Proceedings of the
International Conference on Mechanical Engineering 2007
(ICME2007) 29- 31 December 2007, Dhaka, Bangladesh

ICME07-FL-15

MAGNETOHYDRODYNAMIC (MHD) LAMINAR FREE CONVECTIVE FLOW ACROSS A HORIZONTAL CIRCULAR CYLINDER

Nur Hosain Md. Ariful Azim[1], Abdullah-Al-Mamun[2] and Md. Mustafa Kamal Chowdhury[3]

[1] School of Business Studies, Southeast University, Bangladesh Email: nhmarif@yahoo.com
[2] Institute of Natural Sciences, United International University, Bangladesh, Email: mamun@uiu.ac.bd
[3]Department of Mathematics, Bangladesh University of Engineering and Technology, Bangladesh

ABSTRACT
A time independent and two dimensional free convection flow across a horizontal circular cylinder in the presence of magnetic field has been investigated. The equations governing such flow are transformed to non-dimensional forms using the suitable transformations. The dimensionless equations are then solved numerically with the help of the implicit finite difference method with Keller box scheme. Numerical results are obtained for different values of the magnetic parameter *M* and the Prandtl number *Pr.* The dimensionless skin friction co-efficient, rate of heat transfer, velocity distribution and the temperature profile are shown graphically.

Keywords: Magnetohydrodynamic, Free Convection, Circular cylinder, Finite difference method

1. INTRODUCTION
Many natural phenomena and engineering problems are vulnerable to MHD analysis. Geophysicists encounter MHD phenomena in the interactions of conducting fluids and magnetic fields that are present in and around heavenly bodies. Engineers employ MHD principles in the design of heat exchangers, pumps and flowmeters, in space vehicle propulsion, control and re-entry, in creating novel power generating systems and in developing confinement schemes for controlled fusion. Many researchers investigated the effects of MHD free and forced convection flow both experimentally and theoretically. Luciano[1] investigated the laminar free convection around horizontal cylinder. The author considered the cylinder surface partly isothermal and partly adiabatic. Kuehn and Goldstein [2] determined the numerical solution of the Navier-Stokes's equations on laminar natural convective flow about a horizontal isothermal circular cylinder. Farouk and Guceri [3] investigated the natural convection from a horizontal cylinder. Merkin [4] studied free convection boundary layer flow on an isothermal horizontal circular cylinder. Sparrow and Cess [6] investigated the effect of a magnetic field on free convection heat transfer. Kuiken [7] studied MHD free convection in a strong cross-field. Hossain et. al. [8,9] investigated MHD forced and free convection boundary layer flow near the leading edge. He also investigated MHD forced and free convection boundary layer flow along a vertical porous plate. Wilks [10] studied MHD free convection about a semi-infinite vertical plate in a strong cross-flow. In the cases [1,5], the analyses are related on free convection horizontal cylinder. The authors [6,10] analysed the MHD natural convection flow in various fields.

This study deals with the problem of MHD laminar free convective flow across a horizontal cylinder. Dimensionless equations will be solved numerically using an implicit finite difference method known as Keller box scheme. The results will be presented in terms of the skin friction coefficient, the rate of heat transfer, and the velocity and temperature distributions.

2. FORMULATION OF THE PROBLEM
The steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid across a horizontal cylinder in the presence of a uniformly distributed transverse magnetic field is considered. The flow configuration and the coordinates system are shown in Figure 1.

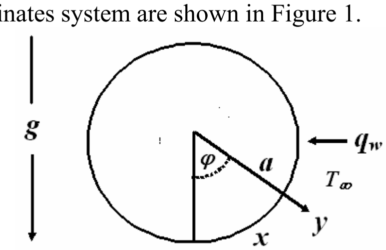

Fig 1: The co-ordinate system

The boundary layer equations governing the convective flow under these assumptions with the Boussineaq approximations can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\pm g\beta(T-T_\infty)-\frac{\sigma u B_0^2}{\rho}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial^2 y} \qquad (3)$$

The appropriate boundary conditions to be satisfied by the above equations are:

$$\mathrm{u}=\mathrm{v}=0,\ \mathrm{T}=\mathrm{T_w}\ \text{ on }\ \mathrm{y}=0 \qquad (4.a)$$

$$\mathrm{u}\rightarrow 0,\ \mathrm{T}\rightarrow \mathrm{T}_\infty\ \text{ as }\ \mathrm{y}\rightarrow\infty \qquad (4.b)$$

The non-dimensional governing equations and boundary conditions can be obtained from equation (1) to (4) using the following non-dimensional quantities:

$$\eta=\frac{y}{a}Gr^{\frac{1}{4}},\quad \xi=\frac{x}{a},$$

$$\psi(x,y)=\nu\xi Gr^{\frac{1}{4}}f(\xi,\eta), \qquad (5)$$

$$\theta(\xi,\eta)=\frac{T(x,y)-T_\infty}{T_w-T_\infty}$$

here η is the free convection similarity variable , ξ is essentially a stretched *x* coordinate, $\psi(x,y)$ is the stream function which satisfies the mass conservation equation and which is related to the velocity components in the usual way as $u=\partial\psi/\partial y, v=-\partial\psi/\partial x$ and

$Gr=\frac{g\beta(T_w-T_\infty)}{\nu^2}a^3$ is the Grashof number.

Using the transformation mentioned in equation (5), the non-dimensional momentum and energy equations are obtained as

$$f'''+ff''-f'^2+\frac{\sin\xi}{\xi}\theta-Mf'=\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \qquad (6)$$

$$\frac{1}{\mathrm{Pr}}\theta''+f\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \qquad (7)$$

where, Prandtl number $\mathrm{Pr}=\frac{\mu C_p}{k}$ and magnetic parameter $M=\frac{\sigma a^2 B_0^2}{\nu Gr^{\frac{1}{2}}\rho}$.

The corresponding boundary conditions become:

$$\left.\begin{array}{l} f'=\frac{\partial f}{\partial\eta}=0 \\ \theta=1 \end{array}\right\}\ on\ \eta=0 \qquad (8.a)$$

$$\left.\begin{array}{l} \frac{\partial f}{\partial\eta}=0\ and \\ \theta\rightarrow 0 \end{array}\right\}\ as\ \eta\rightarrow\infty \qquad (8.b)$$

3. METHOD OF SOLUTION

To get the solutions of the parabolic differential equations (6) and (7) along with the boundary condition (8) , we have employed implicit finite difference method together with Keller- box elimination technique which is well documented and widely used by Cebeci [6].

4. RESULT AND DISCUSSION

In the present investigation we have obtained the solutions of the nonsimilar boundary layer equations governing the laminar free convective flow across a horizontal circular cylinder in the presence of a transverse magnetic field. Detailed numerical solutions have been obtained for a wide range of values of the magnetic parameter *M* (= 0.0, 0.2, 0.5, 0.8, 1.0) and Prandtl number *Pr* (= 1.0, 0.73, 0.2). The values of the Prandtl number *Pr* are taken to 0.73 that corresponds physically the air and 1.0 corresponding to electrolyte a solution such as salt water and 0.2 is used theoretically.

The skin friction coefficient $f''(\xi, 0)$ and the rate of heat transfer $\theta'(\xi ,0)$ are shown graphically for different values of the Prandtl number *Pr* in Figure 2 and Figure 3, respectively with *M* = 0.2. It is clear from Figure 2 that fluid having large Prandtl number has a lower skin friction coefficient $f''(\xi,0)$ and from Figure 3, we observe that the fluid having large Prandtl number has higher rate of heat transfer $\theta'(\xi ,0)$.

The skin friction coefficient $f''(\xi,0)$ and the rate of heat transfer $\theta'(\xi ,0)$ are presented graphically for different values of the magnetic parameter *M* in Figure 4 and Figure 5, respectively. It can be noted from Figure 4 that the skin friction coefficient $f''(\xi, 0)$ is reduced for large magnetic parameter and from Figure 5 it can be seen that the rate of heat transfer $\theta'(\xi ,0)$ decreases monotonically for a particular value of *M* with the increased value of *ξ*. The rate of heat transfer also decreases for the increased value of *M*.

Graphical presentation of velocity distribution $f'(\xi,\eta)$ and temperature distribution $\theta(\xi,\eta)$ for different values of magnetic parameter *M* and Prandtl number *Pr* are presented in Figure 6 to Figure 9, respectively. From the Figure 6 and Figure 7, it can be observed that the velocity profile and temperature profile are getting lower for higher values Prandtl number.

Figure 8 and Figure 9 deal with the effects of different values of the magnetic parameter *M* with *Pr* = 0.73 on the velocity distribution $f'(\xi,\eta)$ and the temperature distribution $\theta(\xi,\eta)$, respectively. From Figure 8, it is revealed that the velocity profile $f'(\xi,\eta)$ decreases with the increase of the magnetic parameter *M* which indicates that the magnetic field retards the fluid motion. Figure 9 shows that the temperature profile increases with the increased value of magnetic parameter *M* which implies that magnetic field produces temperature within the boundary layer.

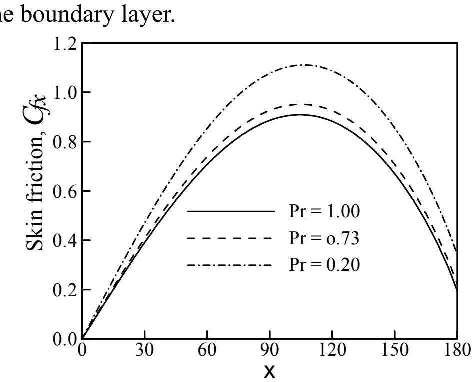

Fig 2: Skin friction for different values of Prandtl number where M = 0.2

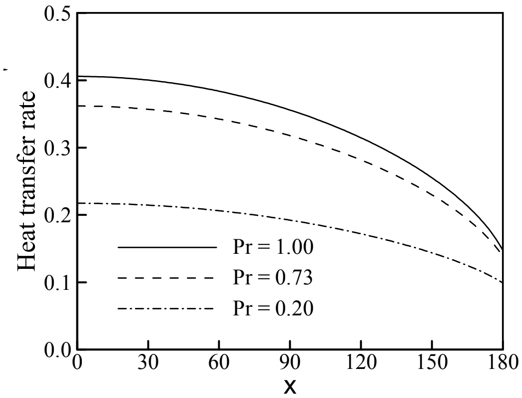

Fig 3: Rate of heat transfer for different values of Prandtl number where M = 0.2

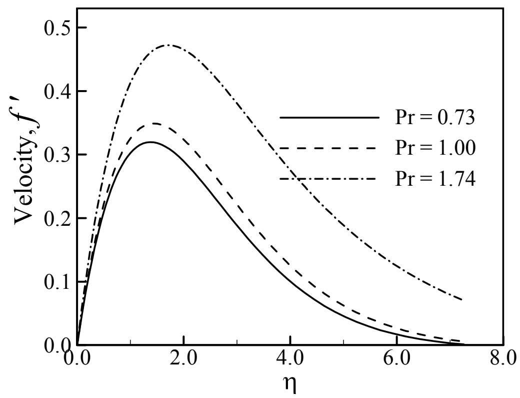

Fig.6: Velocity profiles for different values of Prandtl numbers, where M = 0.2

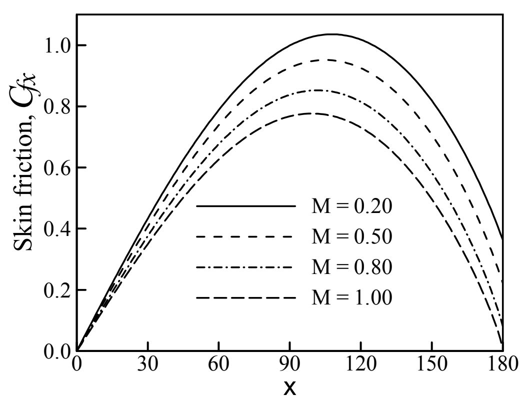

Fig 4: Skin friction for different values of *M* where *Pr* = 0.73

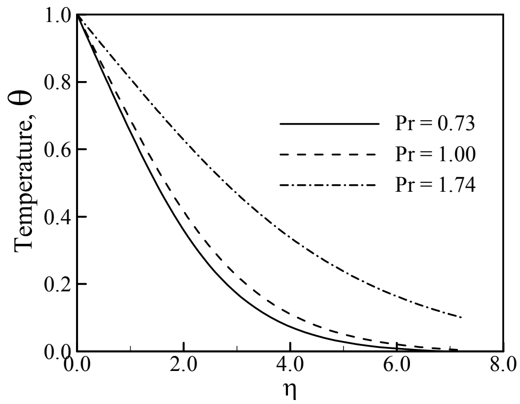

Fig 7: Temperature profiles for different values of Prandtl numbers, where M = 0.2.

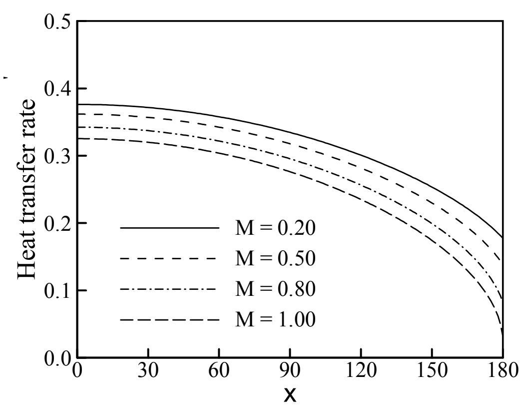

Fig 5: Rate of heat transfer for different values of *M* where *Pr* = 0.73

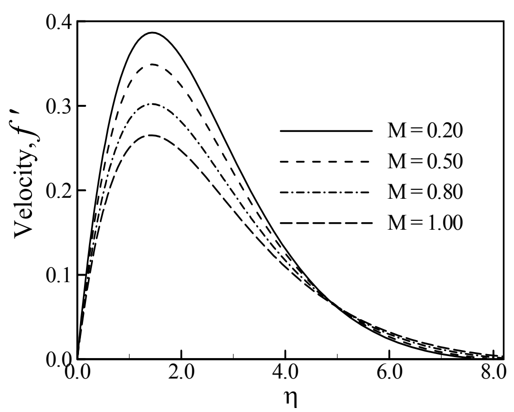

Fig 8: Velocity profiles for different values of M, where Pr = 0.73

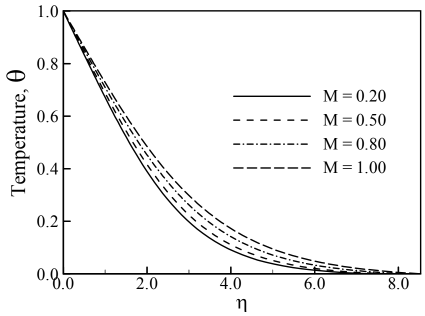

Fig 9: Temperature profiles for different values of *M*, where *Pr*=0.73

5. CONCLUSIONS

Magnetohydrodynamic (MHD) laminar free convective flow across a horizontal cylinder has been investigated introducing suitable transformations. Numerical solutions of the equations governing the flow are obtained by using the implicit finite difference method together with the Keller Box scheme. From the present investigation, the following conclusions may be drawn:

1. The skin friction coefficient and the velocity distribution decrease for increasing value of the magnetic parameter
2. Increased value of the magnetic parameter *M* leads to decrease the rate of heat transfer and increase the temperature distribution.
3. It has been observed that the skin friction coefficient, the temperature distribution and the velocity distribution decrease with the increase of the Prandtl number *Pr* but the rate of heat transfer increase with the increase of the Prandtl number *Pr*.

6. REFERENCES

1. Luciano M. De Socio, "Laminar free convection around horizontal circular cylinders." Int. J. Heat Mass Transfer, Vol. 26, No. 11, pp.1669-1677, (1983).
2. Kuehn T.H. and Goldstein R.J., "Numerical solution of Navier-Stokes equations for laminar natural convection about a horizontal isothermal circular cylinder." Int. J. Heat Mass Transfer, Vol. 23, pp.971-979, (1980).
3. Farouk B. and Guceri S.I., "Natural convection from horizontal circular cylinder-laminar regime." ASME, J. Heat transfer vol. 103, pp.522-526, (1981).
4. Merkin J.H., "Free convection boundary layer on an isothermal horizontal circular cylinder." ASMME/AIChE Heat transfer conference, St. Louis, Mo. August(9-11), (1976).
5. Merkin J.H., "Free convection boundary layer on cylinder of elliptic cross section." J. of Heat transfer. V-99, Aug-1977.
6. Sparrow E.M. and Cess R.D., "Effect of magnetic field on free convection heat transfer." Int. J. Heat Mass Transfer, Vol. 3, pp.267-274, (1961).
7. Kuiken H.K.' "Magnetohydrodynamic free convection in a strong cross-field." J. Fluid Mech. Vol. 40, pp.21-38, (1970).
8. Hossain M.A. and Ahmed M., "MHD forced and free convection boundary layer flow near the leading edge." Int. J. Heat Mass Transfer, Vol. 33 pp.571-575, (1984).
9. Hossain M.A., Alam K.C.A. and Rees D.A.S., "MHD forced and free convection boundary layer flow along a vertical porous plate." Applied Mech. And Engineering, vol.2, No.1, pp.33-51, (1997).
10. Wilks, G, "Magneto hydrodynamics free convection about a semi-infinite vertical plate in a strong cross field." ZAMP, 27, 621-631, 1976
11. Cebeci T. and Bradshaw P.: Physical and computational aspects of convective heat transfer. Springer, New York (1984).

7. NOMENCLATURE

Symbol	Meaning	Unit
c_p	Specific heat at constant pressure	
c_f	Local skin friction	
f	Dimensionless stream function	
g	Acceleration due to gravity	
K	Thermal conductivity of the fluid	
p	Fluid pressure	
q_w	Surface heat flux	
D	Diameter of the cylinder	
T	Temperature in the boundary layer	
T_w	Temperature at the surface	
T_∞	Temperature of the ambient fluid	
U	Velocity component in the x-direction	
V	Velocity component in the y-direction	
X	Distance along the surface	
Y	Distance normal to the surface	
B_0	Applied magnetic field	
β	Co-efficient of thermal expansion	
ψ	Stream function	
η	Dimensionless similarity variable	
ρ	Density of the fluid inside the boundary layer	
ν	Kinematic viscosity	
μ	Viscosity of the fluid	
θ	Dimensionless temperature	
σ	Electrical conductivity	
2φ	Angle formed by the adiabatic surface at the axis of the cylinder.	