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THEORETICAL AND NUMERICAL OPTIMIZATION OF AUTOFRETTAGE IN STRAIN HARDENED THICK-WALLED CYLINDERS.

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ABSTRACT

The optimum autofrettage pressure and the optimum radius of elastic-plastic junction of strain-hardened thick walled cylinders have been studied theoretically by finite element modeling. Equivalent von-mises stress is used as yield criterion. Influence of autofrettage on stress distribution and load-bearing capacity of a cylinder is studied and it has been observed that optimum autofrettage pressure is not a constant value rather depends on the working pressure. It has also seen that to reduce the flow stress within the wall of the cylinder, the autofrettage pressure must be greater than the working pressure. For a particular working pressure, the effect of the ratio of outside to inside radius (b/a=k) on the optimum autofrettage pressure is also observed. Maximum von-mises stress developed at different autofrettage pressure is compared for different material models. It has also seen that the number of autofrettage stages has no effect on the maximum pressure carrying capacity of the cylinder.

Keywords: Autofrettage, thick wall cylinder, elastic plastic junction.

1. INTRODUCTION

The ever increasing industrial demand for axisymmetric pressure vessels which have applications in chemical, nuclear, fluid transmitting plant, power plant and military equipment, have concentrated the attention of designers on this particular area of engineering. Therefore, the prevention of pressure vessel failure to enhance safety and reliability has received considerable attention. On the other hand, the increasingly scarcity of materials and higher costs have led researchers not to confine themselves to the customary elastic regime but attracted their attention to the elastic-plastic along with optimization approach which offer more efficient use of materials. So the main concern is to increase the safety factor without increasing its weight. This purpose can be fulfilled successfully by means of autofrettage process. In this technique, the cylinder is subjected to an internal pressure so that its wall becomes partially plastic. The pressure is then released and the resulting residual stress lead to a decrease in the maximum von mises stress in the working loading stage. That means the increase in the pressure capacity of the cylinder in the next loading stage [1 & 2]. A key problem in the analysis of autofrettage process is to determine the optimum autofrettage pressure and corresponding radius of elasto-plastic boundary where the maximum equivalent von mises stress in the cylinder becomes minimum. The analysis of residual stresses and deformation in an autofrettaged thick-walled cylinder has been given by Chen [3] and Franklin and Morrison [4]. Harvey's report [6] gave only a concept about autofrettage but detail result was missing. Brownell and Young [7], and Yu [8] proposed a repeated trial calculation method to determine the optimum radius of elastic plastic junction which was a bit too tedious and inaccurate; moreover this method is based on the first strength theory which is in agreement with brittle materials. But pressure vessels are generally made from ductile materials [9 & 10] which are in excellent agreement with the third or the fourth strength theory [11 & 13]. The graphical method presented by Kong [12] was also a bit too tedious and inaccurate. Based on the third and the fourth strength theory, Zhu and Yang [14] presented an analytic equation for optimum radius of elastic-plastic juncture, opt r in autofrettage technology. Ghomi & Majzoobi [15] proposed set of equations that used for determining optimum radius of elastic plastic junction. In the present work, Zhu & Yang's equations based on fourth strength theory are employed to predict the optimum autofrettage radius. Numerical simulation is done by using ANSYS for calculating optimum autofrettage pressure.

2. ANALYTICAL APPROACH

Bi-Linear elasto-plastic behavior has been considered in this work.

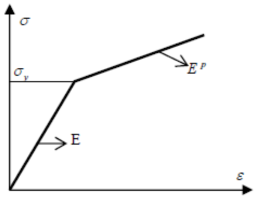


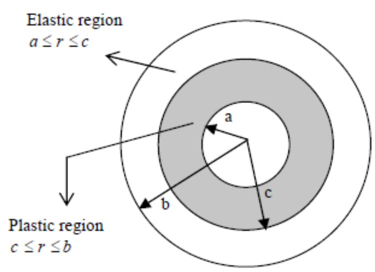
Fig 1. Bi-linear stress strain curve

The model, shown in figure 1 is described as follows:

$$ó = ó_y + E^p å \quad (1)$$

In which ó is the effective stress, $ó_y$ is the initial yield stress, E^p is the slope of the strain hardening segment of stress strain curve , å is the effective strain.

Cylinder is subjected to autofrettage pressure and become partially plastic.



Ghomi & Majzoobi [16] proposed set of equations for determining radial and hoop stresses at different location along the cylinder wall in autofrettaged cylinder.

For the elastic-plastic material with linear strain hardening: In plastic region, a= r = C, for an internal pressure P :

$$\sigma_r = \frac{\sigma_y}{2}\left[\left(1+\frac{c^2}{b^2}\right)+in\frac{c^2}{r^2}+\left(1-\nu^2\right)^{Ep}E\frac{c^2}{r^2}-\frac{c^2}{b^2}\right]\Big/\left[1+\left(1-\nu^2\right)^{E^p}E\right] \quad (2)$$

$$\sigma_0 = \frac{\sigma_y}{2}\left[\left(1+\frac{c^2}{b^2}\right)-in\frac{c^2}{r^2}+\left(1-\nu^2\right)^{Ep}E\left(\frac{c^2}{r^2}-\frac{c^2}{b^2}\right)\right]\Big/\left[1+\left(1-\nu^2\right)^{E^p}E\right]$$

(3)

When the cylinder is pressurized to the autofrettage pressure and the pressure is removed, the residual stress distribution across the wall of the cylinder can be expressed as follow [16]:

For elasto-plastic material:

Residual stresses in plastic region a = r = c :

$$\sigma_{rr} = \frac{\sigma_y\left(\frac{b^2}{r^2}-1\right)\left[1-\frac{c^2}{b^2}+2in\frac{c}{a}+\left(1-\nu^2\right)\frac{E^p}{E}\left(\frac{c^2}{a^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-\nu^2\right)\frac{E}{E}\right]} - \frac{\sigma_y\left[1-\frac{c^2}{b^2}+2in\frac{c}{r}+\left(1-\nu^2\right)\frac{E^p}{E}\left(\frac{c^2}{r^2}-\frac{c^2}{b^2}\right)\right]}{2\left[1+\left(1-\nu^2\right)\frac{E^p}{E}\right]} \quad (4)$$

$$\sigma_{èr} = \frac{\sigma_y\left(\frac{b^2}{r^2}+1\right)\left[1-\frac{c^2}{b^2}+2In\frac{c}{a}+(1-\nu^2)\frac{E^p}{E}\left(\frac{c^2}{a^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-\nu^2\right)\frac{E^p}{E}\right]} + \frac{\sigma_y\left[1+\frac{c^2}{b^2}-2in\frac{c}{r}+\left(1-\nu^2\right)\frac{E^p}{E}\left(\frac{c^2}{r^2}+\frac{c^2}{b^2}\right)\right]}{2\left[1+\left(1-\nu^2\right)\frac{E^p}{E}\right]} \quad (5)$$

Residual stresses in elastic region c = r = b:

$$\sigma_{èr} = \frac{\sigma_y\left(\frac{b^2}{r^2}-1\right)\left[1-\frac{c^2}{b^2}+2In\frac{c}{a}+(1-\nu^2)\frac{E^p}{E}\left(\frac{c^2}{a^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-\nu^2\right)\frac{E^p}{E}\right]} - \frac{\sigma_y c^2\left(b^2-r^2\right)}{2b^2r^2} \quad (7)$$

$$\sigma_{èr} = \frac{-\sigma_y\left(\frac{b^2}{r^2}+1\right)\left[1-\frac{c^2}{b^2}+2In\frac{c}{a}+(1-\nu^2)\frac{E^p}{E}\left(\frac{c^2}{a^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-\nu^2\right)\frac{E^p}{E}\right]} + \frac{\sigma_y c^2\left(b^2-r^2\right)}{2b^2r^2} \quad (8)$$

If the cylinder is loaded again by the internal working pressure, by superposing the residual stresses due to autofrettage and the working pressure, the final stress distribution in the wall of the cylinder will becomes:

For elastic-plastic material:

Overall stresses in plastic region a = r = c:

$$\sigma_{rf} = \frac{\sigma_y\left(\frac{b^2}{r^2}-1\right)\left[1-\frac{c^2}{b^2}+2\ln\frac{c}{a}+\left(1-\nu^2\right)\frac{E^p}{E}\left(\frac{c^2}{a^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-\nu^2\right)\frac{E^p}{E}\right]} - \frac{\sigma_y\left[1-\frac{c^2}{b^2}+2\ln\frac{c}{r}+\left(1-\nu^2\right)\frac{E^p}{E}\left(\frac{c^2}{r^2}+\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-\nu^2\right)\frac{E^p}{E}\right]} - \frac{p\left(\frac{b^2}{r^2}-1\right)}{k^2-1} \quad (9)$$

$$\sigma_{\grave{e}f} = \frac{-\sigma_y\left(\frac{b^2}{r^2}+1\right)\left[1-\frac{c^2}{b^2}+2\ln\frac{c}{a}+\left(1-v^2\right)\frac{E^p}{E}\left(\frac{c^2}{a^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-v^2\right)\frac{E^p}{E}\right]}$$

$$+\frac{\sigma_y\left[1+\frac{c^2}{b^2}-2in\frac{c}{r}+\left(1-v^2\right)\frac{E^p}{E}\left(\frac{c^2}{r^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-v^2\right)\frac{E^p}{E}\right]}+\frac{p\left(\frac{b^2}{r^2}+1\right)}{k^2-1}$$

(10)

Overall stresses in elastic region c = r = b :

$$\sigma_{rf} = \frac{\sigma_y\left(\frac{b^2}{r^2}-1\right)\left[1-\frac{c^2}{b^2}+2in\frac{c}{a}+(1-v^2)\frac{E^p}{E}\left(\frac{c^2}{a^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-v^2\right)\frac{E^p}{E}\right]}$$

$$-\frac{\sigma_y c^2\left(b^2-r^2\right)}{2b^2r^2}-\frac{p\left(b^2-1\right)}{k^2-1} \qquad (11)$$

$$\sigma_{\grave{e}f} = \frac{-\sigma_y\left(\frac{b^2}{r^2}+1\right)\left[1-\frac{c^2}{b^2}+2in\frac{c}{a}+(1-v^2)\frac{E^p}{E}\left(\frac{c^2}{a^2}-\frac{c^2}{b^2}\right)\right]}{2\left(k^2-1\right)\left[1+\left(1-v^2\right)\frac{E^p}{E}\right]}+$$

$$\frac{\sigma_y c^2\left(b^2+r^2\right)}{2b^2r^2}+\frac{p\left(b^2-1\right)}{k^2-1} \qquad (12)$$

According to tresca yield criterion, the equivalent stress σ_{eq} can be defined as: $\sigma_{eq} = \sigma = \sigma_{\grave{e}}-\sigma_r$

$$\sigma_{eq} = \sigma = \sigma_{\grave{e}}-\sigma_r \qquad (13)$$

If the cylinder is intended to remain elastic throughout the loading process of the cylinder, then the equivalent stress should not exceed the yield stress of the material. i.e.:

$$\sigma_{eq} = \sigma_{\grave{e}}-\sigma_r = \sigma_y \qquad (14)$$

A simple case study:

Let's consider an Aluminum cylinder where internal radius, a= 0.01 m and external radius, b = 0.02m. Material properties are summarized in table 1.

Table 1: Material properties

	óy (MPa)	E(GPa)	E^p (GPa)	í
Al	90	72	1.75	0.33

2.1 RESIDUAL STRESS PATTERN

This cylinder is subjected to an internal pressure so that its wall becomes partially plastic and the pressure is then released. Ghomi & Majzoobi [16] proposed set of equations for determining radial and hoop stresses at different location along the cylinder wall in autofrettaged cylinder. By using the equations the resulting residual stress pattern is shown in figure 2:

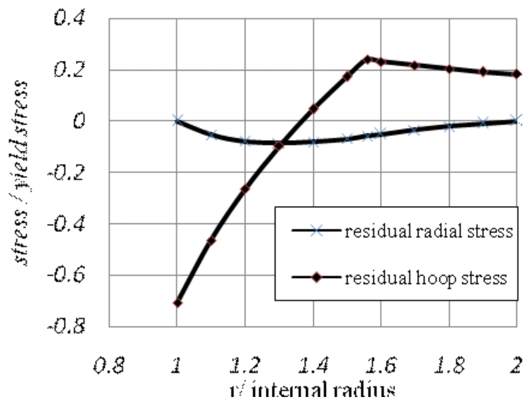


Fig 2. Residual Stress Distribution

From the figure, it is observed that residual compressive hoop stress occurs in near-bore region while residual tensile hoop stress occurs at outer portion. The resulting residual compressive hoop stress leads to a decrease in the maximum value of the von mises stress in the next loading stage.

2.2 Comparison Of Stresses With And Without Autofrettage

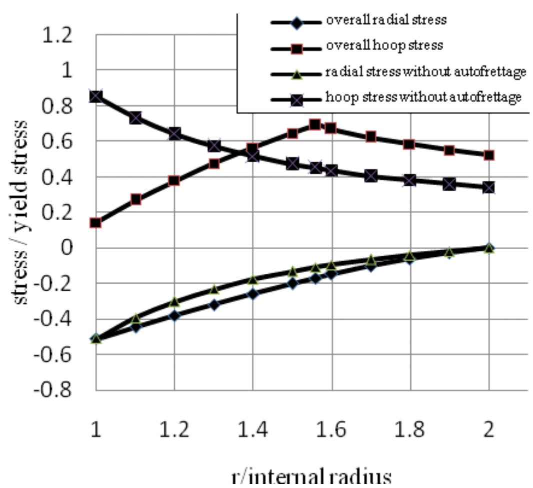


Fig 3. Comparison of Stresses with autofrettage and without Autofrettage.

By using Lame's equation for thick wall cylinder the stress pattern is obtained. If the same cylinder undergoes autofrettage process then the overall stress pattern will change, which is shown on the graph. From figure 3, following points are observed:

Because of compressive hoop stress at inner bore, the resultant hoop stress becomes significantly lower in the autofrettaged cylinder than the original hoop stress developed without autofrettage process at the same cylinder. Radial stress doesn't vary significantly after autofrettage process. The cylinder which undergoes autofrettage process has maximum stress occurring at the point of elasto-plastic junction.

2.3 OPTIMUM ELASTIC PLASTIC RADIUS

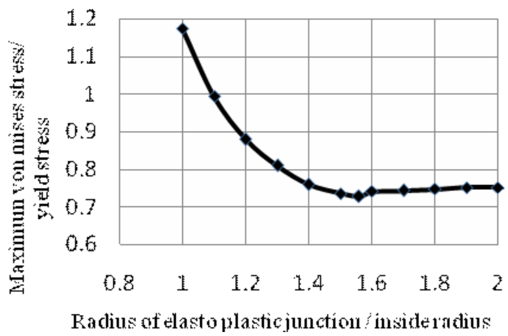


Fig 4. Maximum Von Mises Stress At Different Radius of Elasto Plastic Junction

When the ratio of radius of elasto plastic junction to inside radius equals to one (there is no autofrettage), then the maximum von mises stress exceeds yield stress, thus the material fails. Maximum von mises stress started to decrease as the radius of elastic plastic junction increases. After attaining a certain value of elastic plastic junction, maximum von mises stress started to increase. The point at which maximum von mises stress is minimum, is the optimum radius of elasto-platic junction.

2.4: Zhu & Yang Model For Optimum Elastic Plastic Radius

Zhu & Yang has developed an equation to determine *opt r* which we can calculate just using a pocket calculator.

(a) based on third strength theory,

$r_{opt} = a \exp (p_w / \sigma_y)$ (2)

(b) based on fourth strength theory,

$r_{opt} = a \exp (v3p_w / 2\sigma_y)$ (3)

Ghomi & Majzoobi deduced r_{opt} by using MATLAB.
For determining optimum radius of elastic plastic junction "Ghomi & Majzoobi's model" and "Zhu &Yang's model" are compared. It has been observed that these values vary between 5-7% only.

Sample calculation:

In this case study a=0.01m, b= 0.02m, working pressure p_w = 46 MPa.
From Zhu & Yang's model,

Based on third strength theory r_{opt} = 0.01667 m.

Based on fourth strength theory r_{opt} = 0.0156 m.

From Ghomi & Majzoobi's model (fig. 4), it is observed that r_{opt} is occurring in between 0.015 to 0.016 m. Indeed there is no significant variation between these two models.

Zhu Yang model based on fourth strength theory is considered for calculating *opt r* that simplifies the calculation.

3. NUMERICAL RESULTS

Single cylinder with the dimensions; a=0.1 m, b=0.2 m and an elastic plastic material's model with σ_y = 800 MPa; Modulus of elasticity E = 207 GPa**;** Slope of the strain hardening segment E^p = 4.5 GPa; í = 0. 29; were used for numerical modeling. The two pressure limits Py1 and Py2 can be computed as follows [1 & 17]:

$$Py1 = ó_y (1-1/k^2)/v3$$

$$=347 \text{ MPa}$$

$$Py2 = ó_y \ln(k)$$

$$=555 \text{ MPa}$$

If the autofrettage pressure is lower than 347MPa, then there will be no autofrettage effect. If the pressure is higher than 555MPa, then there will be converse effect. That means, instead of increasing, pressure capacity of the cylinder will decrease.
On this paper, effect of following factors is checked in autofrettage process. The considered factors are :
1. Working pressure; 2. Value of k (b/a); 3. Material model (elastic perfectly plastic and elastic plastic with different slope of strain hardening segment); 4. Autofrettage Stages.

3.1 Working Pressure

The cylinders were subjected to autofrettage pressures ranging from 350 MPa to 650 MPa. After removing the autofrettage pressure (AP), the cylinders were subjected to the working pressures(WP) of 100, 200, 300 and 400 MPa. From the numerical simulations, the curve of the von-Mises stress distribution was obtained for each autofrettage and working pressure (WP). From the curve, the value and the position of maximum von-Mises stress (MVS) were extracted. This stress and its position were then plotted versus autofrettage pressure for each working pressure. The results are shown in figure 5.

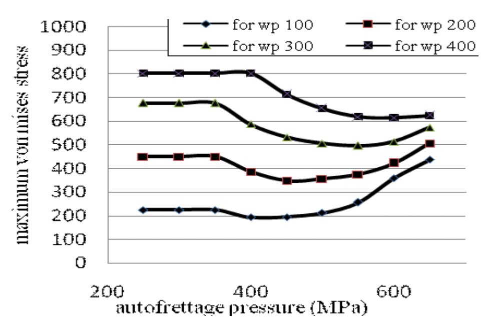


Fig 5. Variation of MVS versus autofrettage pressure at four working pressures.

It is observed that for each working pressure, the MVS remains constant up to an autofrettage pressure which is nearly equal to Py1 . The curve then begins to decline to a certain point thereafter, begins to rise or remains constant. It can be seen that for all working pressures, the rising portion of the curves end at a point which is nearly equal to Py2.

From the numerical results, it can be concluded that: (i) The MVS depends on the working pressure and for any WP, the best AP lies between Py1 and Py2 ; (ii) For autofrettage pressures lower than Py1 and higher than Py2 the MVS remains unchanged; (iii) The position of MVS moves towards the outer radius as AP increases, (iv) For working pressure less than 300MPa, the autofrettage effect starts when the autofrettage pressure attain a value of 350 MPa. For working pressure 400MPa it is also observed that autofrettage pressure should be more than 400MPa to initiate the autofrettage effect. This means autofrettage pressure must be greater than the working pressure.

Table 2: Effect of working pressure at maximum von mises stress

WP (MPa)	MVS Without autofrettage (MPa)	MVS With autofrettage (MPa)	%Reduction Of MVS
100	225	193	14.22
200	450	348	22.67
300	676	496	26.62
400	840	615	26.78

For constant value of K, percent reduction of MVS is observed for different working pressures. From the table, it is observed that percent reduction of MVS is higher at higher working pressure. This means the autofrettage effect is more beneficial at higher working pressure.

3.2 Value Of K (B/A)

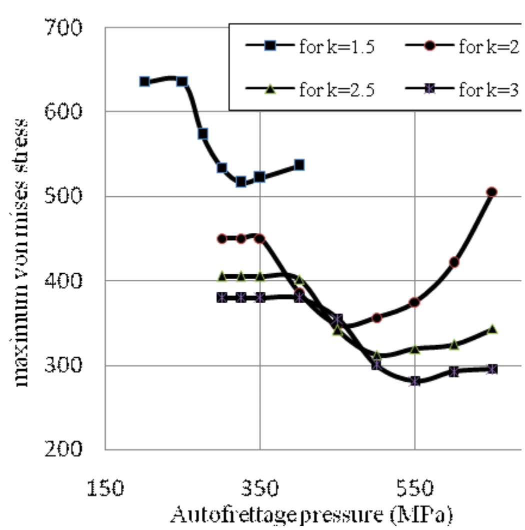


Fig 6. Effect of K (b/a) On Optimum Autofrettage Pressure

For constant working pressure (200 MPa) with different K values (inside radius constant), von mises stresses are observed for different autofrettage pressure. From graph, it is seen that optimum autofrettage pressure increases along with the k value.

Table 3: Effect of k (b/a) on MVS

K(b/a)	MVS Without autofrettage (MPa)	MVS With autofrettage (MPa)	%Reduction Of MVS
1.5	636	516	18.88
2.0	450	348	22.67
2.5	405	312	22.97
3.0	380	281	26.05

For constant working pressure, percent reduction of MVS is observed for different value of K. From the table 3, it is observed that the percent reduction of MVS is higher at higher values of K. This means the autofrettage effect is more beneficial with the increase of the thickness of the cylinder wall.

3.3. Material model

For working pressure of 200 MPa, the cylinder is subjected to autofrettage pressure ranging form 250 to 700 MPa. Here, the material of cylinder wall is varied form elastic perfectly plastic (E^p= 0) to elastic plastic with different slope of strain hardening segment (E^p= 4.5, E^p= 30, E^p= 50). From the numerical simulations, the curve of von-Mises stress distribution was obtained for each autofrettage pressure and different material models. From the curve, the value and the position of the maximum von-Mises stress (MVS) were extracted.

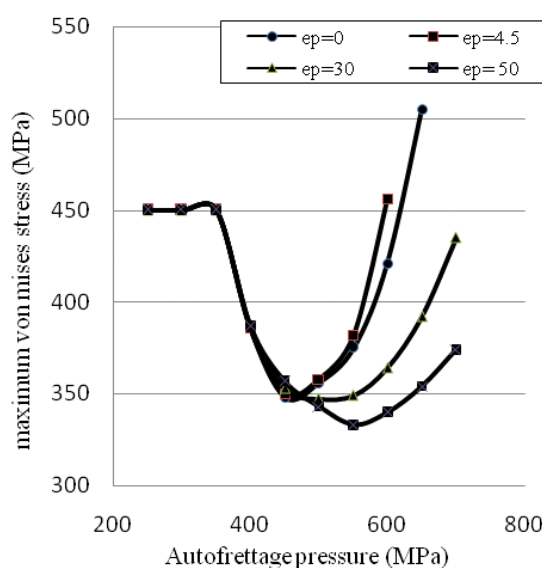


Fig 7. Effect of Material Model on Optimum Autofrettage Pressure

From the graph, it is observed that von mises stress varies in nominal manner in between Py1 (347MPa) and

Py2 (555 MPa). Variation becomes significant after exceeding Py2. The optimum autofrettage pressure is higher at higher value of the slope of strain hardening segment. The resultant von mises stress decreases as the slope of the strain hardening segment increases. So if the cylinder wall material has higher value of slope of strain hardening segment, then the autofrettage process can give us much more benedictions.

3.4. Autofrettage Stages

Consider a cylinder with working pressure of 300 MPa, where autofrettage pressure is 500 MPa. At first step, the autofrettage is done in three loading stages and at second step; autofrettage is done in nine loading stages. From the numerical simulation, it is observed that in both cases the MVS is 505 MPa and the stress pattern is almost similar. So there is no effect of loading stages on autofrettage process.

4. CONCLUSION

From the analytical and numerical results the following conclusions can be drawn:

1. In autofrettaged cylinder, maximum stress does not occur at inner bore instead, it occurs at the radius of elastic plastic junction. As the autofrettage pressure increases the point of maximum stress moves toward the outer bore.
2. Optimum autofrettage pressure is not a constant value rather it depends on the working pressure. The optimum autofrettage pressure increases along with the increase of working pressure.
3. For same working pressure, increasing the ratio of outer to inner radius K leads to an increase in the optimum autofrettage pressure.
4. It has also been observed that if the slope of strain hardening segment increases then the optimum autofrettage pressure also increases.
5. Because of autofrettage, percent reduction of maximum von mises stress increases for higher K value and for higher value of the slope of strain hardening segment.
6. Number of autofrettage stages has no effect on pressure capacity.

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6. NOMENCLATURE

Symbol	Meaning	Unit
a	Internal radius	(m)
b	External radius	(m)
$\sigma_{\theta r}$	Residual hoop stress	(MPa)
σ_{rr}	Residual radial stress	(MPa)
σ_{rf}	Overall radial stress	(MPa)
$\sigma_{\theta f}$	Overall hoop stress	(MPa)
P_w	Working pressure	(MPa)

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