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CONJUGATE EFFECTS OF VISCOUS DISSIPATION AND PRESSURE WORK ON MHD NATURAL CONVECTION FLOW ALONG A VERTICAL FLAT PLATE WITH HEAT CONDUCTION

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ABSTRACT

In this paper, the effects of viscous dissipation and pressure stress work on magnetohydrodynamic (MHD) free convection flow along a vertical plate has been investigated. Magnetohydrodynamic free convection flow and heat conduction due to wall thickness *b* have been considered in this investigation with a goal to attain similarity solutions of the problem posed, the developed equations are made dimensionless by using suitable transformations. The non-dimensional equations are then transformed into non-linear equations by introducing a non- similarity transformation. The resulting non-linear similar equations together with their corresponding boundary conditions based on conduction and convection are solved numerically by using the finite difference method along with Newton's linearization approximation. Numerical results for the details of the velocity profiles, temperature profiles, skin friction coefficient and the surface temperature distributions are shown both on graphs and in tabular form for different values of the parameters entering into the problem.

Keywords: Free convection, magneto hydrodynamics, viscous dissipation and pressure work.

1. INTRODUCTION

Natural convection heat transfer has gained considerable attention because of its numerous applications in the areas of energy conservations cooling of electrical and electronics components, design of solar collectors, heat exchangers and many others. The main difficulty in solving natural convection problems lies in the determination of the velocity field, which greatly influences the heat transfer process. Ackroyd [1], studied the stress work effects in laminar flat plate natural convection flow. Gebhart [2] has shown that the viscous dissipation effect plays an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotational speeds and also in strong gravitational field processes on large scales (on large planets) and in geological processes. Joshi and Gebhart [6] have shown that the effects of pressure stress work and viscous dissipation in some natural convection flows. With this understanding Takhar and Soundalgekar [10] have studied the effects of viscous and Joule heating on the problem posed by Sparrow and Cess [9], using the series expansion method of Gebhart [2]. Zakerullah [11] investigated the viscous dissipation and pressure work effects in axisymmetric natural convection flows. Pozzi and Lupo [8] have shown the coupling of conduction with laminar natural convection along a flat plate. Alam et al. [3] studied the effect of pressure stress work and viscous dissipation in natural convection flow along a vertical flat plate with heat conduction. Alim et al. [4] investigated the Joule heating effect on the coupling of conduction with magnetohy-drodynamic free convection flow from a vertical flat plate.

The transformed non-similar boundary layer equations governing the flow together with the boundary conditions based on conduction and convection were solved numerically using the Keller box (implicit finite difference) method, along with Newton's linearization approximation method. The effects is of the Prandtl number *Pr,* the viscous dissipation parameter *N*, the magnetic parameter *M* and pressure work parameter G_e on the velocity and temperature fields as well as on the skin friction and surface temperature are studied

2. MATHEMATICAL FORMULATION

Let us consider a steady, laminar, two dimensional, incompressible free convection and mass transfer flow along a side of a vertical flat plate of thickness '*b'* insulated on the edges for which pure conduction is occurred and with a temperature T_b maintained on the other side (Fig.1). The physical model and the co-ordinate system are shown in Fig.1. The *X*-axis is taken along the vertical flat plate in the upward direction and the *Y*-axis normal to the plate.

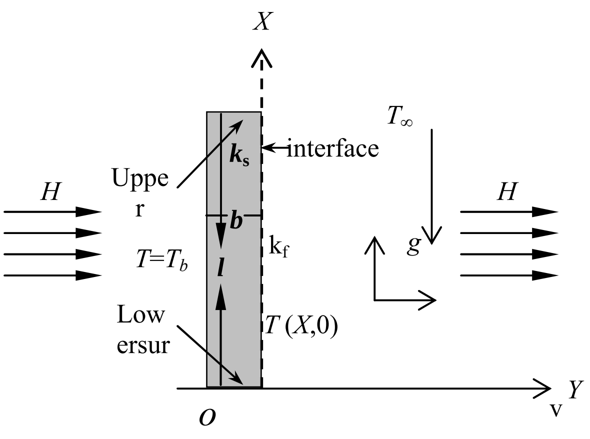

Fig.1: Physical configuration and coordinates system

In this analysis, $\vec{H}=\mu_e H_0$, μ_e being the magnetic permeability of the fluid, H_0 is the transverse magnetic field strength. The Boussinesq approximation $\rho=\rho_\infty\left[1-\beta\left(T-T_\infty\right)\right]$, then the basic equations become:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad \textbf{(1)}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\nu\frac{\partial^2 U}{\partial Y^2} +g\beta\left(T-T_\infty\right)-\frac{\sigma H_0^2 U}{\rho} \qquad \textbf{(2)}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{\kappa}{\rho\ C_P}\frac{\partial^2 T}{\partial Y^2} +\frac{\nu}{C_P}\left(\frac{\partial U}{\partial Y}\right)^2+\frac{T\beta\ U}{\rho C_P}\frac{\partial P}{\partial X} \qquad \textbf{(3)}$$

The symbols have got their meaning as mention in the Nomenclature. The appropriate boundary conditions to be satisfied by the above equations are

$$U=0\ ,\ V=0\ \ \textbf{at}\ \ Y=0$$
$$U\to 0\ ,\ T\to T_\infty\ \ \textbf{as}\ Y\to\infty \qquad \textbf{(4)}$$

The coupling conditions

$$\frac{k_s}{k_f}\frac{\partial T_{so}}{\partial Y}=\left(\ \frac{\partial T}{\partial Y}\ \right)_{Y=0} \qquad \textbf{(5)}$$

should be maintained for the temperature and the heat flux be continuous at a solid fluid interface as suggested by Miyamoto [7], Where k_s and k_f are the thermal conductivity of the solid and the fluid respectively. The temperature T_{so} in the solid as given by Pozzi and Lupo [8] is

$$T_{so}=T(X,0)-\left\{T_b-T\ \ (X,0)\right\}\frac{Y}{b} \qquad \textbf{(6)}$$

where T (X, o)is the unknown temperature at the interface to be determined from the solutions of the equations. We observe that the equations (1) - (3) together with the boundary conditions (5) - (6) are non-linear partial differential equations, which have been solved numerically..

3. TRANSFORMATION OF THE GOVERNING EQUATIONS

Equations (1)–(3) may now be non-dimensionalized by using the following dimensionless dependent and independent variables:

$$x=\frac{X}{L}\ ,\ y=\frac{Y}{L}d^{1/4}\ ,\ U=\frac{\nu}{L}ud^{1/2}$$
$$V=\frac{\nu}{L}d^{1/4}v\ ;\quad \theta=\frac{T-T_\infty}{T_b-T_\infty} \qquad (7)$$
$$L=\frac{\nu^{2/3}}{g^{1/3}}\ ,\ d=\beta(T_b-T_\infty)$$

L has been defined in terms of ν and g, which are the intrinsic properties of the system. The reference length along the 'Y' direction has been modified by a factor $d^{-1/4}$ in order to eliminate this quantity from the dimensionless equations and the boundary conditions.

For exterior conditions, we know hydrostatic pressure, $\partial P/\partial X=-\rho_e g$ and $\rho=\rho_e$, and the pressure work parameter $G_e = (g\beta L)/C_P$. Using the above relations (7) the non-dimensional form of the governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (8)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta \qquad (9)$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2} +N\left(\frac{\partial u}{\partial y}\right)^2-G_e\frac{\left\{T_\infty+\left(T_b-T_\infty\right)\theta\right\}}{\left(T_b-T_\infty\right)} \qquad (10)$$

Where $P_r = \mu C_p/\kappa_f$ is the Prandtl number, $M=\sigma {H_0}^2L^2/\mu d^{1/2}$ is the magnetic parameter and $N=\nu^2 d/L^2 c_p\left(T_b-T_\infty\right)$, the dimensionless viscous dissipation parameter.

The corresponding boundary conditions (4) - (5) take the following form:

$$u=v=0\ ,\ \theta-1=p\frac{\partial\theta}{\partial y}\ \text{ at }\ y=0$$
$$u\to 0,\ v\to 0\ \text{ as } y\to\infty \qquad (11)$$

where P is the pressure and p is the conjugate conduction parameter given by p = (κ_f/κ_s) (b/L) $d^{1/4}$. Here the coupling parameter 'p' governs the described problem. The order of magnitude of 'p' depends actually on b/L and κ_f/κ_s, $d^{1/4}$ being the order of unity. The term b/L attains values much greater than one because of L being small. Therefore in different cases 'p' is different but not always a small number. In the present investigation we have considered p = 1 which is accepted for b/L of o κ_f /

κ_s. To solve the equations (9) – (10) subject to the boundary conditions (11), the following transformations are introduced for the flow region,

$$\psi = x^{4/5}(1+x)^{-1/20} f(x,\eta),$$
$$\eta = yx^{-1/5}(1+x)^{-1/20}, \qquad (12)$$
$$\theta = x^{1/5}(1+x)^{-1/5} h(x,\eta)$$

Here η is the dimensionless similarity variable and ψ is the stream function which satisfies the equation of continuity and $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$ and h (η, x) is the dimensionless temperature. Then the equations (9) and (10) transformed to the following non dimensional forms:

$$f''' + \frac{16+15x}{20(1+x)} ff'' - \frac{6+5x}{10(1+x)} f'^2$$
$$-Mx^{2/5}(1+x)^{1/10} f' \qquad \textbf{(13)}$$
$$+h = x\left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x}\right)$$

$$\frac{1}{\Pr} h'' + \frac{16+15x}{20(1=x)} fh'$$
$$-\frac{1}{5(1+x)} f'h + Nxf''^2$$
$$-G_e\, x\left\{\left(\frac{1+x}{x}\right)^{1/5}\left(\frac{T_\infty}{T_b - T_\infty}\right) f' + hf'\right\} \qquad \textbf{(14)}$$
$$= x\left(f' \frac{\partial h}{\partial x} - h' \frac{\partial f}{\partial x}\right)$$

In the above equations the primes denote differentiation with respect to η . The boundary conditions (11), take the following form

$$f(x,0) = f'(x,0) = 0, \quad h'(x,0) =$$
$$-(1+x)^{1/4} + x^{1/5}(1+x)^{1/20} h(x,0) \qquad \textbf{(15)}$$
$$f'(x,\infty) = 0 \;,\; h'(x,\infty) = 0$$

The solutions of the above equations (13) and (14) together with the boundary conditions (15) enable us to calculate the skin friction τ and the rate of heat transfer θ at the surface in the boundary is

$$\tau = \mu\{x^{2/5}(1+x)^{-3/20} f''(,x,0)\} \qquad (16)$$

4. METHOD OF SOLUTION

The numerical methods used is finite difference method together with Keller box Scheme which is described in detail in the book by Cebecci and Bradshow [13] and widely used by Hossain and Alim [5,6].

5. RESULTS AND DISCUSSION

Here we have investigated the effects of viscous dissipation and pressure work on free convection flow along a vertical flat plate in presence of magnetohydrodynamics and heat conduction due to wall thickness b. Solutions are obtained for the fluids having Prandtl number Pr = 0.05, 0.10, 0.72, 1.00,1.74 and for a wide range of the values of the viscous dissipation parameter N = 0.2, 0.4, 0.6, 0.9 the magnetic parameter M =1.0, 1.3, 1.5, 1.7, 2.0 and the pressure work parameter Ge = 0.1, 0.4, 0.6, 0.9. If we know the values of the functions $f(x, \eta,)$, h (x, η) and their derivatives for different values of the Prandtl number Pr and the magnetic parameter M, we may calculate the numerical values of the surface temperature θ (x, 0) and the velocity gradient f'' (x, 0) at the surface that are important from the physical point of view.

Fig.2 (a) and Fig.2 (b) deal with the effect of the viscous dissipation parameter N (=0.2, 0.4, 0.6, 0.9) for different values of the controlling parameters Pr = 0.72, M = 1.0 and Ge = 0.4 on the velocity profiles $f'(x, \eta)$ and the temperature profiles h (x, η). From Fig. 2(a), it is revealed that the velocity profile $f'(x, \eta)$ increases slightly with the increase of the viscous dissipation parameter N which indicates that viscous dissipation increases the fluid motion slowly. Small increment is shown from Fig.2 (b) on the temperature profile h (x, η) for increasing values of N with the same controlling parameter.

Same results are observed from Fig. 3(a) and Fig. 3(b) for velocity profiles $f'(x, \eta)$ and temperature profiles h (x, η) respectively for different values of the pressure work parameter G_e (= 0.1, 0.4, 0.6, 0.9) with others controlling parameters Pr = 0.72, N = 0.07, M = 0.50.

Fig.4 (a) and 4(b) deal with the effect of the magnetic parameters M (=1.0, 1.3, 1.5, 1.7, 2.0) with others controlling parameters Pr = 0.72, N = 0.40 and Ge = 0.50 on the velocity profiles $f'(x, \eta)$ and the temperature profiles h (x, η). From Fig. 4(a), it is revealed that the velocity profile $f'(x, \eta)$ decreases with the increase of the magnetic parameter M which indicates that magnetic parameter decreases the fluid motion. But it is observed that the temperature profiles increases with the increase of magnetic parameter M that is shown in Fig.4 (b).

Numerical values of the skin friction coefficient coefficient f'' (x, 0) and the surface temperature θ (x, 0) are depicted graphically in Fig.5 (a) and 5(b) respectively against the axial distance x for different values of the viscous dissipation parameter N (=0.2, 0.4, 0.6, 0.9) for the fluid having Prandtl number Pr = 0.72 and others fixed parameter. From Fig. 5(a) and Fig. 5(b), it can be observed that increase in the value of the viscous dissipation parameter N leads to increase of the skin friction coefficient f'' (x, 0) and the surface temperature θ (x, 0).

In Fig.6 (a), the skin friction coefficient f'' (x, 0) and Fig.6 (b), the surface temperature θ (x, 0) are shown graphically for different values of the magnetic parameter M (=1.0, 1.3, 1.5, 1.7, 2.0). In Fig. 6(a), it can be observed that increase in the value of the magnetic parameter M (=1.0, 1.3, 1.5, 1.7, 2.0) leads to decrease the value of skin friction coefficient f'' (x, 0). Similar results hold in surface temperature distribution θ (x, 0) shown in Fig.6 (b) for same values of magnetic parameter
M (=1.0, 1.3, 1.5, 1.7, 2.0).

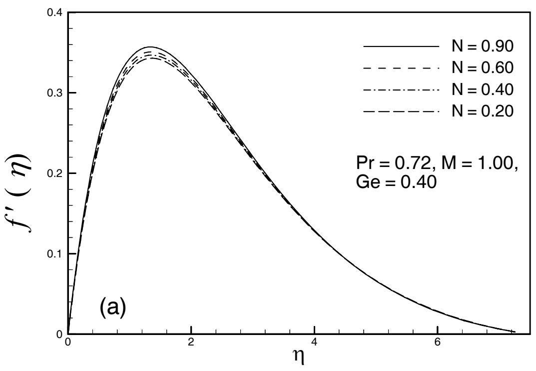

Fig. 2(a): Variation of dimensionless velocity distribution f' (x,η) against dimensionless distance η for different values of viscous dissipation parameter *N* with other fixed values.

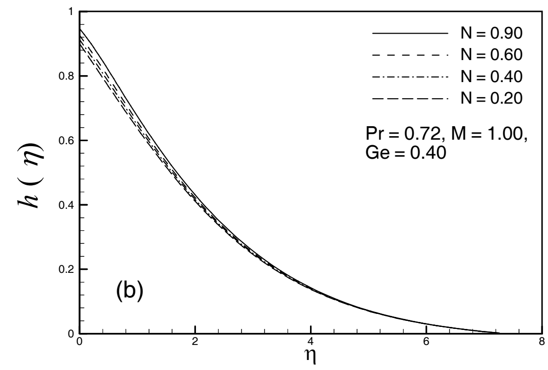

Fig. 2(b): Variation of dimensionless temperature distribution $h(x,\eta)$ against dimensionless distance η for different values of viscous dissipation parameter *N* with other fixed values.

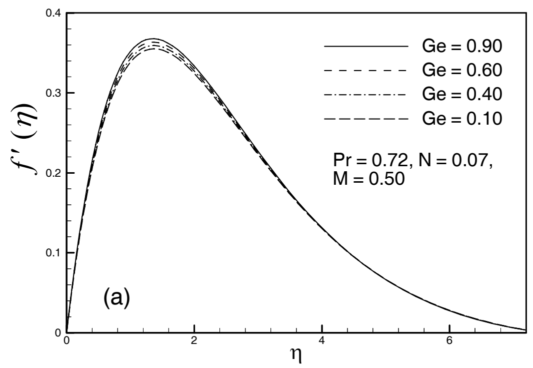

Fig. 3(a): Variation of dimensionless velocity distribution f' (x,η) against dimensionless distance η for different values of viscous pressure work parameter *Ge*.

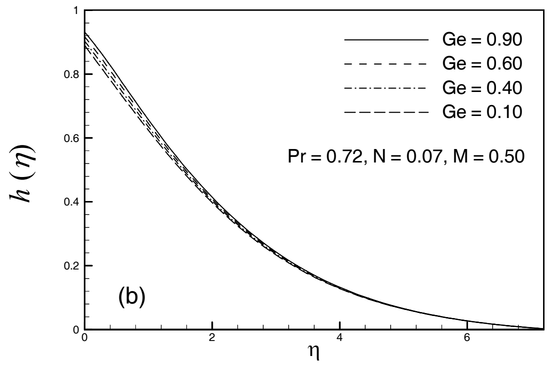

Fig. 3(b): Variation of dimensionless temperature distribution $h(x,\eta)$ against dimensionless distance η for different values of viscous pressure work parameter *Ge*.

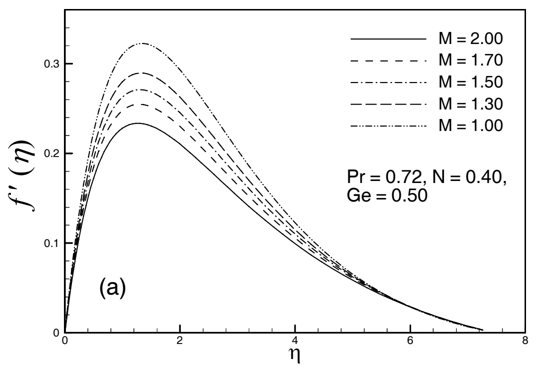

Fig.4(a):Variation of dimensionless velocity distribution f' (x,η) against dimensionless distance η for different values of viscous magnetic parameter *M* with other fixed values.

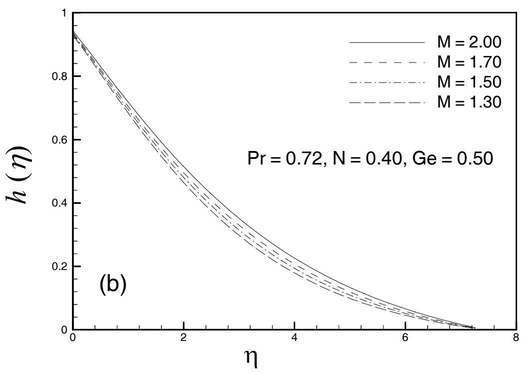

Fig.4(b):Variation of dimensionless temperature distribution $h(x,\eta)$ against dimensionless distance η for different values of viscous dissipation parameter *N* with other fixed values.

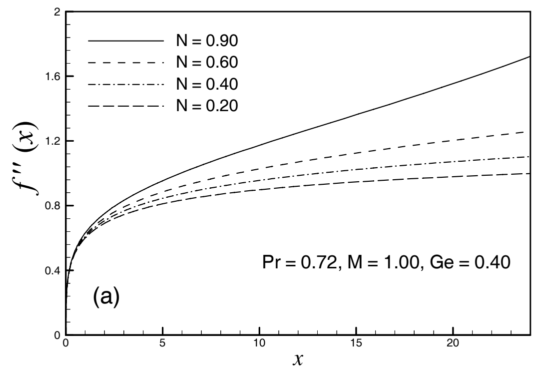

Fig.5(a): Variation of skin friction coefficient $f''(x, 0)$ with dimensionless distance x for different values of viscous dissipation parameter N with Pr = 0.72, M = 1.00 and Ge = 0 .40.

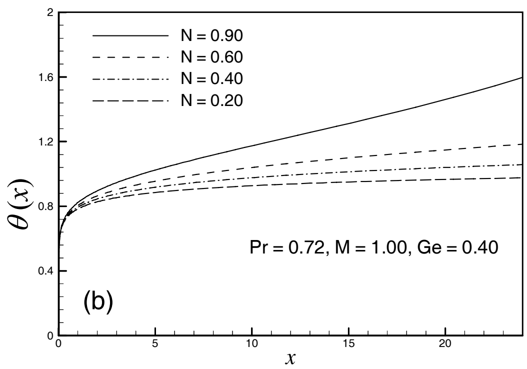

Fig. 5(b): Variation of surface temperature $\theta(x, 0)$ with dimensionless distance x for different values of viscous dissipation parameter N with Pr = 0.72, M = 1.00 and Ge = 0 .40.

6. CONCLUSIONS

The effect of viscous dissipation N and pressure work parameter Ge for Prandtl number Pr (= 0.1, 0.73, 1.0, 1.74) on the magnetohydrodynamic (MHD) natural convection boundary layer flow with heat generation from a vertical flat plate has been studied introducing a new class of transformations. From the present investigation, the following conclusions may be drawn:

- The skin friction coefficient, the surface temperature, the velocity and the temperature profiles increase for increasing value of the viscous dissipation parameter *N*.
- The skin friction coefficient, the surface temperature, the velocity and the temperature profiles increase for increasing values of the pressure work parameter *Ge*.
- The skin friction coefficient, the surface temperature, the velocity profiles decrease with the increasing values of magnetic parameter *M,* but the temperature profiles increase with the increase of magnetic parameter.

It has been observed that the temperature profiles and the velocity profiles over the whole boundary layer decrease with the increase of the Prandtl number *Pr*.

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8. NOMENCLATURE

Symbol	Meaning
b	Plate thickness
C_P	Specific heat at constant pressure
d	$\left(T_b - T_\infty\right)/T_\infty$
f	Dimensionless stream function
g	Acceleration due to gravity
Ge	Pressure work parameter.
h	Dimensionless temperature

L	Reference length, $\nu^{2/3} / g^{1/3}$
l	Length of the plate
N	Viscous dissipation parameter
p	Coupling parameter, $p = \left(k_f / k_s\right)\left(b / L\right)d^{1/4}$
Pr	Prandtl number
M	Magnetic parameter
T	Temperature
T_b	Temperature at outside surface of the plate
T_s	Solid temperature
T_∞	Fluid asymptotic temperature
U, V	Velocity components
u ,v	Dimensionless velocity components
X, Y	Cartesian coordinates
x, y	Dimensionless Cartesian coordinates as indicated in fig. 1
Greek Symbols	
β	Co-efficient of thermal expansion
Ψ	Stream function
κ_f, κ_s	Fluid and solid thermal conductivities
η	Dimensionless similarity variable
ρ	Density of the fluid
μ , ν	Dynamic and kinematic viscosities of the fluid
θ	Dimensionless temperature
σ	Electrical conductivity