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'ELASTIC-PLATEAU' CONSTITUTIVE RELATION OF FLEXIBLE POLYURETHANE FOAM UNDER IMPACT LOADING

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ABSTRACT

Based on an experimental database obtained from static and dynamic tests a modified version of the well-known model proposed by Maxwell was combined with Sherwood-Frost model to describe the constitutive relation of polyurethane foam more precisely. A new transient velocity equation is introduced to calculate the varying strain rate during crush progress so that strain rate effect can be considered under dynamic loading. Excellent agreement between experimental results and tests confirms the efficacy of the proposed constitutive model. First portion of the constitutive equation, a modified Maxwell model explores the nonlinear constant visco-elastic nature of polyurethane foam.

Keywords: Polyurethane foam, Constitutive model, Transient velocity and Visco-elastic.

1. INTRODUCTION

Polyurethane foam becomes popular for its excellent crashworthy behavior, cellular structure and cost benefit ratio in the field of personal protection, packaging and automotive industry. The analysis and understanding of how polyurethane foam behaves in an impact is very important in the effort to lower the potential harm to use it as an occupant of a vehicle. A full scale test is considered the most reliable source of information regarding crashworthiness and safety of vehicles, high cost of such tests and difficulties of collecting sufficient data result in increasing interest in develop equation so that further test will not be necessary. In this regard, the impact behavior of polyurethane foams is examined from the viewpoint of accurately describes constitutive relation. In the broad sense polyurethane foam is divided into two main families: flexible and rigid polyurethane foam depending on the arrangement of the struts (cell wall) and voids in the foam. In FPF, the strut and void structure allows air to pass through the foam when force is applied. The elasticity of the struts acts as a shock absorber and allows the foam to recover shape after compression/impact in the same manner as shock absorbers perform in a car. On the other hand, rigid foam cells have much the same structure, but windows in the cell walls are closed, restricting airflow. This study was performed with flexible polyurethane foam from which air expelled with great velocity after impact.

Constitutive modeling of polymeric foams has been proposed with various approaches. Based on morphological observations, Qi and Boyce [1] described the stress-strain behavior of thermoplastic polyurethane. Jun et al. [2] presented a rate dependent hydrodynamic constitutive equation for rigid polymeric foams. They focused on modeling of strain rate dependency and temperature effect on polymeric foam subject to high rate impact loading. After decomposition of the foam in two parts: a skeleton and a nonlinear elastic continuum in parallel, Neilsen et al. [3] developed a constitutive theory for rigid polyurethane foam.

The extensive research is one of the most important contributors to the growth of the polyurethane foam in the engineering application. So, this study was attempted to describe more precisely high nonlinear constitutive relation of polyurethane foam, which is valid both for static and impact loading condition.

2. STRESS STRAIN BEHABIOR

The complete compressive stress–strain plots, can be characterized by three main distinguish regions, 'elastic region', 'plateau region' and 'densification region' (Fig. 1). In the elastic region, the foam cell begins to collapse by elastic buckling and stress almost proportionally rises with strain. In the case of impact, foam beneath the impactor is impulsively accelerated to a common velocity with the impactor within this region. A comparatively long region in where collapse progresses at on roughly constant load, until the opposing walls in the cells meet and touch is termed as plateau region. In the densification region, cell bands collapse, cell wall meet and touch.

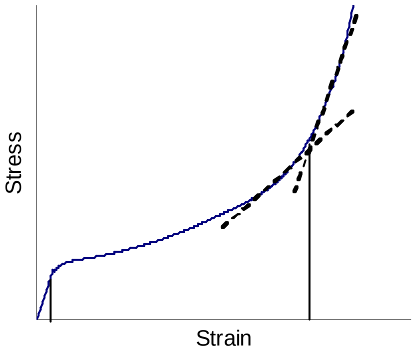


Fig 1: Typical stress-strain curve of polyurethane foam

3. CONSTITUTIVE THEORY

For the effective crush protection foam should absorb or convert all kinetic energy within plateau regime. To represent elastic region and plateau region more precisely entire stress-strain relationship has been decomposed into two parts. First equation, developed by [4] from the Maxwell model, describes the constitutive relation up to elastic region. Then rest of the regions is expressed by Sherwood and Frost [5] constitutive equation that incorporates the effects of foam density, temperature and strain rate.

3.1 Visco-elasicity

A primary characteristic of visco elastic materials (judges from earlier articles) is the influence of previous states of deformation; the current state depends on the strain and strain rate history. The experimental results described indicate that the behaviors observed are amenable to description by a visco-elastic foam model. First portion of the equation (from Maxwell model), based on visco-elasticity), describe up to elastic region given below as function of strain (ε) and strain rate:

$$\sigma_e = E_f \tau \dot{\varepsilon}\left[1-\exp\left(-\frac{\varepsilon}{\tau\dot{\varepsilon}}\right)\right] \tag{1}$$

Here, τ (time constant) is a material property that can be determined by experimentally. E_f can be calculated from the following expression [6]:

$$E_f = E_s\left(\frac{\rho_f}{\rho_s}\right)^2 \tag{2}$$

3.2 Integral power model

The modified power model by Sherwood and Frost model, utilized after elastic region was:

$$\sigma_c = H(T)D(\rho)\left(\dot{\varepsilon}/\dot{\varepsilon}_0\right)^{m(\varepsilon)}\sum_{n=1}^{10} A_n \varepsilon^n \tag{3}$$

Where, A_n is the coefficient values for the shape function which can be determined by curve fitting of the compressive test result. $m(\varepsilon)$ can be expressed by

$$m(\varepsilon) = a + b\varepsilon \tag{4}$$

Finally, suggested constitutive relation of this study is the combination of Eq. 1 and Eq. 3.

4. TRANSIENT VELOCITY EQUATION

During quasi-static compression strain rate remains constant but in the case of impact loading it changes frequently. It is crucial to predict strain rate during impact to calculate stress strain relationship properly. The value of strain rate can be calculated as the ration of impact velocity over the initial specimen length. Thus, the relation between strain rate and velocity at any strain is:

$$\dot{\varepsilon} = \frac{v}{H} \tag{5}$$

Since impact energy is a function of velocity for a particular striker, once transient velocity is found, it will be easy to calculate the remaining impact energy in the striker during the crush progress. Also after how much deflection of object, the impactor will be stop (at that condition V=0) or maximum deflection in an absorber can be predicted if V is known.

For the same initial impact energy and initial impact velocity (v_0) reactive load exerted by the foam varies with stiffness of the foam. Thus deflection and velocity profile strongly depends on the stiffness of the foam. The transient velocity can be calculated from the following equation developed by utilizing energy balance relation.

$$v = \left[v_0^{\,2} - \frac{2gH\varepsilon}{\gamma - 1}\right]^{0.5} \tag{6}$$

The value of γ, depends on the stiffness of the foam, can be evaluated by analyzing experimental data. Again stiffness of the foam depends on stiffness property of the material from which foam was made of and density of the foam. As, all foam under the current study made of from the same material (polyurethane),

The value of γ varies only with density of the foam. In this research, the value of γ was selected in Eq. 6 for the calculation of transient velocity by analyzing experimental data so that the calculated transient velocity can associate with the least rms error. Then, the introduced Eq. (6) has been verified by impact testing results obtained from Instron Dynatup 9250 HV.

5. SPECIMAN AND IMPACTOR

Fig 2: Specimen of the polyurethane foam.

Two different densities (67 kg/m^3 & 90 kg/m^3) cylindrical flexible polyurethane foam specimens (Fig. 2) were subject to both impact and quasi-static loading at room temperature. During the study, mechanical and physical properties of foam almost remain same because the temperature and specification were constant. The important physical and mechanical properties of cell wall of the specimen are summarized in Table1.

Table 1: Mechanical properties of the cell wall.

Density (kg/m^3)	Young's modulus (Gpa)	Yield strength (Mpa)	Poisson's ratio
1200	0.45	40	0.4

A flat circular cross-section ended steel impactor was employed for impact testing whose size (diameter 100 mm) was larger than the specimen size (diameter and height 42 mm and 40mm respectively) so that it could strike the whole specimen. Each impact experiment was performed under the same impactor. Total mass of the impactor with dead weight was 27 kg

6. QUASI-STATIC TEST

The main intension of quasi-static tests was to find out the value of coefficient of shape function, and quantitatively identify density and strain rate effects. Then these values were employed to calculate the stress-strain response under impact loading. The quasi-static tests were carried out in a MTS810 machine whose maximum capacity is 100 KN. A rectangular steel block parallel to the bottom-clamping device was mounted to the top hydraulic actuator of the testing machine in order to ensure a uniform load distribution. There are also optical devices to measure the sample deflection and force transducers for force measurement. Command was given to operate the machine by software that the specimen would be unloaded and the experiment stopped just after 90% deflection of the total height of the specimen. 0.001 s^{-1} and 0.1 s^{-1} strain rates were applied for both densities (67 kg/m^3 and 90 kg/m^3) specimen. For the calculation of visco-elastic constitutive relation, time constant τ (material property) was determined so that the theoretical curve can fit with quasi-static experimental data. The value of τ is: $\tau = 90$. Again for the calculation after elastic region, coefficient values A_n (given in Table 2) of the shape function specified in Eq. (3) were evaluated by curve fitting of 10th order polynomial. Data points for curve fitting were selected from the experimental result of σ vs. ε curve. The experimental relation, which was selected for data points of curve fitting performed under the following quasi-static condition

Table 2: Co-efficient values of the shape function.

n	A_n
0	0.01
1	13
2	-197
3	1687
4	-8656
5	27942
6	-57375
7	73627
8	-55839
9	21937
10	-3098

Density of the specimen, $\varepsilon = 67$ kg/m^3

Lowest Constant strain rate, $\dot{\varepsilon} = 10^{-3}$ s^{-1}

Temperature, $T = 23°$C (room temperature).

Since all tests (both impact and quasi-static) were conducted at the room temperature; temperature effect was same for all specimens. Density effect was determined by comparing 2 experimental results for the same strain rate (10^{-3} s^{-1}) applied on different densities at room temperature, considering that 67 kg/m^3 density specimen has a unity effect. Thus in this study density and temperature effects were:

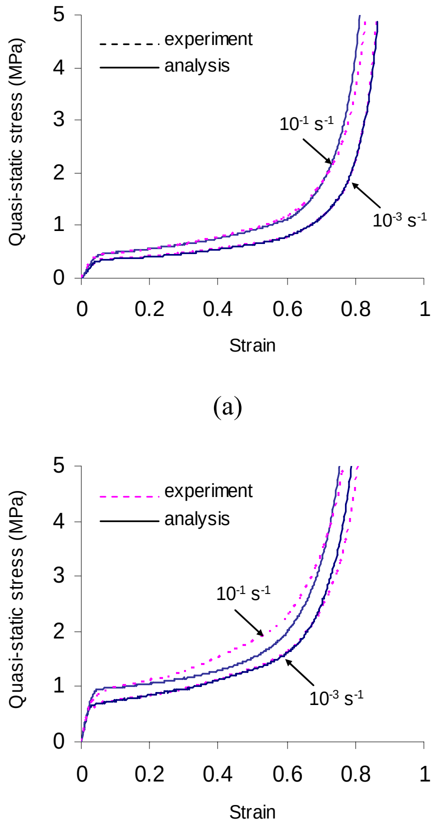
$$D(\rho) = 0.046\rho - 2.059; 67 \le \rho \le 90 \tag{7}$$

$$H(T) = 1; T = 23°\text{C} \tag{8}$$

For evaluating strain rate effect, the best linear fit of m , (specified in Eqs. (3) and (4)) of the polyurethane foam used in this study is,

$$m = 0.05173 + 0.001553\dot{\varepsilon} \tag{9}$$

Finally entire analysis curves in Fig. 3 were calculated up to elastic region using Eq. (1) and after elastic region Eq. (3) was employed with considering density, temperature and strain rates effects.



(a)

(b)

Fig 3: Calculated and experimental stress-strain relationship of polyurethane foam under quasi-static loading when density of the specimen was: (a) 67 kg/m^3 and (b) 90 kg/m^3.

The above-calculated curves for both densities and strain rates almost follow the same path as the experiments. Stress corresponding to overlap point of two equations is the plateau strength. Thus calculated plateau strengths are 0.32 MPa, 0.45 MPa, 0.6 MPa and 0.80 MPa. And the experimental results illustrate almost same plateau strength for corresponding density of the specimen and strain rate (see Fig. 3). Again from Fig. 3 entire curve calculated by two equations follows almost same path as the experimental curve. Above reasons sustain combination of elastic-plateau proposed constitutive relations validity for the quasi-static case. And also interesting to note that by solving two equations plateau strength of polyurethane foam can be found if density and strain rate are known.

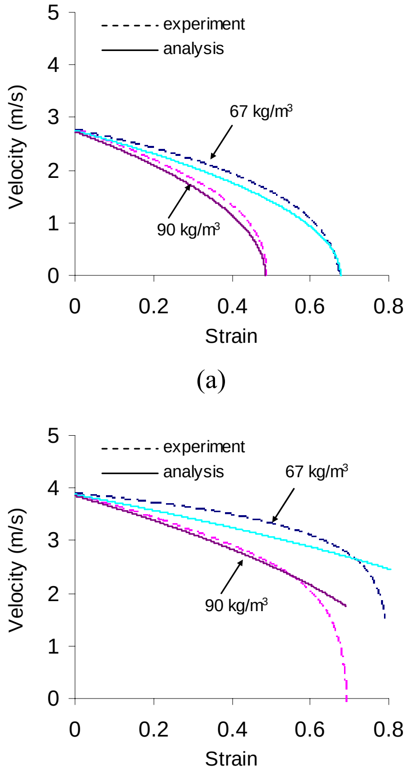
7. IMPACT TEST

Impact testing was performed by a drop tower name Instron Dynatup 9250 HV. Instron Dynatup 9250HV is a structural design to raise a drop weight to a specific height above a specimen and to drop the drop weight to that specimen. The main structural members of the tower comprise a table plate, two columns, two ball screws, four corner extrusions and a drop table. These members form a rigid frame for the components of the tower. The tower drive mechanism is located in the tower drop enclosure. A latch block containing the drop weight hook release mechanism moves up and downs on the tower columns driven by the ball screws. The drop weight is able to move freely on the columns. A rectangular block beneath the tester supports the specimen. On the 9250HV tower, two accelerating springs are mounted on the top plate. When drop weight acceleration is necessary, the latch block moves up towards the top of the tower and the springs protrude through bores in the latch block and bear directly on the top surface of the weight to provide the required acceleration. The control panel, which is the primary control interface, let enable the drop tower and specimen pneumatic clamp fixture and carries out all tower drive and tests functions. The control panel operates in conjunction with the software control console on the computer. It is easy to set up the test and data acquisition functions on the computer. In this study, the impact velocity was input in the machine with the help of a personal computer. Then latch block hook mechanism supported the drop weight while the latch block was moved to the appropriate height for the test. Upon command, the hook released the drop weight which then falls and impacts the specimen. Then, Instron Dynatup impulse data acquisition system that is a combination of hardware and software components captured impact information (impactor velocity, displacement and load as functions of time) at varying speed from impact test.

2 tests were conducted with different velocities for each density specimen. The initial impact velocities of the impactor were 2.75 m/s and 4.00 m/s where as corresponding mass of the impactor was 27 kg.

As polyurethane foam shows viscous behaviour (strain rate sensitivity) under impact load, at first velocity should be predicted at any strain so that corresponding strain rate could be calculated by using Eq. (5). For the calculation of transient velocity, the values of γ (specified in Eq. 6) were 1.07 and 1.05 respectively for 67 kg/m^3 and 90 kg/m^3 density specimen. The values of γ were determined so that the calculated transient velocity can associate with the least rms error and this least overall rms error is 5%. Velocity, load, deflection as a function of time can be directly obtained from the impact testing of a specimen performed by Instron Dynatup 9250 HV. Comparing with the impact experimental data, introduced transient velocity Eq. (6) of the impactor is verified. The velocity-strain curves obtained from the experiments (experiment) and from the Eq. (6) (analysis) are presented in the Fig. 4

Comparison of predictions based on the proposed transient velocity Eq. (6) with 4 experimental velocity profiles for different initial velocities and 2 different densities of foam presented in Fig. 5 shows the validity of that equation. After calculating transient velocity, strain rate was evaluated from Eq. (5). For the impact constitutive relation, Eq. (1) was employed for calculation up to the elastic region and Eq. (3) after the elastic region. The density, strain rate effect follows the same relation as quasi-static since all specimens were made of from same material. Eq 7, Eq 8 and Eq. 9 were used for calculation of density, temperature and strain rate effects. The calculated and experimental stress-strain relationship for the polyurethane foam under impact.

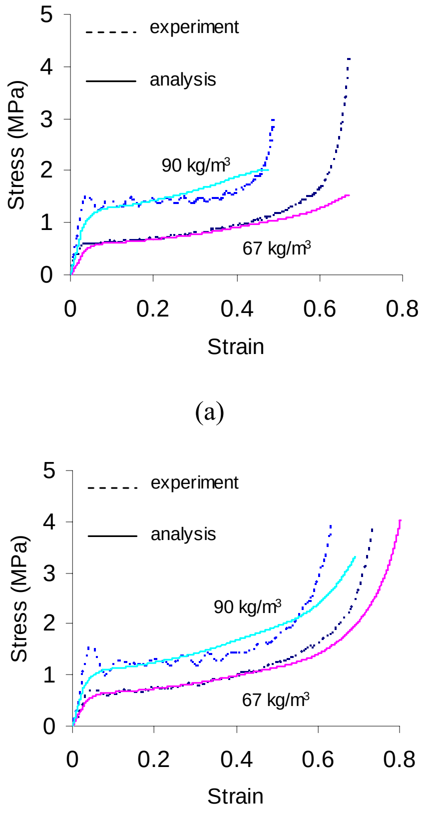


(a)

(b)

Fig 4: Comparison of transient velocity predicted by Eq. (6) with those of experiments when initial velocity of the impactor was (a) 2.75 m/s and (b) 4.00 m/s.

From both quasi-static and impact calculated stress-strain relationship it is unambiguous that there is no discontinuity in the graph and follows the same path as the data almost up to plateau region. Though stress-strain curve of impact loading undergo too many oscillations; flow stress of calculated curve follows the same path as the experiment. However, in the densification region little variation was observed.



(b)

Fig 5: Calculated and experimental stress-strain response of polyurethane foam under impact loading when initial velocity of the impactor was (a) 2.75 m/s and (b) 4.00 m/s.

8. CONCLUSIONS

By employing fundamental approach, combination of elastic-plateau constitutive model has been suggested. Results produced by this constitutive relation show evidence of good correlation with experiments thus substantiating its validity. First portion of the constitutive model adequately describes the non-linear constant visco-elastic nature of the polyurethane foam. Another portion of the model evidences of dependence of power type strain rate both under static and dynamic loading. By solving two equations, plateau stress can be calculated. Thus constitutive model decomposes the foam behavior into a visco-elastic part and rate dependent integral power model part. The parameters determined by polynomial curve fitting are based on measured quasi-static data. A newly transient velocity equation adequately describes the velocity profile.

9. REFERENCES

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10. NOMENCLATURE

Symbols	Meaning
a, b	material constants
E_f	elastic modulus of foam
E_s	elastic modulus of solid
ρ_f	density of foam
ρ_s	density of solid material
σ_e	stress up to elastic region
σ	stress after elastic region
ε	strain
ε	strain rate
τ	time constant
γ	contact dynamic resistive co-efficient
v	transient velocity of the impactor
v_0	initial velocity of the impactor
H	height of the specimen
g	acceleration of the gravity
n	integer number
A_n	co-efficient of shape function