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FINITE DIFFERENCE ANALYSIS OF SHORT SMA COLUMNS WITH TENSION- COMPRESSION ASYMMETRY

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ABSTRACT
A new numerical scheme based on finite difference technique has been devised and used to trace the load-deflection curves (equilibrium configuration paths) of a column that has highly non-linear and prominent non-symmetric responses in tension and compression. Next, Thompson's two theorems are used to interpret the critical load for the column from those load-deflection curves.
The devised method can be used to calculate buckling loads of any short column with material nonlinearity. To utilize the fruitfulness of the devised method, however, experimentally obtained stress-strain curves of superelastic shape memory alloy (SMA), having significantly non-symmetric responses in tension and compression, have been used for determining the buckling response of short columns. A rectangular cross-section is selected for analysis of the column, loaded beyond the linear stress-strain relations. Comparison shows excellent agreement between the present results of finite difference technique with those obtained from Timoshenko's method for inelastic buckling of a both ends hinged column.

Keywords: Non-linear stress-strain curves, Non-symmetric stress-strain property, Critical load, Finite difference technique, Load-deflection curves

1. INTRODUCTION
Instability of structures is always a challenging topic for practicing engineers. Therefore, calculation of buckling loads by different numerical schemes is extensively reported in the literature; one of the major reasons of such studies is the discrepancy of results between the experimental buckling loads and their predicted values. Those different numerical methods include finite element, finite difference, as well as strength of materials approach etc. A few of such studies are listed in the reference. For example, Bert and Ko [1] used finite difference technique and calculated buckling loads of columns constructed of bimodular material, which has a different Young's modulus in tension than it has in compression. Gadalla and Abdalla [2] predicted buckling behavior of compression members with variability in material and/or section properties based on eigen solutions. Earlier Li [3] dealt with multi-step non-uniform columns by analytical approach.

Commercial FEM code ANSYS was used for comprehensive analysis of slender as well as short columns made of stainless steel and shape memory alloy [4] - [6]. It was pointed out in those studies that though Euler's slender column formula can be used for ideal cases, inclusions of actual stress-strain relations, which are non-linear, become necessary if one needs to rigorously study the postbuckling path even for a very slender column. Moreover, in those of our previous studies, tensile and compressive stress-strain curves were used separately for simulation purpose. Consequently, it was concluded that for numerical predictions of response of short beams/columns made of steel or, shape memory alloy (SMA), simultaneous use of non-linear stress-strain curves in tension and compression becomes essential in some cases [4]- [6].

It should be noted that though commercial FEM code ANSYS has material model like Mooney-Rivlin, that can use both of these tension-compression stress-strain curves simultaneously, but with substantial modifications while evaluating Mooney-Rivlin constants [5]. Since, such modifications are not always desirable; therefore, in their next study Rahman, Akanda and Hossain [7] and Hossain [8] used both tensile and compressive stress-strain curves simultaneously. In these two studies, a strength of materials approach, termed as Timoshenko's method [9], was used to calculate the buckling load of both ends hinged columns with non-symmetric response in tension and compression. It should be mentioned here that this strength of materials method is suited only for both ends hinged columns. A complete study, however, should also include all practically possible boundary conditions of columns.

Bending of superelastic shape memory alloy (SMA) beams have been reported by Auricchio and Sacco [10], Rejznar et al.[11] and Raniecki [12]. In our immediate previous study, a numerical scheme based on integration technique was used to comprehensively deal with the bending of a tapered shape memory alloy (SMA) beam that has highly non-linear stress-strain relations and also shows non-symmetric response in tension and compression, [13].

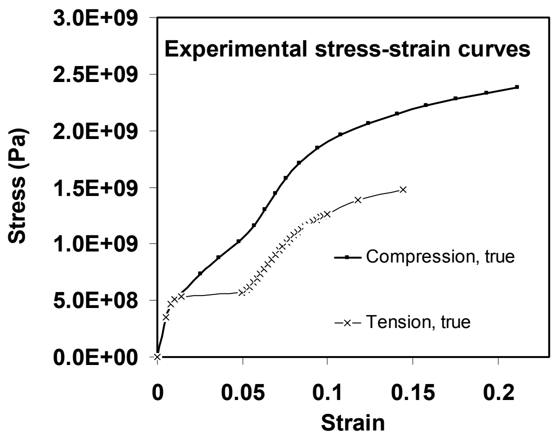

Fig 1: Stress-strain curves for the super elastic SMA in compression and tension [14]

Observing the above-mentioned facts, the present study concentrates on a suitable numerical scheme in order to find buckling load of any short column by simultaneously using column's highly non-linear stress-strain curves in tension and compression for all practically possible end conditions. Therefore, stress-strain curves that are asymmetric in tension and compression as shown in Fig.1 are used for simulation purpose so that the usefulness of the devised numerical scheme can be demonstrated. The numerical scheme is based on finite difference technique associated with a special iteration scheme. Although buckling of columns have been studied by finite difference technique by other researchers as mentioned, the present study incorporates highly non-linear (and asymmetric) stress-strain curves for large values of strains which, we believe something new as far as numerical analysis is concerned for short columns. Moreover, the present study relies on the self-developed complete computer code which is rather simple, straightforward but, efficient and therefore, no commercial software is needed for detail analysis.

Such a study will be practically important as well because tension-compression asymmetry becomes prominent for large inelastic bending of any short column, used in numerous structural applications. From the curves of Fig. 1, it is seen that the tension compression asymmetry becomes prominent if the strain exceeds 1%.

2. MATHEMATICAL MODELING

The basic equations for the analysis of beam-columns can be derived by considering the beam subjected to an axia compressive force P and to a distributed lateral load of intensity q. The equation is

$$(EIy'')'' + Py'' = q \text{ ....}(1)$$

Using the finite difference expressions for the second order derivative,

$$\frac{d^2y}{dx^2} = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \text{.....................}(2)$$

We obtain the governing equation as

$$y_{i+2} - \left(\frac{2a_{i+1} + 2a_i}{a_{i+1}} - A\right) y_{i+1} + \left(\frac{a_{i+1} + 4a_i + a_{i-1}}{a_{i+1}} - 2A\right) y_i - \left(\frac{2a_i + 2a_{i-1}}{a_{i+1}} - A\right) y_{i-1} + \frac{a_{i-1}}{a_{i+1}} y_{i-2} = B \text{ .... }.(3)$$

Where

a_i *is EI at grid i,*

$$A = \frac{Ph^2}{a_{i+1}} \text{ and } B = \frac{qh^4}{a_{i+1}}$$

Eq (3) can be used for any column, in slight bent shape, having variable geometric and material properties.

Boundary conditions, for both ends clamped:

$$y = 0 = \frac{dy}{dx} \quad at\ x = 0\ and\ x = L......(4)$$

Boundary conditions, for both ends hinged:

$$y = 0 = \frac{d^2y}{dx^2} \quad at\ x = 0\ and\ x = L......(5)$$

Finite difference expressions with order of error h^2 are used for boundary conditions as well as governing equations.

$$\sigma_c = \frac{P}{bh_t} = -\frac{1}{\Delta}\int_{\varepsilon_1}^{\varepsilon_2} \sigma d\varepsilon \text{ ....}(6)$$

$$E'' = \frac{12}{\Delta^3}\int_{\varepsilon_1}^{\varepsilon_2} \sigma(\varepsilon - \varepsilon_0) d\varepsilon \text{ ....}(7)$$

$$M = \frac{E'' I}{\rho} \text{ ....}(8)$$

M-Δ, and E" -Δ relations are known directly by

Computer code using Eq (6) to Eq (8) and Fig 1.

3. RESULTS AND DISCUSSION

To solve Eq. (1) it is very important to select the optimum value of *h*. From fig-2 it is clear that shape varies a very little with grid size. Therefore we have selected 90 grids for our computation.

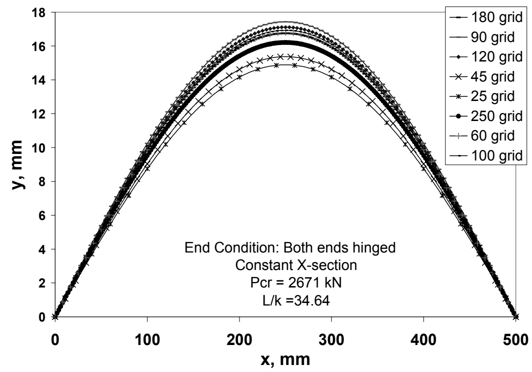

Fig 2: Buckled shapes of the column for different grid sizes

To correctly predict the deformed shapes, E'' for different values of P must be available (as shown in Fig. 3).Using this value, Eq(1) is solved to find the maximum buckling load.

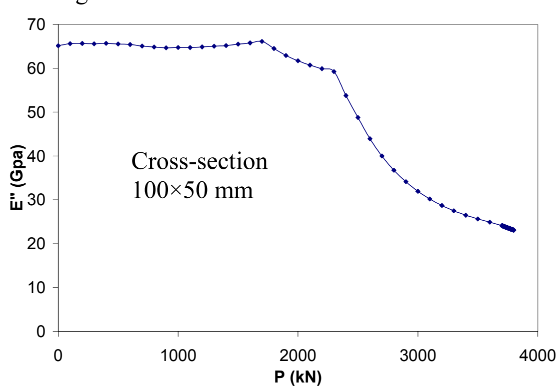

Fig 3: Variation of E with P

By this method buckling load can be determined from the P-δ curves. The solutions do not converge anymore as at the point of instability; deformation becomes too large. Some results are also generated for comparison purpose by Timoshenko's method [13]. By this latter method, critical load only for a both ends hinged column can be determined from the peak point of slenderness ratio versus midspan deflection curve.

Now column with an eccentric load is modeled to compare the value with Timoshenko's method [8]and [9]. Based on Fig 3 and M-Δ , E" - Δ relations we calculated E" at every grid point to find the critical load. Critical load for an eccentricity of 5mm is found to be 1625 kN which is 1635 kN by Timoshenko's method [8].

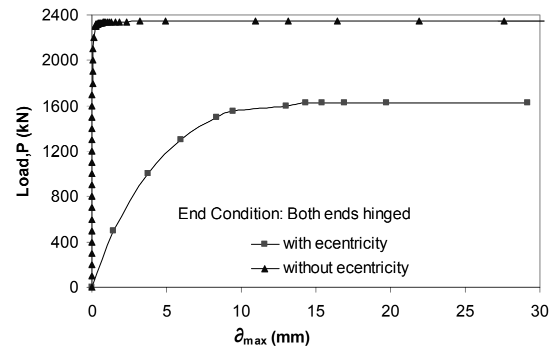

Fig 4: Load deflection curve with eccentric axial loading

The above comparison proves the soundness of the method used.

Loading path of any column (including a superelastic SMA column) can be predicted by the method devised. The unloading path of a superelastic SMA column, however, can't be predicted without the unloading stress-strain curves. For the short SMA columns considered in this study permanent deformations are likely to occur upon unloading which are not easy to predict even by the available commercial softwares. Moreover, from our experience it is obvious that predictions of unloading paths of highly slender superelastic SMA columns can be important and interesting [14] and[[6]. But predictions of buckling are much more important for short SMA columns than the predictions of their unloading paths [5]. Therefore, the paper concentrates on the calculations and presentations of critical and nonsymmetric stresses for the short SMA columns.

Based on Fig 3. for *L/k*= 38, 34.65, 28 Figs. 5, 6 and 7 show the P-δ curve, shape of the buckled column and corresponding bending moments. Critical load for both ends clamped columns varies with l/k ratios. For l/k=38, P_{cr}=3501 kN and for l/k=28, P_{cr} =4458 kN and 3779 kN when l/k=34.65. Critical load is increased when slenderness ratio is decreased, but not following Euler's formula.

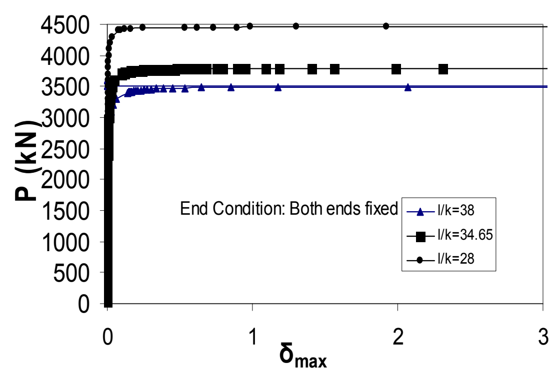

Fig 5: Load deflection curve for column with different l/k

The results are finally compared below with the value obtained for SMA wire of circular cross section with diameter 2 mm [14].

Table 1: Comparison of σ_c

L/k	σ_c (MPa)	
	Present Study Cross-section (100×50 mm)	Reference [14] Φ =2 mm
28	891.6	946.5
38	700	575

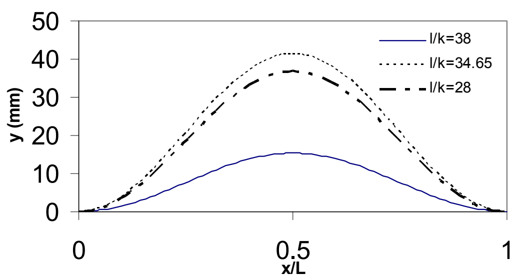

Fig 6: Deflected shape for l/k = 28, 34.65 and 38 corresponds to critical load

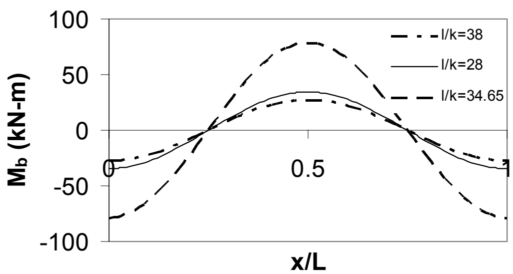

Fig 7: Moment curve for l/k = 28, 34.65 and 38 corresponds to critical load

6. CONCLUSIONS

A highly reliable and accurate numerical scheme is demonstrated in order to predict buckling load of short columns having nonlinear and asymmetric $\sigma - \varepsilon$ curve in tension and compression.

The computer code based on FD technique can be used to reliably predict buckling response of any column with any type of boundary conditions and loading.

7. REFERENCES

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7. NOMENCLATURE

Symbol	Meaning	Unit
E:	Modulus of elasticity	Gpa
x:	Axial distance	mm
h_t:	height of rectangular cross-section	mm
M_b:	bending moment	kN-m
P:	axial load on column	kN
e:	load eccentricity	mm
L:	column length	mm
ε:	strain	
σ:	Stress	Pa
δmax:	Maximum lateral deflection	
Δ:	h/ ρ	
y	lateral deflection	mm