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STRESS ANALYSIS OF STEEL PLATE HAVING HOLES OF VARIOUS SHAPES, SIZES AND ORIENTATIONS USING FINITE ELEMENT METHOD

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ABSTRACT
Steel is widely used in machine parts, structural equipment and many other applications. In many steel structural elements, holes of different shapes and orientations are made with a view to satisfy the design requirements. The presence of holes in steel elements creates stress concentration, which eventually reduce the mechanical strength of the structure. Therefore, it is of great importance to investigate the state of stress around the holes for the safety and proper design of such elements. In this paper, the effects of hole size, shape and orientation on the stress-strain distribution are investigated. Finite element method, one of the popular numerical techniques, is used for the solution of two-dimensional elastic plates incorporating hole located at the centre of the plate. Results from finite element method have been compared with the analytical results for different shapes.

Keywords: Finite Element Method, Stress Concentration.

1. INTRODUCTION
Almost all structures consist of assembly of simple elements, which are connected to each other by joints. Joints or connections that are usually made in steel structures are mechanical fastening using bolts or rivets. In the mechanical fastening, holes are made to place the bolts or rivets; these make the structure weak and susceptible to failure. Therefore, it is necessary to investigate the state of stress around the holes for the safety and proper design of such structures.

From the point of view of the above facts, it is of great importance to understand the behavior of the steel structures with holes. For the solution of the problem several methodologies can be followed, however, all of these methods can be classified in the following three general categories: experimental, analytical and numerical method. Though experimental methods give the most reliable results, it is very costly, as it requires special equipments, testing facilities etc. Analytical solution of every problem is almost impossible because of complex boundary conditions and shapes. For this reason the numerical methods had become the ultimate choice by the researchers in the last few decades. Invention and rapid improvement of the computing machines, i.e. sophisticated high performance computers, also played an important role for the increasing popularity of the numerical methods.

Stress analysis of a steel structure with holes requires the solution of partial differential equations. There are various numerical methods available for the solution of partial differential equations. Among them most popular methods are: Finite Element Method (FEM) and Finite Difference Method (FDM). The finite element method is a numerical technique for obtaining approximate solution to a wide variety of engineering problems. Although originally developed to study stresses in complex airframe structures, it has since been extended and applied to the broad field of continuum mechanics because of its diversity and flexibility as an analysis tool. The finite difference model of a problem gives a point wise approximation to the governing equations. This model is improved as more points are used. With finite difference techniques we can treat some fairly difficult problems; but for example, when we encounter irregular geometries or an unusual specification of boundary conditions, we find that finite difference techniques become hard to use.

Unlike the finite difference method, which envisions the solution region as an array of grid points, the finite element method envisions the solution region as built up many small, interconnected sub regions or elements. A finite element model of a problem gives a piece wise approximation to the governing equations. Since these elements can be put together in a variety of ways, they can be used to represent exceedingly complex shapes.

2. DESCRIPTION OF THE PROBLEM

The geometry of the problem is shown in Fig 1. The material of the plate is high strength alloy steel; Poisson's ratio ν=0.33, Young's modulus E=200 GPa. Uniform tensile load (σ_x) is applied to the plate's lengthwise direction.

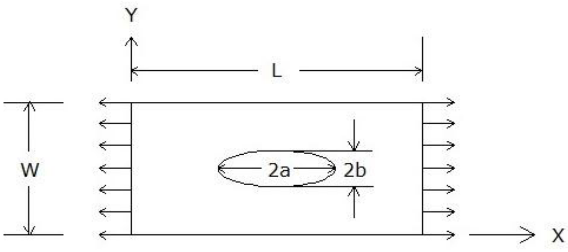

Fig 1. Rectangular plate with centrally located elliptical hole under uniform tensile loading

3. EQUATIONS USED

Stress analysis of an elastic body is usually three dimensional problem. But, most of the practical problems appear in the state of plane stress or plane strain. Stress analysis of three-dimensional bodies under plane stress or plane strain can be treated as two-dimensional problems. The solution of two-dimensional problems require the integration of the different equations of equilibrium together with the compatibility equations and boundary conditions. If body force is neglected, the equations to be satisfied are

$$\partial\sigma_x / \partial x + \partial\sigma_{xy} / \partial x = 0 \quad (1)$$
$$\partial\sigma_y / \partial y + \partial\sigma_{xy} / \partial x = 0 \quad (2)$$
$$(\partial^2 / \partial x^2 + \partial^2 / \partial y^2)(\sigma_x + \sigma_y) = 0 \quad (3)$$

Substitution of stress components by displacement components u and v into Eqs. (1) to (3) makes Eq. (3) redundant and Eqs. (1) and (2) transforms to

$$\partial^2 u/\partial x^2 + (1-v)/2(\partial^2 u/\partial y^2) + (1+v)/2\,(\partial^2 v/\partial x\partial y) = 0 \quad (4)$$

$$\partial^2 v/\partial y^2 + (1-v)/2(\partial^2 v/\partial x^2) + (1+v)/2(\partial^2 u/\partial x\partial y) = 0 \quad (5)$$

Now the problem is to find u and v from a two dimensional field satisfying the two elliptical partial differential Eqs. (4) and (5).
Instead of determining the two functions u and v the problem can be reduced to solving a single function ψ(x,y), which can be determined by satisfying Eqs. (4) and (5). The displacement potential function ψ(x,y) can be defined as

$$u = \partial^2 \psi/\partial x\partial y \quad (6.a)$$

$$v = -[(1-v)\,\partial^2 \psi/\partial y^2 + 2\partial^2 \psi/\partial x^2]/(1-v) \quad (6.b)$$

By the above definitions the displacement components u and v satisfies Eq. (4) and the only condition reduced from Eq. (5) that the function ψ(x,y) has to satisfy is

$$\partial^4 \psi/\partial x^4 + 2\,\partial^4 \psi/\partial x^2\partial y^2 + \partial^4 \psi/\partial y^4 = 0 \quad (7)$$

So, now the problem is to evaluate a single function ψ(x,y) from the bi-harmonic Eq. (7), satisfying the boundary conditions specified at the boundary

4. RESULTS AND DISCUSSION

The solution of the displacement and stress components u_x, u_y , σ_x, σ_y and σ_{xy} for elliptical hole and σ_x for square hole are obtained. Their distributions along some selected sections are described below. Also the changes of maximum stress for elliptical and square hole with angular orientations and with sizes are described.

4.1 Solution for Rectangular Plate with Elliptical Hole

a) Distribution of σ_x

Figure 2 shows a problem of rectangular plate with elliptical hole at the centre (a/b=2) and two ends of plate are subjected to uniform tensile stress along X-direction and other two ends are kept free. At the extreme right end, the magnitude of normalized axial stress equals to unity, i.e. actual stress is equal to the applied stress.

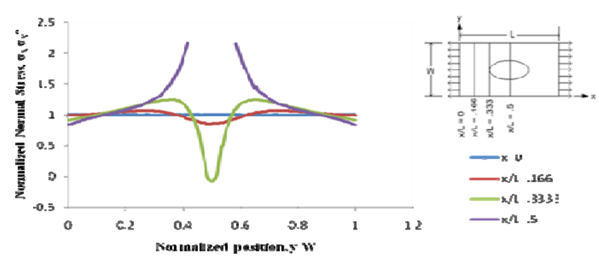

Fig 2. Distribution of normalized axial stress component at different sections of the plate.

Maximum compressive stress exists at two ends of elliptical hole on its major axis. Then it propagates towards the ends at a declining rate. From section y/W=0.42 to section y/W=0.58 compressive stress is generated. Most critical section is x/L=0.5. At x/L=0.5 section the magnitude of axial stress is more than 2 times of the applied stress.

b) Distribution of σ_y

Figure 3 illustrates the normalized axial normal stress (σ_y / $\sigma_{x\circ}$) with respect to y-axis (y/W). At x/L=0 throughout the section the value is positive. The value of tensile stress then decreases and compressive stress dominates over the later sections. Compressive stress develops due to the effect of hole. The effect of hole starts at the section x/L=0.333 where the major axis of hole exists in the corresponding section. At this section maximum compressive stress develops at the point where the major axis touches the section. The value of compressive stress develops is much more than the developed axial stress through applied load. The value of compressive stress decreases at the further outer sections from the hole. And it diminishes at the boundary of the plate. Again it is seen that after crossing the section x/L=0.333 along X-direction, the value of tensile stress increases. This is due to the effect of minor axis of hole. At the section x/L=0.5 the maximum tensile stress occurs because the ends of minor axis (vertical axis) of elliptical hole touches with this section.

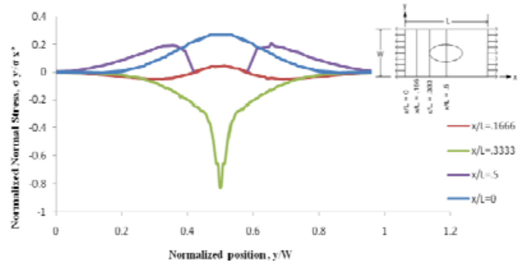

Fig 3. Distribution of normalized lateral stress component at different sections of the plate.

c) Distribution of σ_{xy}

Figure 4 illustrates the normalized shear stress (σ_{xy} / σ_{x°) with respect to y-axis (y/W). The trends of these graphs except the end section and middle section of the plate are wave shaped. As the hole is located at the centre of the plate, so due o the effect of hole half of the sections goes positive value and others half goes negative value. With the development of normal stress, shear stress also developed in the plate but this value is much smaller compared to normal stress. As the load is applied over a ductile material a small amount of shear stress developed which is not dominant over normal stress but for brittle material the value of shear stress is dominant over normal stress. If we apply load over brittle material, then the value of shear stress will be considerable.

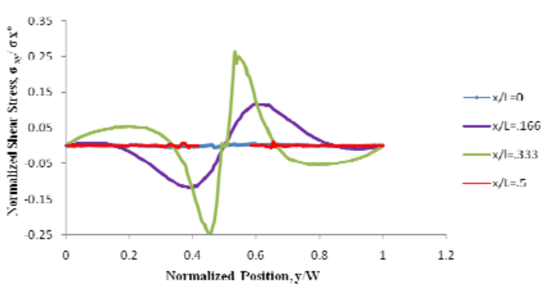

Fig 4. Distribution of normalized shear stress component at different sections of the plate.

d) Distribution of u_x

Figure 5 represents the normalized displacement component (u_x/W) with respect to y-axis (y/W). From the graph it is seen that displacement at any section reaches maximum at y/W=0.5.As area is small here, less material is present here. So bonding force between molecules is weaker and as a result axial displacement will be higher. Again the value is higher in the outer layer. From the graph, it is also seen that after x/L=0.5 the value of displacement is negative. This is due to the fact that, uniform tensile load is applied over the both ends of the plate.

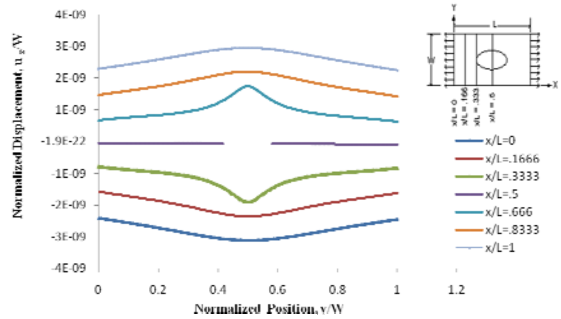

Fig 5. Distribution of normalized displacement component (u_x/W) at different sections of the plate.

e) Distribution of u_y

Figure 6 illustrates the normalized displacement component along y-axis (u_y/W) with respect to y-axis (y/W). With the development of σ_x a small amount of σ_y develops over the plate. Due to the effect of lateral normal stress, lateral normal strain develops. From the graph it is seen that for the half of each section the value is positive and for other sections value is negative. And value will be zero at y/W=0.5. Actually the deformation along y-direction starts from the position of y/W=0.5. Then it propagates through the both lateral ends of the plate. The two halves of each section from y/W=0.5 tends to extend along the positive and negative direction of Y-axis. They oppose each other at the midpoint of each section. For this reason, the value of lateral strain is zero at midpoint of each section. Again it is seen that, as we go towards the inner sections of the plate the value of lateral displacement increases due to the hole effect.

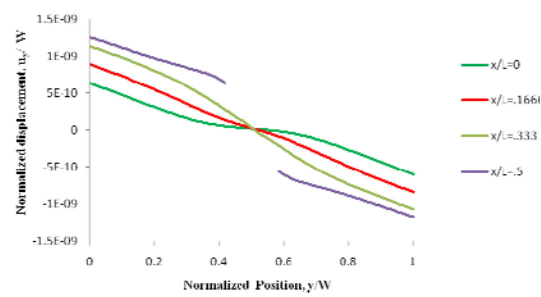

Fig 6. Distribution of normalized displacement component (u_y /W) at different sections of the plate.

f) Variation of Maximum Stress in the Plate Due to the Variation of Size of Hole

Figure 7 shows the variation of maximum longitudinal stress developed over a rectangular plate with the size variation of elliptical hole inserted into the plate. From the graph it is seen that with the increase of b/a ratio, maximum stress increases for 0 degree angle. For other angles maximum stress also decreases with decrease of b/a ratio, but after a certain b/a ratio instead of decrease of maximum stress, the value increases with decrease of b/a ratio. This is due to the fact that for large decrease of b/a means effect of one axis of hole, rejects the effect of other axis of hole. As a result, the desired value will not be found.

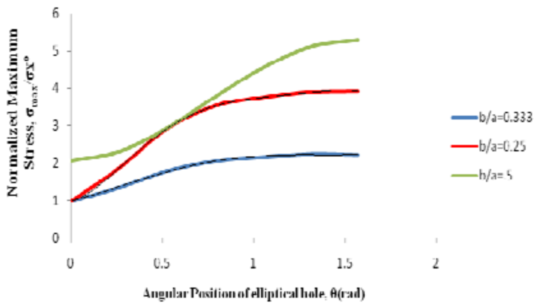

Fig 7. Variation of maximum stress in the plate due to the variation of size of hole.

g) Variation of Maximum Stress Due to the Orientation of Elliptical Hole

Figure 8 represents the variation of maximum axial stress due to variation of angular orientation of the elliptical hole inside the plate (a/b=2 and L/W=1). From the graph it is seen that the value of maximum stress not only depends on hole size but also depends on the angular position of hole which is located at the centre of the plate. With increase of angle, the value of maximum stress increases and it reaches maximum at 90 degree. At 0 degree the area under applied load is higher compared to other orientations. As a result, the value of maximum stress will be lower. This maximum stress exists at the ends of the vertical axis of the hole. At 90 degree, the position of major axis interchanges with the position of the minor axis. As a result, the area under the applied load decreases and maximum stress increases.

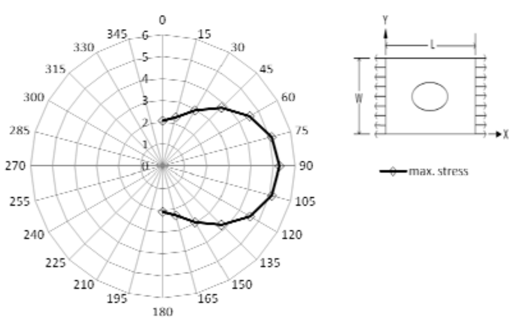

Fig 8. Variation of maximum stress due to the orientation of elliptical hole.

Many computational efforts are applied for the development of equation, for orientation of hole. All equations are developed based on the well known equation for elliptical hole at 0 degree angular orientation which is

$$\frac{\sigma_{max}}{\sigma} = (1 + 2\frac{b}{a}) \quad (8)$$

For a/b=2, the equation which gives less error than other equations is an exponential equation is represents as

$$\frac{\sigma_{max}}{\sigma} = [1 + 2 \times \frac{b}{a} \times e^{(-..5366\,\theta^6 + 2.2419\,\theta^5 - 2.8179\,\theta^4 + .0012\,\theta^3 \; + 1.7398\,\theta^2 + 0.6013\,\theta)}] \quad (9)$$

For a/b=3, the equation found from the finite element method is

$$\frac{\sigma_{max}}{\sigma} = [1 + 2\frac{b}{a}(-4.842\,\theta^3 + 10.93\,\theta^2 + 0.251\,\theta + 1.034)] \quad (10)$$

4.2 Solution for Rectangular Plate with Square Hole

a) Variation of Stress at Different Cross Sections

Figure 9 shows the normalized axial normal stress (σ_x/σ_{x^o}) with respect to y-axis (y/a) for a rectangular plate with square hole at the centre .Two ends of plate are subjected to uniform tensile stress along X-direction and other two ends are kept free. In order to present the graphs of stress in dimensionless form, actual values of stress are divided by the applied stress.. At the extreme right end, the magnitude of normalized axial stress equals to unity, i.e. actual stress is equal to the applied stress. In all sections except (x/L=0) the axial normal stress exceeds the applied stress and there is also compressive stress exists. In a material molecules are arranged in parallel layers. Due to the presence of hole all layers cannot move at a same rate. Maximum compressive load exists at two extremities of square hole along longitudinal axis. Then it propagates through the plate and its value declines towards the ends. The value of compressive stress goes to zero at ends of the plate and only applied load exists there. Stress concentration at sharp edge at the square hole, so we get the maximum value of tensile stress there. The value of stress increases up to the sharp edge, then it begins to decrease due to the flat region of square from which the compressive stress originates.

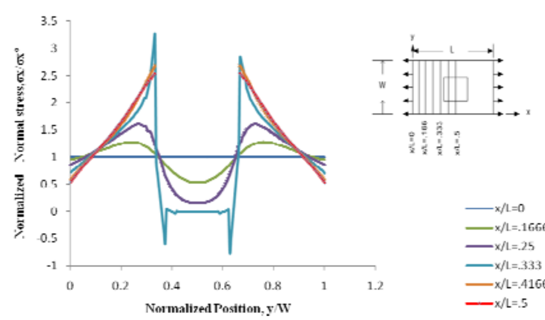

Fig 9. Distribution of normalized axial stress along the plate having square hole at the center.

b) Variation of Maximum Stress due to Orientation of Square Hole

Figure 10 represents variation of maximum stress due to variation of angular orientation of the square hole inside the plate (L/A=3 and L/W=1). It is seen that maximum stress increases with increasing angle of hole. It reaches maximum value at 30 degree and after that value decreases. As it is a square hole the value of maximum stress at 0 degree and 90 degree are same.

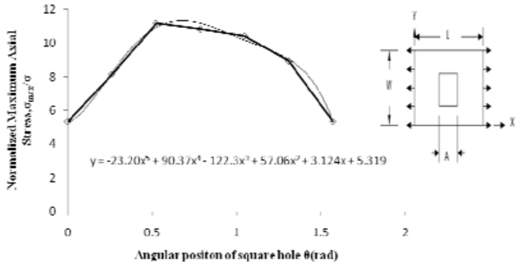

Fig 10. Variation of maximum stress due to orientation of square hole.

4.3 Variation of Maximum Stress due to Orientation of Rectangular Hole

Figure 11 represents Cartesian graph for maximum stresses developed in a plate with rectangular holes (A/B=2 and L/W=1) located at centre with various angular orientation. From the graph it is seen that it follows a fourth order polynomial

$$Y = -2.29x^4+7.066x^3-9.66x^2+7.279x+2.393 \quad (11)$$

The value of maximum stress increases up to 30 degree then decreases up to 45 degree. After having a maximum value at 60 degree it again decreases.

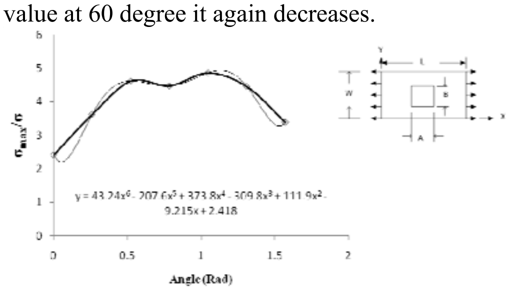

Fig 11. Variation of maximum stress due to orientation of rectangular hole.

5. CONCLUSION

In this study finite element method is used for the solution of two-dimensional problems of rectangular plate having centrally located holes of various shapes. Finite element results are carried out by using the commercial software COMSOL 3.3. Results are presented in the form of non-dimensional graphs. Effects of hole shape are critically analyzed from the results of the finite element method and analytical methods. The following conclusions are drawn in regard to the present study. Comparisons between the results by the present finite element method and the analytical solution technique yield good agreement. At 0° angular position of elliptical hole, maximum stress occurs at the two ends of hole on its minor axis. At 90° angular position of elliptical hole, maximum stress occurs at the two ends of hole on its major axis. For elliptical hole, with the increase of plate length to width ratio the maximum stress at all angular position increases. Moreover, with the increase of elliptical hole major diameter to minor diameter ratio, the maximum stress at all angular position increases. In case of rectangular plate under uniform tensile loading, square hole experiences more longitudinal stress than elliptical hole. If the holes are made to rotate, the value of maximum stress increases with rotation. For square hole, the maximum value of maximum stress occurs at 45° angular position and it decreases with increase of plate length to hole length ratio. In case of rotation of square hole, for very small or very large plate length to hole length ratio, the highest value of maximum stress is found at 30° of rotation.

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7. NOMENCLATURE

Symbol	Meaning	Unit
x,y	Rectangular co-ordinate	
E	Modulus of Elasticity	(Pa)
ν	Poisson's Ratio	
u, v	Displacement component in the x and y direction	(mm)
σ_x, σ_y, σ_{xy}	Stress component in the x, y direction and xy plane	(Pa)
ψ	Potential function	
W	Width of the plate	(mm)
L	Length of the plate	(mm)
a, b	Elliptical hole dimension	(mm)
A, B	Rectangular hole dimension	(mm)

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