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A COMPARATIVE STUDY OF TRACK BUCKLING PARAMETERS OF CONTINUOUS WELDED RAIL

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ABSTRACT

Track buckling has been considered as a major concern for Continuous Welded Rail (CWR) since the introduction of CWR on railway networks. Both static and dynamic loadings on track can cause track buckling. Track can be subjected to static loads such as tensile and compressive loads due to temperature differential, and misalignment due to maintenance operations and cumulative deflections. Dynamic parameters include vehicle loading on track under different speeds, Net Axle Lateral to Vertical (L/V) force ratio and bogie centre spacing. A number of research studies have been carried out to investigate the effect of different parameters of track buckling to establish a safety operating criteria. Railways around the world follow their own specific standards to safeguard against track buckling. However, all the studies differ in environment and operational characteristics. Hence unification of knowledge concerning different track buckling parameters is important. In the present study a comparative analysis of different parameters of track buckling is presented. An experimental design has been proposed to investigate variation of Rail Neutral Temperature (RNT) and dynamic effect on track buckling.

Key word: Continuous Welded Rail, Track Buckling Parameters, Safety Criteria

1. INTRODUCTION

Continuous welded rail (CWR) has replaced the jointed rail for the last four to five decades to reduce maintenance cost and improve ride comfort. High compressive forces, weakened track conditions and vehicle loads are major issue concerning buckling [1]. Compressive force is mainly caused by thermal force generated by the temperature differential between Rail Neutral Temperature (RNT) and actual rail temperature. The thermal force can be obtained from the following formula

$$F = \alpha E A_r (T_N - T_R) \quad (1)$$

Symbols are defined in nomenclature section.

The rail is constrained by sleepers, fasteners and ballast (Fig. 1). A well constrained track prevents the longitudinal and lateral movements of the rail caused by thermal force and train load. Track condition can be weakened by initial misalignment, lack of consolidation of ballast, inappropriate fastening, sleeper types etc. A weak track cannot provide the required resistance to the loads that cause buckling. Vehicle load and speed also play an important role in promoting buckling. Currently empirical formulae are mostly used to analyse safe operating criteria. Prud'homme developed a formula [2] for allowable axle load, which is followed by many of the railways of the world.

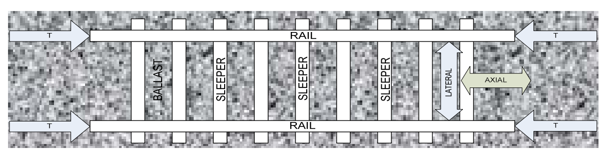

Fig 1. Compressive thermal force on a track [3]

The Prud'homme limit is however only applicable for vehicle approval and since it had been established in 1950s and 1960s, the value is somewhat conservative. The present track structure is stronger than that of that time.

2. TRACK BUCKLING PARAMETERS

Track buckling potential is characterized by upper ($T_{B, MAX}$) and lower ($T_{B, MIN}$) buckling temperatures. It can be noted from Fig 2 that, if the track is subjected to a temperature of $T_{B, MAX}$, the energy required to buckle the track will be zero, ie the track will buckle spontaneously. After buckling, the track reaches a stable state where buckling temperature is $T_{B, MIN.}$ Fig 2 also shows that buckling energy requirement at $T_{B, MIN}$ is much higher than that at $T_{B, MAX.}$

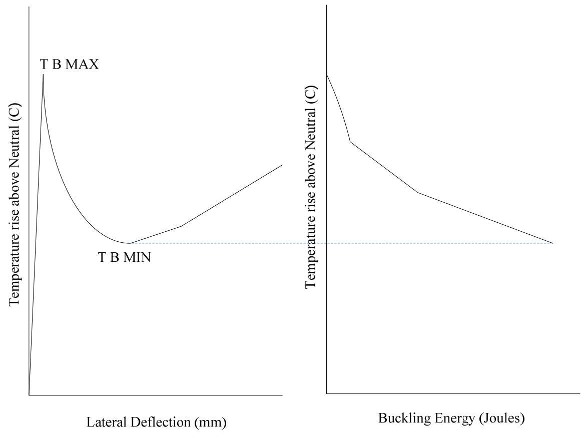

Fig 2. Buckling temperature and energy [4]

2.1 Initial Misalignments

Misalignments present in the track play an important role in triggering track buckling. It is considered that rail is manufactured to an initial straightness, which typically permits maximum defect amplitudes of 0.5 mm over 2 m of rail length [2].

F. Birmann and F. Raab [5] show that straight tracks with smaller lateral imperfections buckled at much higher temperature increases than those tracks with noticeable lateral imperfections. Again, one train can create a small amplitude line defect which can increase the buckle proneness for the next train coming.

Samavedam et al. [6] show that negative deflection occurs under locomotive axles while the central bending wave under the hopper car increases the potential for lateral deflection. This is more severe for larger bogie centre spacings. The upper buckling temperature condition has been found to be more sensitive to misalignment than lower buckling temperature. It has been found that both upper and lower buckling temperatures increase with the increase of half wavelength and decrease with the increase of amplitude. Esveld's [7] investigation shows similar trends considering the fixed relationship between amplitude and half wavelength.

2.2 Lateral Resistance

The track must have the ability to resist lateral forces generated by train passing. Amans and Suavage [6] developed some empirical formulae for determining lateral resistance of a track, which do not include unloaded ballast resistance, sleeper-ballast friction coefficients, curvature and thermal load, effect of track misalignment, dynamic and multiple loads.

Samavedam et al. [6] sorted peak lateral resistance as the primary parameter in studying the effects of lateral resistance on buckling strength. It has also been observed that upper buckling temperature increases more rapidly than lower buckling temperature for higher peak lateral resistance. Track lateral resistance is highly variable as it is affected by many track conditions and maintenance parameters, including ballast section, consolidation, and maintenance.

Esveld [4] shows that a non-linear relationship exists between lateral resistance and vertical loading. Elasto-plastic with softening behaviour is typical for well consolidated ballast, while the bi-linear model is usually used for describing ballast lateral behaviour. A 3D elastic beam element is used for the analyses and lateral resistance is investigated in relation to displacement.

Kish et al. [8] show the effect of lateral resistance on residual deflection considering multiple train passes. It has been found that residual deflection decreases non-linearly (softening) with the increase of peak lateral resistance. Esveld [7] states that vertical vibration caused by passing trains needs to be considered in determining lateral resistance.

2.3 Sleeper Ballast Friction

Sleeper ballast friction is the main contributor to lateral and longitudinal resistance. Friction coefficients between the ballast and sleeper end, sleeper bottom and sleeper side provide the resistance to track movement. Samavedam et al. [6] studied the effect of roughness of sleeper in relation to lateral resistance of the track. Lateral resistance of the track has been expressed by the following equations in [6].

$$F = F_b + F_s + F_e \tag{2}$$

$$F_b = \mu_f Q \tag{3}$$

$$F_{P\,dynamic} = \begin{Bmatrix} \left[F_P - \mu_f Q \right] \text{ for uplift} \\ \left[F_P + \mu_f R_V(x) \right] \text{ otherwise} \end{Bmatrix} \tag{4}$$

Symbols are defined in nomenclature section.

Uplift of track occurs when the sum of vertical deflection forces caused by vehicle loads and the self-weight of the track is less than zero. It has been found that increasing the coefficient of roughness increases both upper and lower buckling temperature. For timber sleepers, track roughness factor increases with time as ballast tends to incise and lock itself into the timber. On the other hand, concrete sleepers become smooth over time and hence roughness factor is reduced. It has also been observed that base lateral resistance contributes most to the lateral resistance.

Kish et al. [9] show an analytical equation of friction coefficient considering vertical load influence over concrete and timber sleepers.

$$\mu = \mu_2 + (\mu_1 - \mu_2)e^{-\beta R} \tag{5}$$

μ_1, μ_2 and β vary for concrete and timber sleepers. While the static value of lateral resistance increases with level of consolidation, value of friction coefficients were found to be decreasing with the number of train passes. Thus the consequence of vehicle passing is to affect the lateral resistance significantly. It has been observed that a reduction of 5% in friction coefficient happens in every 100 train passes.

Kish et al. [9] use Beam on Elastic Foundation (BOEF) to obtain vertical loads on sleepers. The BOEF assumes that the rail is supported by a single layer with uniform stiffness and it experiences a two- dimensional state of stress. Chrismer's [10] analysis shows that the actual distribution of vertical sleeper loads can be better predicted by layer elastic modelling which considers multiple horizontal layers to represent the ballast, subballast and subgrade layers. Vertical force on the sleeper under the axle has been found to be approximately 10% greater using the layer elastic model than in BOEF method.

2.4 Rail Properties

Analysis by Tew et al. [11] shows the result of Miyai and concludes that lateral stability of any track section decreases with the increase of rail size. However they found one anomaly with the 53 kg/m rail which has higher horizontal moment of inertia compared to that of other larger rail sizes. This property helps to resist lateral bending. This effect was also verified by Railways of Australia (1988) by using Association of American Railroad's (AAR) track buckling model.

Samavedam et al. [6] show that track buckling temperature decreases with the increase of rail size more rapidly than the lower buckling temperature. This is because of the fact that increased area contributes to more thermal force and thereby reduces the effect of increasing bending stiffness. Again, though smaller rail section shows better buckling strength that does not allow for lower bending stiffness and hence maximum axle load of smaller rail. Hence an optimization between wheel load and fatigue is necessary to select an appropriate rail size.

2.5 Torsional Resistance

D L Bartlett of British Rail [5] shows that buckling load is proportional to torsional resistance of the rail to sleeper connection. New Zealand Railways carried out tests on torsional resistance of various types of fasteners [12]. However the tests were conducted with new timber sleepers and variation of fastener resistance with sleeper age was not conducted.

Samavedam et al. [6] examined concrete and timber sleepers with different fastener systems (McKay Safelok and Pandrol fastener) showing that the timber sleeper fastening systems are much stiffer than concrete Pandrol and McKay systems. It has been shown that sleeper type or condition has no significant influence on torsional resistance, while fastener type is a significant parameter. Again, lower buckling temperature is more sensitive to torsional resistance increase compared to the negligible change of upper buckling temperature. Hence buckling strength of the track is not consistently affected by this phenomenon. European Rail Research Institute (ERRI) developed a simulation software 'CWERRI' by which Van [13] verifies the result of Samavedam et al. [6].

2.6 Curvature

Samavedam et al. [6] show that upper buckling temperature decreases more rapidly than lower buckling temperature with the increase of curvature. Strong, weak and medium track have been considered for the test. It has been found that progressive buckling can occur at 7 degree or higher curvature for weak track. However the model does not consider effects of non-uniformly distributed ballast resistance along the track, missing sleepers and fasteners, variation of track gauge and differing neutral temperatures between two sites [14].

Esveld [7] study uses CWERRI model to observe curvature effect on track buckling and it has been found that, although the characteristics are similar to the study of [6], the temperature range is different. Different track conditions in USA and Europe are probably behind this difference.

2.7 Longitudinal resistance

Thermal gradient, dynamic braking and rail creep generate longitudinal forces in the rail. Track must provide adequate longitudinal resistance to restrict longitudinal movement. Ballast mass between the sleepers and rail to sleeper connection friction (from toe load grip on the rail foot and the use of rail anchor devices) provide longitudinal resistance. For initial longitudinal displacement up to 6.35 mm (0.25 inch), Samavedam et al. [6] assume the following relationship between longitudinal resistance (f) and displacement (u).

$$f = k_f \, u \qquad (6)$$

Samavedam et al. [6] show that lower buckling temperature increases with the increase of longitudinal resistance, while upper buckling temperature remains almost constant. Esveld [7] investigation shows the same trend of longitudinal resistance characteristic using CWERRI software. If the track has no/little misalignment, longitudinal resistance will have little effect on buckling.

Grissom et al. [15] considered the effects of the torsional stiffness of the rail fasteners, the lateral bending stiffness of the cross- sleeper, and the track gauge to model the lateral response to temperature increases and approximated the axial resistance curve by the following formula

$$f(x) = f_0 \tanh\,[\mu\, u(x)] \qquad (7)$$

2.8 Temperature

Currently empirical relations have been used to determine rail temperature from the ambient temperature. F. Birmann and F. Raab [5] concluded that there is an accumulation of permanent lateral track deformations due to reversal of temperature over a period of time, which increases buckle potential.

Kish et al. [8] show that residual deflection varies linearly with temperature difference, and 20- 40 % increase in the residual deflection can result by a ΔT of 28^0 C. Van [13] states that buckling is not only based on maximum temperature at which buckling starts, but also on minimum temperature after buckling, which can be found with a post-buckling computation.

In all cases rail neutral temperature affects the result most significantly. However a potential difficulty is the variable nature of neutral temperature. RNT tends to shift downward over time due to the effects of traffic, rail movement and track maintenance. Longitudinal stiffness plays an important role in controlling neutral temperature variations. Pandit [5] describes the theoretical formula of neutral temperature using longitudinal strain in rail

$$T_N = T_L + \frac{1}{\alpha}\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right\} \quad (8)$$

On a curve of radius R, if the track is shifted by an amount equal to C, the above formula can be changed to

$$T_N = T_L + \frac{1}{\alpha}\left\{\frac{\partial u}{\partial x} + \frac{C}{R} + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right\} \quad (9)$$

From equation (8) and (9) it is clear that if the displacements u, v and w cause compressive strains (-ve) neutral temperature (T_N) will be lower than the laying temperature (T_L). Rail longitudinal movement can be caused by train action (acceleration and braking) or wheel rolling action. Track lateral shift may occur due to hunting motion of bogie or curving. Non-uniform vertical settlement of ballast caused by vertical wheel load can generate longitudinal strain in the track, which further changes the rail neutral temperature.

2.8 Track Foundation Vertical Stiffness

Samavedam et al. [6] show the effect of track foundation vertical stiffness on track buckling potential, and it has been observed that upper buckling temperature increases with the increase of vertical stiffness. Lower buckling temperature also increases with the increase of stiffness, but this increase is less sensitive to that of upper buckling temperature. However an initial downward slope is visible in both the temperatures due to the complex relationship between the vehicle induced uplift wave and buckling lengths.

3. VEHICLE PARAMETERS

Axle load, Net Axle L/V ratio, bogie or truck centre spacing (TCS) and number of passes contribute to the static track buckling parameters. Samavedam et al. [6] show that upper buckling temperature decreases with the increase in axle load, while lower buckling temperature remains almost constant. It has been found [6] that longer TCS provide higher safety margins against possible explosive buckling.

Kish et al. [8] show that deflections over 5 mm are likely to be unstable after 20 passes. Samavedam et al. [2] observed that track tends to stabilize after several passes of a constant load. But after reaching a critical value of load, residual deflection tends to increase. Hence lateral load calculation is recommended.

Lateral load does not remain constant when a train passes over any misalignment in the track [8]. Spirals on curves, gauge narrowing, switch points, and other discontinuities can also produce large dynamic axle loads spread over relatively small wavelengths.

4. SAFETY CRITERIA

Rail longitudinal force needs to be less than the critical buckling load. From the theory of engineering mechanics, critical load for the buckling of rail acting as a column can be determined using the following formula

$$P_{cr} = \frac{\pi^2 EI}{l^2} \quad (10)$$

However the above formula does not consider lateral resistance of the track. Wen Pei et al. [16] show the following relation considering lateral resistance of the track using the beam column principle..

$$P_{cr} = 2\sqrt{Elk_0\left[1-\left(1+\frac{8}{3\pi}\right)\frac{H}{d}+\left(\frac{7}{16}+\frac{4}{3\pi}\right)\frac{H^2}{d^2}\right]} \quad (11)$$

In order to maintain rail longitudinal force within safe limits, measurement of RNT is necessary. Most of the railways use a fixed value of RNT with one or two degree allowance. RNT varies due to rail longitudinal movement, radial breathing in curves, track vertical settlement and maintenance activities. RNT may even vary between two rails (up to 5^0 C) due to destressing operations.

Samavedam et al. [6] show buckling temperatures are functions of peak lateral resistance and the amplitude of misalignment for a given track curvature and rail size. From this analysis, the allowable temperature rise can be determined based on the criteria setup for different track strengths. Minimum required lateral resistance (MRL) for a given allowable rail temperature has been derived as a function of track curvature for rail size and amplitude of misalignment. This theory does not allow for the variable nature of ballast resistance with track deflection.

Table 1: Lateral limit load and L/V ratio with respect to axle load

	Lateral Limit Load (KN)			Axle L/V Ratio			
Axle Load (KN)	75	150	225	75	100	150	225
Prud'homme limit	35	60	85.0	0.47	0.43	0.40	0.38
85% Prud'homme limit	29.7	51	72.2	0.40	0.37	0.34	0.32
A & S Concrete Sleepered Track	38.5	66	93.5	0.51	0.48	0.44	0.42
A & S Wood Sleepered Track	29.7	51	72.2	0.40	0.37	0.34	0.32
A & S Recently maintained Track	35	60	85.0	0.47	0.43	0.40	0.38
Kish et al [12] 6 deg. Curve with Misalignment	38.6	59.6	80.6	0.52	0.46	0.40	0.36
Kish et al [12] 6 deg. Curve	40.8	61.8	83	0.54	0.48	0.41	0.37
Kish et al [12] Tangent track with Misalignment	41.9	62.2	82.4	0.56	0.49	0.41	0.37
Kish et al [12] Tangent track	44.6	64.8	85.1	0.59	0.51	0.43	0.38
US X 2000 Vehicle	37.5	75	113	0.50	0.50	0.50	0.50
FRA 96/ 03 [2]	68.4	91.1	114	0.91	0.76	0.61	0.51
SNCF for Tamped Track	54.8	85.5	116	0.73	0.65	0.57	0.52
SNCF for Consolidated Track	85.3	133	180	1.14	1.01	0.88	0.8

A & S – Amans and Suavage, FRA- Federal Railroad Administration, SNCF- French National Railways

Railway industries use wheel L/V, axle sum L/V, net axle L/V and truck side L/V for vehicle certification. Net axle L/V for high speed trains is used here for comparing different studies. Table 1 shows the comparison among different studies on determining lateral limit loads in relation to various axle loads. Prud'homme limit considers an indefinite number of axle passes with no allowance for misalignment. It seems impractical, as it requires rigorous maintenance activity. Again, Prud'homme considered the constant lateral force only but high lateral forces over short wavelengths can occur in field conditions [9]. 85% Prud'homme limit considers thermal and curvature effects. The Amans and Suavage investigation specifies different limits for different types of sleeper and track construction. Amans and Suavage developed a semi empirical formula which can take into account misalignment amplitude for dynamic loads. Kish et al. [9] show different limit loads for a 6 degree curve and tangent track with and without misalignment. USDOT's [2] investigation reveals a higher L/V ratio than the current US practice. However, the USDOT [2] study deals with stationary loads only. SNCF developed lateral limit loads for dual block concrete sleepers which are much higher than that for the mono-block sleeper track.

Investigations through all studies except for the US X2000 vehicle presented in Table 1 show that allowable net axle L/V decreases with the increase of axle load. The Kish et al. [9] investigation on L/V ratio over several numbers of passes using the Track Residual Deflection Analysis (TREDA) model determined a safe ratio of 0.37. This analysis assumes constant lateral load and track with no initial line defects. However, TREDA can take into account variable lateral resistance and line defects.

A probabilistic approach has been proposed by Kish [17]. It determines probability of buckling at a given rail temperature and thus railroads can perform trade-offs between different maintenance practices for a given level of buckling probability using 'CWR Safe' software. However 'CWR Safe' does not consider parameters related to grade, train braking and acceleration, longitudinal movement etc.

Zarembski et al [18] propose a site specific track buckling risk analysis methodology which uses the available track database of the railway industry. However this model is site specific and requires experienced personnel for that site to develop an accurate model.

5. EXPERIMENTAL DESIGN

At present the track stability is managed in different railway industries by using some empirical formulas and experiences where dynamic effects are not quantified. However effect of different parameters and dynamic effect are needed to be incorporated in the track stability management tool. The present experimental design has been proposed (Fig 3 and Fig 4) to investigate the parametric effects (static and dynamic) on different track conditions.

A curved track with switches and cutting (Fig 3) has been considered appropriate for the test as it can handle radius, irregular track structure and non-uniform temperature effect on revenue track. Rail Stress Modules (RSM) will be used to monitor data in the revenue track. Details of RSM technology can be found in [19]. Through this investigation, it is aimed to develop a suitable RNT measuring and analysing tool for Australian track conditions in addition to observing the effect of track radius, braking zone, and irregular track structure and temperature distribution on the variation of RNT.

Fig 4 shows the setup to monitor change of axial stress along with lateral and vertical load due to dynamic effect. A trigger is proposed to record data during train passing. The setup in fig 4 will be placed in a braking zone to monitor change of strain due to braking, and thereby variation of RNT can be observed.

Currently Design Neutral Temperature (DNT) is used to keep track stable (neither buckle nor break) throughout the year, based temperature of the region. However dynamic effect and different track constructions are not included in DNT selection. From the present experimental design it is aimed to find a better method of selecting DNT.

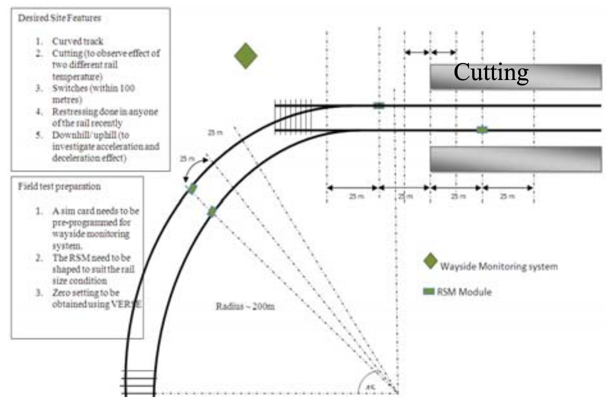

Fig 3. Monitoring of RNT for a long duration

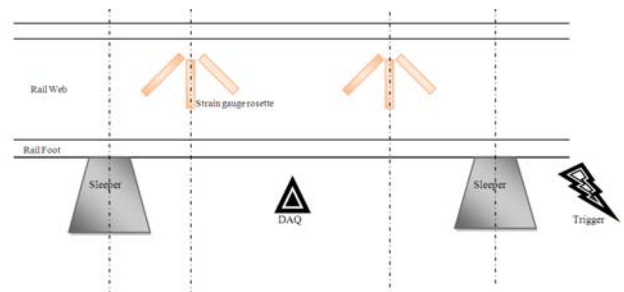

Fig 4. Monitoring dynamic effect on track buckling parameters

6. CONCLUSION

The track is managed by using some empirical relationships among different parameters, most of which were developed 30-40 years ago. It is necessary to incorporate the improved understanding of different parameters on the track stability management tool. A comparative study on track buckling parameters has been carried out. It is necessary to consider different track construction standards and vehicle parameters over a specific track to determine safe operating practice.

An experimental design has been proposed to investigate effect of dynamic parameters on different track conditions. The experimental setup will help to generate an improved track stability analysis with the inclusion of dynamic effect on track buckling potential in different track constructions.

Each track buckling parameter has its own individual effect on track shift. In revenue track, each parameter often depends on other parameters. Correlations among different parameters need to be established to provide better maintenance and inspection guidelines for the railway industry.

7. ACKNOWLEDGEMENT

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R3.112- Track Stability Analysis.

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9. NOMENCLATURE

Symbol	Meaning	Unit
F	Longitudinal Force	N
α	Coefficient of thermal expansion	$^0C/m$
E	Modulus of Elasticity	N/m^2
A_r	Area of rail	m^2
T_N	Neutral Rail Temperature	^0C
T_R	Rail Temperature	^0C
T_L	Rail Laying Temperature	^0C
$T_{B\,MAX}$	Upper buckling Temperature	^0C
$T_{B\,MIN}$	Lower buckling Temperature	^0C
μ_f	Coefficient (index of bottom roughness)	Dimensionless
F_L	Lateral Resistance	N/m
F_b	Base lateral resistance	N/m
F_s	Side lateral resistance	N/m
F_e	End shoulder lateral resistance	N
F_P	Peak Lateral Resistance	N/m
Q	Weight of sleeper	N
R_v	Distributed vertical forces between sleeper and ballast	N
μ_1	Friction coefficient (index) at zero vertical load	Dimensionless
μ_2	Friction coefficient (index) at large vertical load (>89 KN)	Dimensionless
β	In terms of 1/KN	1/KN
u	Rail displacement axial direction	m
v	rail displacement in lateral direction	m
w	rail displacement in vertical direction	m
C	Track shift due to curving	m
$\frac{\partial u}{\partial x},\frac{\partial v}{\partial x},\frac{\partial w}{\partial x}$	Tensile strains in x direction	Dimensionless
R	Radius of the track	m
k_f	Longitudinal stiffness	MPa
f	Longitudinal resistance	N
f_0	Maximum axial resistance	N
μ	Constant for fitting the expression to the non-linear test data	Dimensionless
I	Moment of Inertia	mm^4
k_0	Unit Lateral resistance force before the rail is lifted	N
d	depth of rail clip covered into the sleeper and ballast	mm
H	lift height	mm
l	Column Length	m