

## FREE AND FORCED VIBRATION EXPERIMENTS ON A CROSSBEAM SYSTEM

Anirban Mitra, Prasanta Sahoo and Kashinath Saha

Department of Mechanical Engineering, Jadavpur University, Kolkata, India

### ABSTRACT

In the present paper free and forced vibration experiments are carried out on a crossbeam system, made up of two slender beams in contact with their longitudinal axes perpendicular to each other. The experiments performed are the first of its kind in case of two beam systems. The free vibration experimentation is carried out by exciting the system with the blow of a soft rubber hammer, whereas study of forced vibration behaviour involves excitation by an external harmonic force having a specific magnitude, generated by a function generator and delivered to the system through an electrodynamic oscillator. The results of the free vibration experiments are presented in terms of the natural frequencies of the system and the response of the system to external excitation is presented in amplitude-frequency plane. Also the changes in system response due to change in position of the supporting beam has been observed.

**Keywords:** Crossbeam System, Free Vibration, Forced Vibration.

### 1. INTRODUCTION

A beam is one of the basic structural elements, which is extensively used in many branches of modern civil, mechanical, and aerospace engineering separately or in association with other beams or plates to satisfy different structural requirements like stiffness enhancement, light weight, low cost, material saving etc. Therefore, dynamic response analysis of single beam and beam systems has always been an area of immense interest to researchers.

One of the earlier works in this field was by Srinivasan [1, 2], who employed the Ritz-Galerkin technique to solve the governing nonlinear differential equation of dynamic equilibrium for free and forced vibration of a simply supported beam. Ray and Bert [3] carried out experimental studies to verify the analytical solutions for the nonlinear vibrations of simply supported beam. Yamamoto et al [4] conducted experimental analysis for a beam subjected to harmonic excitation, with a view to validate their analytical results for forced vibrations and subharmonic oscillations. They utilised ball bearings to simulate simply supported boundary conditions and used two magnets to provide the external excitations. The effects of large amplitude vibrations on dynamic strain and fundamental mode shape of a clamped-clamped beam were investigated analytically and experimentally by Bennouna and White [5, 6]. Leissa [7] described an exact method for determining the vibratory displacements of an Euler-Bernoulli beam subjected to distributed excitation forces Chen et al [8] performed experiments on a pre-stretched clamped beam in order to compare their results of finite element model for the nonlinear random

response under acoustic and thermal loads. In this set up also permanent magnets and excitor coils were used to apply external excitation. Experimental investigations on multimode responses in a cantilever beam were undertaken by Tabaddor and Nayfeh [9]. Ribeiro et al [10] experimentally investigated the phenomenon of internal resonance for the case of a clamped beam. Lee and Feng [11] carried out experiments using an electromagnetic shaker to determine the dynamic response of a beam with a frictional joint. Geometrically non-linear vibration of a hinged-hinged beam excited transversely with a harmonic excitation by an electromagnetic exciter was investigated experimentally by Ribeiro and Carneiro [12].

Forced vibration study of beam systems consisting of more than a single beam has also been a popular area of research. Ewing and Mirsafian [13] put forward an analytical model for forced vibration of a system of two Euler-Bernoulli beams joined with a nonlinear rotational torsional spring with linear and cubic stiffness. Oniszcuk [14] applied the classical modal expansion method for analysing undamped forced transverse vibrations of an elastically connected complex simply-supported double-beam system. It is evident from the review of existing literature that experimental investigations of a single beam under different loading and boundary conditions have been carried out extensively. But such experiments for a beam system are quite rare and the system response has not been studied.

In the present paper experiments are carried out on a two beam system, termed as crossbeam, which consists of two perpendicular slender beams in contact so as to form a 'cross', as shown in figure 1(a). The experiments

are devised to validate the sanctity of a theoretical model presented in a separate work. Such crossbeam systems are quite common in civil, mechanical and construction engineering applications. They also can effectively be used separately or with plate elements to construct marine and aerospace structures. At no loading condition there is no interaction between the beams of the system, though they are in surface contact. But when some transverse load is applied on the upper beam (Beam-1), it pushes down on the lower beam (Beam-2), which in turn provides a resistance to deformation of beam-1. It is assumed that external loading in the form of transverse harmonic excitation acts on beam-1 only and the purpose of beam-2 is to give the system a stiffening effect. The contact point between the two beams can be specified by two coordinates,  $x_r$  and  $y_r$ , as shown in figure 1(b). In the present paper, the contact point is assumed at the mid-span of beam-2, i.e.,  $y_r = 0.5L_2$ . However  $x_r$  can vary along the span of the beam and the effect of this change on the natural frequencies and forced vibration response of the system has been studied. The external harmonic excitation is always provided at the point of contact between the two beams.

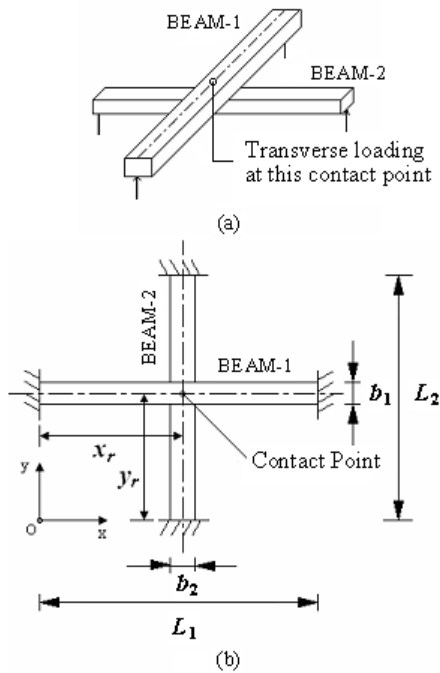
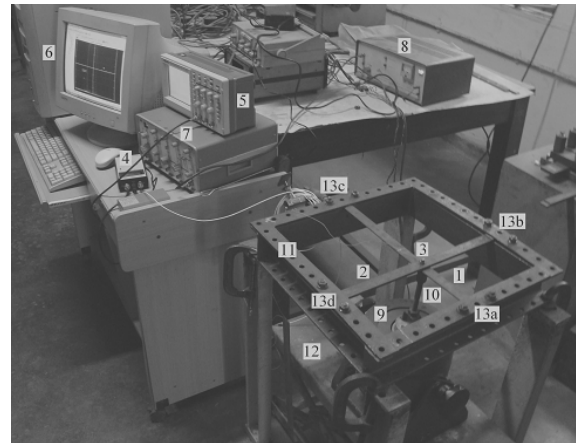


Fig. 1: Crossbeam system with significant dimensions

## 2. EXPERIMENTAL DETAILS

An experimental set up, as shown in figure 2, is prepared to carry out free and forced vibration experiments on the crossbeam system. Figure 3(a) shows the schematic diagram of the free vibration set up with indications of the major components and the schematic diagram of figure 3(b) refers to forced vibration set up. The free vibration experimentation is carried out by exciting the system with the blow of a soft rubber hammer, whereas experimental study of forced vibration behaviour involves excitation by a harmonic force

having a specific magnitude. To achieve this, the set up is slightly modified and a function generator, a power amplifier and an electrodynamic oscillator are introduced. The following section provides a brief description of the set up and test procedure.



- |                                       |   |
|---------------------------------------|---|
| 1. Beam-1                             | 8. Power Amplifier  |
| 2. Beam-2                             | 9. Electrodynamic oscillator                                |
| 3. Accelerometer on the contact point | 10. Rubber padded end of oscillator rod                     |
| 4. Coupler                            | 11. Channel frame   |
| 5. Digital storage oscilloscope       | 12. Mechanical jack   |
| 6. Desktop computer                   | 13(a-d). Fixed ends ensured by bolting tightly to the frame |
| 7. Function generator                 |   |

Fig 2. Photograph of the experimental set up

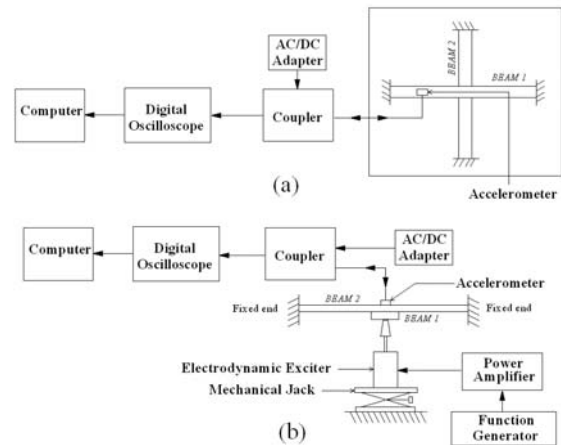


Fig. 3. Schematic diagram of experimental setup for (a) Free vibration and (b) Forced vibration experiments on crossbeam system

### 2.1 Experimental Set-up

Two slender beams are positioned perpendicularly and their ends are bolted firmly to the channel frame so as to simulate clamped boundary condition. The frame along with the crossbeam system is rigidly fixed to a heavy base with C-clamps. The accelerometer (Manufacturer: Kistler Instrument Corporation, Type: 8728A500, acceleration range:  $\pm 500g$  ( $g = 9.80665 \text{ m/s}^2$ ), frequency range: 1 Hz–10 kHz ( $\pm 5\%$ )) is a shear mode piezoelectric sensor and is mounted on the beam at a suitable position using Petro-Wax adhesive material.

The locations of the accelerometer are carefully selected to avoid any nodal points. The mass of the accelerometer (1.6 grams) is significantly less than the mass of the crossbeam system and hence it can be assumed that system response is not significantly affected by the effect of mass loading of the accelerometer. The accelerometer is connected to a coupler (Manufacturer: Kistler Instrument Corporation, Type: 5114, Frequency response: 0.07 Hz – 60 kHz ( $\pm 5\%$ )), which provides the constant current power supply to the impedance converter of the accelerometer and decouples the DC bias voltage from the output signal. The coupler provides the electrical interface between the accelerometer and the display device, which is a digital storage oscilloscope (Manufacturer: Tektronix Inc., Model: TDS 210) with the following specifications: peak detect bandwidth: 50MHz, sample rate range: 50 samples/s–1Gigasamples/s, record length: 2500 samples, and lower frequency limit: 10 Hz. It has the capability to transform a time domain signal into frequency domain through a Fast Fourier Transform (FFT) module. The oscilloscope is connected to a desktop computer through RS-232 communication ports. The data acquired through the oscilloscope are sent to the computer and saved to its hard disk using WSTRO Wavestar software for storage and off-line post-processing.

The external excitation for the forced vibration experimentation is provided through a function generator (Manufacturer: Aplab, Model: 2119, frequency output range: 0.0002 Hz – 20 MHz (in 10 ranges)). It can generate continuous sinusoidal signal of a specific frequency with a precision of  $\pm 3\%$  of the full range and ensures minimal distortion of the waveform. The generated harmonic signal is amplified using a 50W power amplifier (Manufacturer: VEB Metra Meß- und Frequenztechnik, Type: LV 102, frequency response: 3 Hz – 40 kHz) and sent to an electrodynamic oscillator (Manufacturer: VEB RFT Messelektronik, Model: ESE 221, Type: 11077), which provides the excitation to the crossbeam system. A rod with a soft rubber pad at one end is screwed to the top plate of the oscillator. The oscillator is placed on the platform of a mechanical jack and positioned appropriately below the crossbeam system. At the time of forced vibration experiment the oscillator is lifted with the help of the jack to touch the crossbeam system at the desired location. The contact between the lower beam and the padded rod should ensure that contact is maintained throughout the cycle of excitation and yet contribute minimum pre-stressing to the system. In case of free vibration experiment the jack is lowered so that there is no contact between the oscillator rod and the beam and excitation is provided by striking the system with a soft rubber hammer.

## 2.2 Test Procedure

Experiments are carried out to determine the free and forced vibration characteristics of the crossbeam system. The set up is readied by making the electrical connections for computer, oscilloscope, coupler and accelerometer and also for function generator, power amplifier and electrodynamic exciter. The accelerometer is mounted on the test specimen at a predefined location

using adhesive. A two-wire cable between the accelerometer and the coupler is used and the signal and power share the same line. Output from the coupler is connected to one of the channels of the oscilloscope. The oscilloscope is set to 'math' mode and 'auto' trigger mode is kept on. It is then kept ready by pressing the 'RUN' button as the system is hammered to provide disturbance and it captures the signal from the vibrating beams and plots the signal in frequency domain. The data is then transferred to the computer hard disk through the RS-232 ports and saved under an appropriate file name and format. The oscilloscope captures and plots the signal from the vibrating system in frequency-amplitude plane. Using horizontal and vertical cursors the amplitude (in dB) and frequency (in Hz) of the signal are read from the display and the data is tabulated in an excel sheet in the computer. The free vibration experiment is carried out first and for the subsequent forced vibration test the mechanical jack is lifted to put the oscillator in contact with the crossbeam system. A sinusoidal signal of a specified frequency, lower than the natural frequency of the system, is used as the initial external excitation. Now the excitation frequency is increased gradually by turning the dial of the function generator and the procedure of noting frequency and amplitude is repeated. The excitation frequency is increased beyond resonance till the amplitude of the signal falls to a sufficiently low level. The whole experimental procedure is repeated for different positions of the supporting beam (beam-2).

## 3. RESULTS AND DISCUSSION

The experiments carried out on the crossbeam system have the objective of determining the natural frequency of the system and to observe the system response due to an external harmonic excitation. Also the changes in system response due to change in position of the supporting beam (beam-2) has been observed.

The specimens used to construct the crossbeam system are two slender beams of rectangular cross-section. Figure 4 shows the schematic diagram of one such beam with representative dimensions. Different cross-sectional dimensions used for the present experiments are mentioned in Table 1. The material of the beams is mild steel.

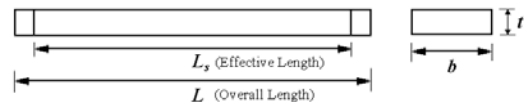


Fig 4. Schematic diagram of a slender beam

Table 1. Beam dimensions in mm

Specimen No.	$L$	$L_s$	$b$	$T$
A1	500	400	20.513	2.753
A2	500	400	18.367	5.373
B1	500	400	24.680	3.000
B2	500	400	24.720	5.000

The results of the free vibration experiments are presented in terms of the natural frequencies of the system, whereas the response of the system to external

excitation is presented in amplitude-frequency plane. The ordinate represents maximum amplitude of vibration and abscissa represents the frequency in Hertz. The amplitude data noted from the oscilloscope is in dB, which corresponds to the ratio of output and input voltages to the oscilloscope, as given by the following relation.

$$dB = 20 \log_{10} (V_o/V_i)$$

To validate the experimental procedure a free vibration experiment is carried out on a single beam and the results are compared with commercial finite element package ANSYS (version 11.0) and also with analytical results. The comparison of the natural frequencies from the three different methods is compared in Table 2. From the table it can be seen that the analytical and simulation results are in good agreement, but the experimental results differ from them. The difference can be attributed to insufficiency in replicating stretching boundary conditions of the system.

Table 2. Comparison of natural frequencies of a single beam obtained through experimental, analytical and simulation (ANSYS) methods.

Specimen No.	Free Vibration Frequency (Hz)			Mode
	Experimental	Analytical	ANSYS	
A1	75.0204	91.4552	91.482	1 <sup>st</sup>
	217.793	251.9252	252.160	2 <sup>nd</sup>
	435.196	493.3565	494.280	3 <sup>rd</sup>
A2	136.672	178.4944	178.52	1 <sup>st</sup>
	409.237	491.6796	491.98	2 <sup>nd</sup>
	876.492	962.8785	964.09	3 <sup>rd</sup>

Table 3 presents the natural frequencies of the crossbeam system with different combinations of the constituent beams. The effect of shift in position of the supporting beam (beam-2) is also studied and the results in terms of natural frequencies are shown in Table 4. It is evident from the results that as beam-2 shifts towards one of the ends the natural frequencies of the system increase. It can be concluded that the off-set position of beam-2 increases the overall stiffness of the system and hence an increase in the natural frequencies is observed.

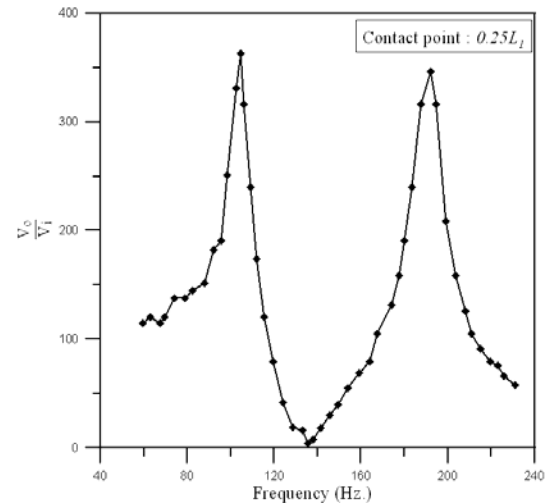
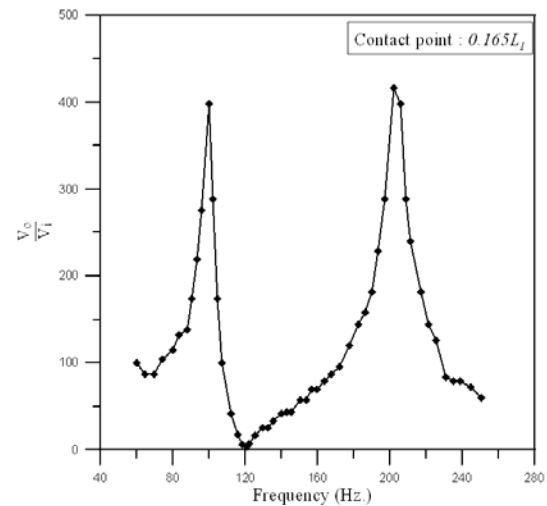
Table 3. Free vibration frequency (1<sup>st</sup> Mode) of a crossbeam system with different cross-sectional dimensions

Sl. No.	Specimen Arrangement	Free Vibration Frequency (Hz) (1 <sup>st</sup> Mode)
1	A1-A1	77.8998
2	A2-A2	139.917
3	A1-A2	123.693
4	A2-A1	126.938

Table 4. Free vibration frequency (1<sup>st</sup> Mode) of a crossbeam system for change in position of beam-2

Sl. No.	Specimen Arrangement	Beam-2 position	Free Vibration Frequency (Hz) (1 <sup>st</sup> mode)
1	A1-A1	0.5	77.8998
2		0.35	84.7548
3		0.125	87.9997

The system behavior under forced vibration is also studied for different positions of beam-2 along the span of beam-1. Results are presented in figures 5 and 6 as frequency-amplitude plots, where frequency is in Hertz and amplitude is proportional to the output voltage of the oscilloscope. Figure 5 presents the plots for crossbeam system with specimen arrangement B1-B1, whereas Figure 6 shows the same for arrangement B1-B2. Both the figures include frequency-amplitude plots of the system for five different positions of beam-2 as mentioned in the figure legends.



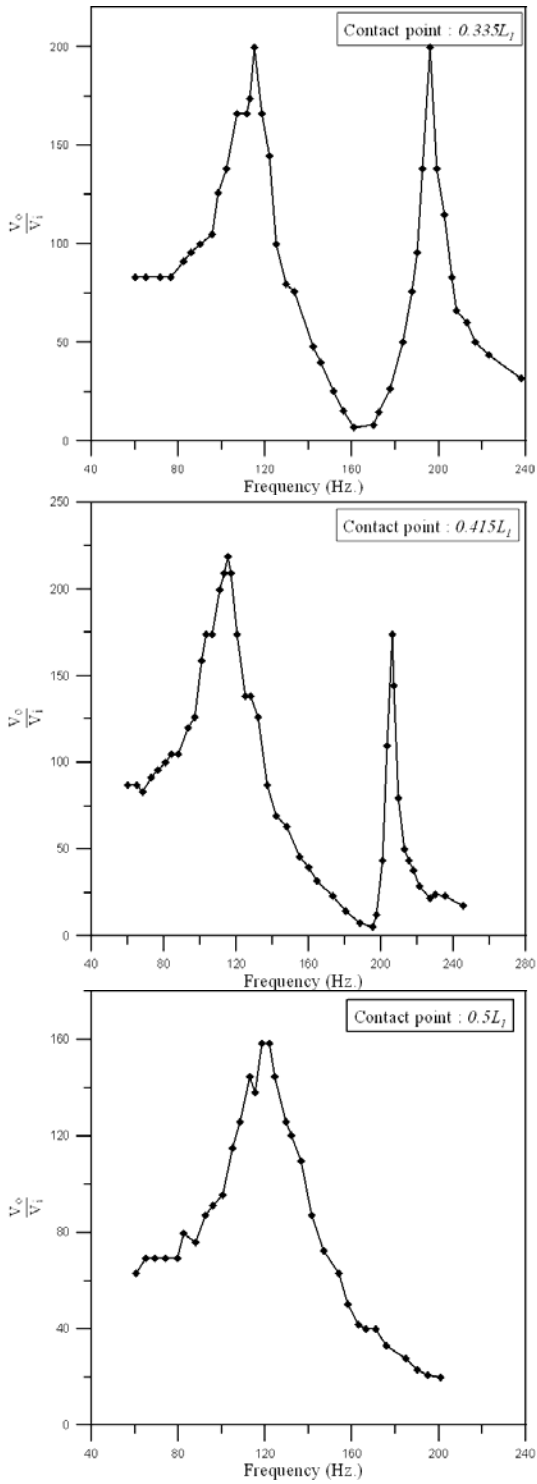
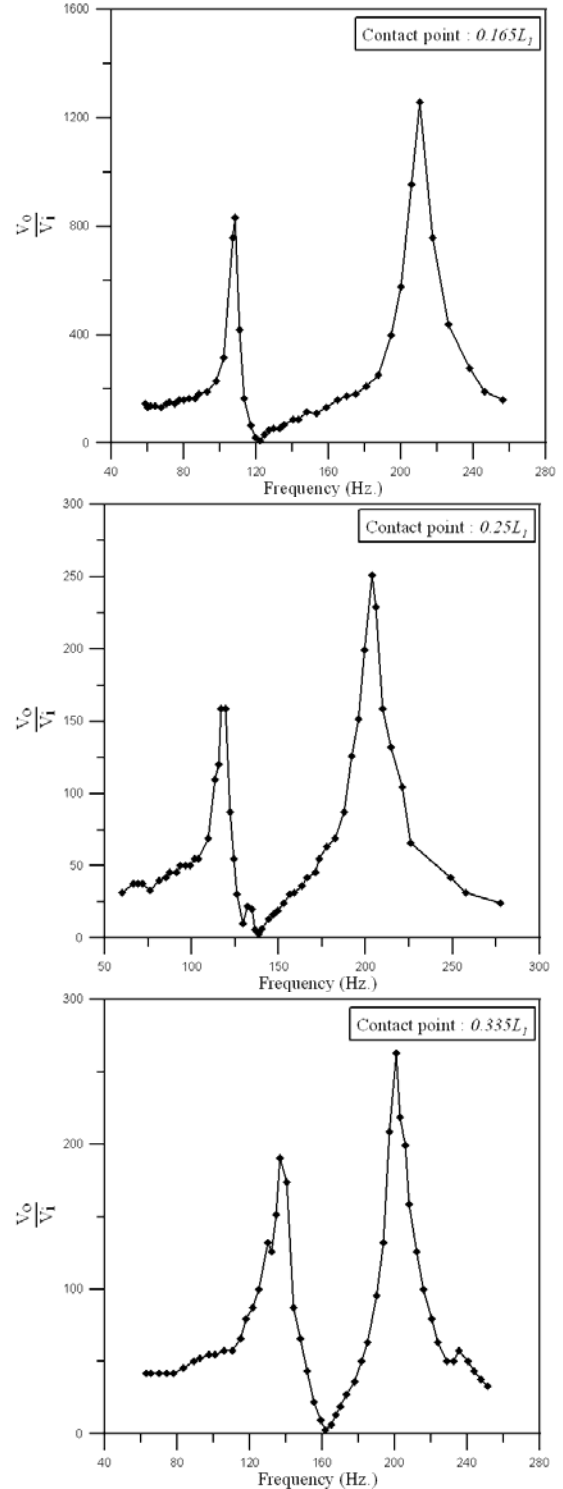


Fig 5. Frequency amplitude plots for forced vibration experiment for crossbeam system with specimen arrangement B1-B1

All the corresponding plots in figures 5 and 6 exhibit similar characteristics. However the particular case of contact point at the middle ( $x_r = 0.5L_j$ ) shows marked difference. The plots show that for a symmetric configuration of the system only one amplitude peak is obtained, whereas for an off-set position of the

supporting beam (beam-2) two peaks are present. It is seen from the plots that with increase in excitation frequency the amplitude response increases and attains a peak value. This corresponds to the resonant frequency. After this particular value, with further increase in excitation frequency the amplitude drops sharply. However, if frequency value is still increased another amplitude peak is obtained. It is also apparent from the plots that the distance between the two peaks increases as beam-2 shifts outward.



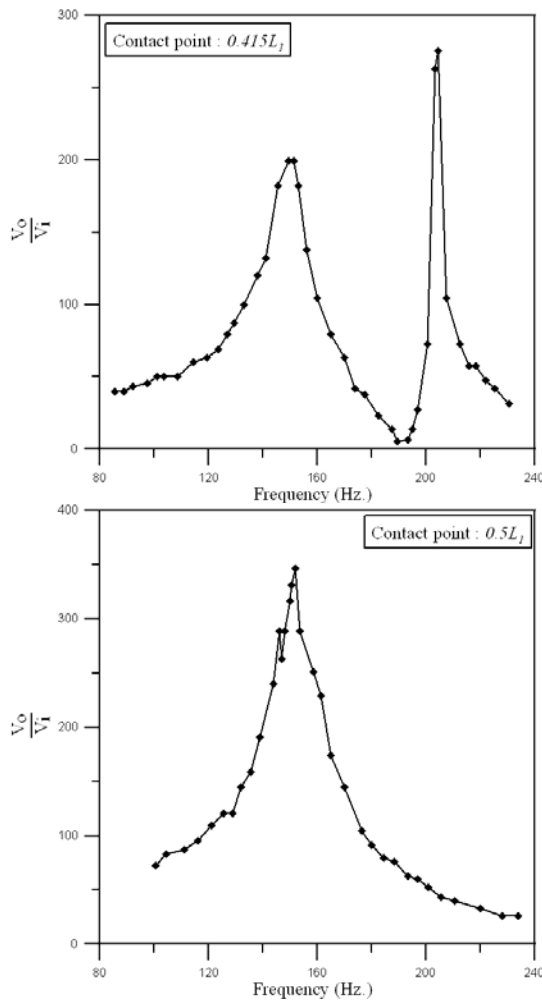


Fig 6. Frequency amplitude plots for forced vibration experiment for crossbeam system with specimen arrangement B1-B2

#### 4. CONCLUSION

The present paper undertakes an experimental free and forced vibration analysis of a two beam system under harmonic excitation. The experimental study undertaken is the first of its kind for a two beam system. A set up simulating clamped boundary conditions is developed and experimentations are carried out for different dimensions of the cross-section of the beams. The effect for shift in position of the lower beam is also observed. The results for free vibration case are presented in terms of natural frequencies of the system. Validation of the experimental process is carried out by comparing the results obtained from free vibration test on a single beam with analytical and simulation results. The results for forced vibration experiments are presented in terms of frequency-amplitude plots. The results indicate a shift in the resonant frequency which calls for further analytical study.

#### 5. ACKNOWLEDGEMENT

The first author acknowledges the research support received from AICTE, India, vide File No.: 1-10/RID/NDF/PG/(17)2008-09 Dated : 13.03.2009.

#### 6. REFERENCES

1. Srinivasan, A. V., 1965, "Large Amplitude Free Oscillations of Beams and Plates", American Institute of Aeronautics and Astronautics Journal, 3:1951-1953.
2. Srinivasan, A. V., 1966, "Nonlinear Vibrations of Beams and Plates", International Journal of Nonlinear Mechanics, 1:179-191.
3. Ray, J. D. and Bert, C. W., 1969, "Nonlinear Vibrations of a Beam with Pinned Ends", Journal of Engineering for Industry, Transactions of the ASME, 91: 977-1004.
4. Yamamoto, T., Yasuda, K. and Aoki, K., 1981, "Subharmonic oscillations of a slender beam", Bulletin of the Japan Society of Mechanical Engineers, 24(192): 1011-1020.
5. Bennouna, M. M. and White, R. G., 1984, "The Effects of Large Vibration Amplitudes on the Dynamic Strain Response of a Clamped-Clamped Beam with consideration on Fatigue Life", Journal of Sound and Vibration, 96: 281-308.
6. Bennouna, M. M. and White, R. G., 1984, "The Effects of Large Vibration Amplitudes on the Fundamental Mode Shape of a Uniform Clamped-Clamped Beam", Journal of Sound and Vibration, 96(3): 309-331.
7. Leissa, A. W., 1989, "Closed Form Exact Solutions for the Steady State Vibrations of Continuous Systems subjected to Distributed Exciting Forces", Journal of Sound and Vibration, 134: 435-453.
8. Chen, R. R., Mei, C. and Wolfe, H., 1996, "Comparison of Finite Element Non-linear Beam Random Response with Experimental Results", Journal of Sound and Vibration, 195(5): 719-737.
9. Tabaddor, M. and Nayfeh, A.H., 1997, "An Experimental Investigation of Multimode Responses in a Cantilever Beam", Journal of Vibration and Acoustics, Transactions of the American Society of Mechanical Engineers, 119: 532-538.
10. Ribeiro, P., Alves, L. and Marinho, J., 2001, "Experimental Investigation on the Occurrence of Internal Resonances in a Clamped-Clamped Beam", International Journal of Acoustics and Vibration, 6: 169-173.
11. Lee, Y. and Feng, Z. C., 2004, "Dynamic Responses to Sinusoidal Excitations of Beams with Frictional Joints", Nonlinear Science and Numerical Simulation, 9: 571-581.
12. Ribeiro, P. and Carneiro, R., 2004, "Experimental Detection of Modal Interaction in the Non-linear Vibration of a Hinged-Hinged Beam", Journal of Sound and Vibration, 277: 943-954.
13. Ewing, M. S. and Mirsafian, S., 1996, "Forced Vibration of Two Beams Joined with a Non-linear Rotational Joint: Clamped and Simply Supported End Conditions", Journal of Sound and Vibration, 193(2): 483-496.
14. Oniszczuk, Z., 2003, "Forced Transverse Vibrations of an Elastically Connected Complex Simply Supported Double-Beam System", Journal of Sound and Vibration, 264: 273-286.