*/*h* constant. In the curves K_t vs. *C*/*H* where *r*/*h* is fixed, the value of K_t is maximum for the maximum value of *H*/*h* for a given *C*/*H*. Again for a constant value of *H*/*h* the value of K_t increases as the applied load area increases.

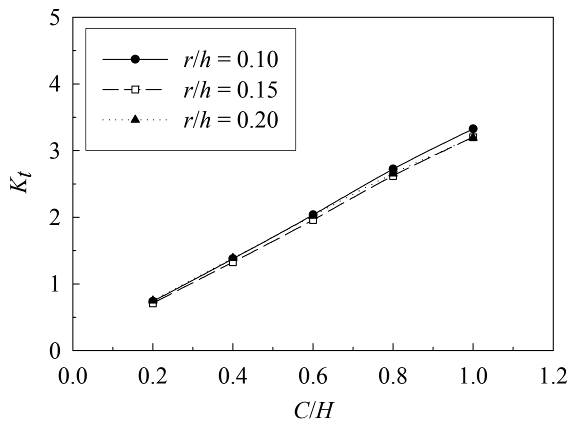

Fig 14: K_t as a function of C/H for H/h = 1.15

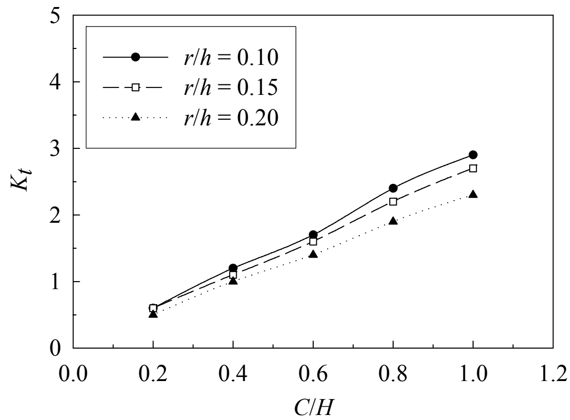

Fig 15: K_t as a function of C/H for H/h = 1.2

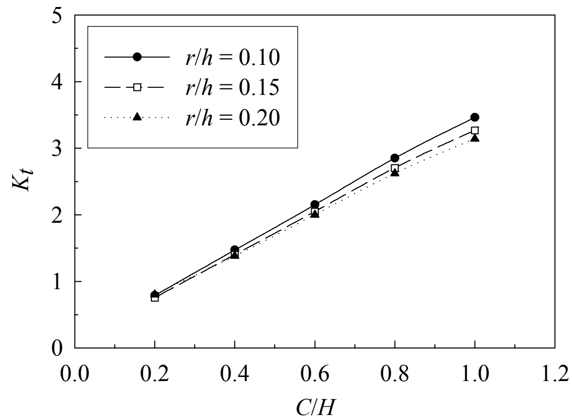

Fig 16: K_t as a function of C/H for H/h = 1.5

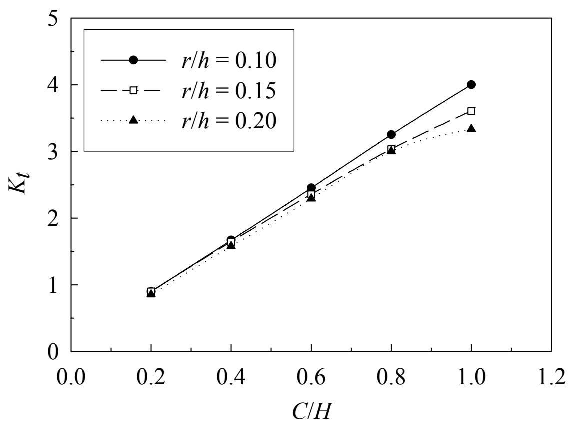

Fig 17: K_t as a function of C/H for H/h = 2.0

4. USAGE OF THE GRAPHS

The Fig 10 to Fig 13 can be utilized for a comprehensive knowledge on the variations of the value of the theoretical stress concentration factor (TSCF) for various notch radiuses. These results verify the accuracy of the results determined for the similar cases by FEM. The Figs 5 to 9 give a vivid idea about the deformations and developed stresses at various sections. The Figs 14 to 17 show the variation of TSCF with respect to area of the applied load.

5. CONCLUSIONS

The present study provides the solutions of theoretical stress concentration factor in short flat tension plate having opposite U-shaped notches by using finite difference technique. The results found for notch root show good agreement with the FEM solution.

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7. NOMENCLATURE

Symbol	**Meaning**	**Unit**
K_t	Stress Concentration Factor	-
x, y	Rectangular co-ordinates	(mm)
E	Elastic modulus of the material	(Pa)
ν	Poisson's ratio	(mm/mm)
u, v	Displacement component in the x and y-direction	(mm)
σ_x, σ_y, σ_{xy}	Stress component in the x-direction, y-direction and xy-plane	(Pa)
r	Notch radius	(m)
σ_{max}	Maximum localized stress	(Pa)
σ_{nom}	Nominal stress	(Pa)
H	Wide dimension of the plate	(m)
h	Narrow dimension of the plate	(m)
L	Total plate length	(m)
C	End length on which the applied load is distributed	(m)
S	Uniform traction	(Pa)