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FLOW CHARACTERISTICS THROUGH A ROTATING CURVED DUCT WITH SQUARE CROSS SECTION

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ABSTRACT
In this paper, a comprehensive numerical study is presented for the thermal flow through a rotating curved duct with square cross section. Numerical calculations are carried out over a wide range of the Taylor number $0 \le Tr \le 3000$ for the Dean numbers $Dn = 1000$ and $Dn = 2000$ with the Grashof number $Gr = 500$, where the outer wall is heated and the inner one cooled. Spectral method is used as a basic tool to solve the system of non-linear differential equations. The rotation of the duct about the center of curvature is imposed, and the effects of rotation (*Coriolis force*) on the flow characteristics are investigated. As a result, multiple branches of asymmetric steady solutions with two- and four-vortex solutions are obtained. Linear stability of the steady solutions is then investigated. When there is no stable steady solution, time evolution calculations of the unsteady solutions are obtained, and it is found that there occur only periodic and multi-periodic solutions where the solution is unstable.

Keywords: Curved Square Duct, Secondary Flow, Steady Solutions, Dean Number, Taylor Number.

1. INTRODUCTION
The study of flows and heat transfer through a curved duct is of fundamental interest because of its importance in chemical, mechanical and biological engineering. Due to engineering applications and their intricacy, the flow in a rotating curved duct has become one of the most challenging research fields of fluid mechanics. Since rotating machines were introduced into engineering applications, such as rotating systems, gas turbines, electric generators, heat exchangers, cooling system and some separation processes, scientists have paid considerable attention to study rotating curved channel flows. The readers are referred to Berger *et al*. [1] and Nandakumar and Masliyah [2] for some outstanding reviews on curved duct flows.

One of the interesting phenomena of the flow through a curved duct is the bifurcation of the flow because generally there exist many steady solutions due to duct curvature. Many researches have performed experimental and numerical investigations on developing and fully developed curved duct flows. An early complete bifurcation study of two-dimensional (2D) flow through a curved channel was conducted by Winters [3]. However, an extensive treatment on the flow characteristics for both the isothermal and non-isothermal flows through curved duct with rectangular cross section was performed by Mondal [4].

The flow through a rotating curved duct is another subject, which has attracted considerable attention because of its importance in engineering devices. The fluid flowing in a rotating curved duct is subjected to two forces: the Coriolis force due to rotation and the centrifugal force due to curvature. For isothermal flows of a constant property fluid, the Coriolis force tends to produce vortices while centrifugal force is purely hydrostatic. When a temperature induced variation of fluid density occurs for non-isothermal flows, both Coriolis and centrifugal type buoyancy forces can contribute to the generation of vortices (Wang and Cheng [5]). These two effects of rotation either enhance or counteract each other in a non-linear manner depending on the direction of wall heat flux and the flow domain. Therefore, the effect of system rotation is more subtle and complicated and yields new; richer features of flow and heat transfer in general, bifurcation and stability in particular, for non-isothermal flows. Recently, Mondal, Alam and Yanase [6] performed numerical prediction of non-isothermal flows through a rotating curved square channel with the Taylor number $0 \le Tr \le 2000$ for the Grashof number $Gr = 100$.

In the present paper, a comprehensive numerical study is presented for the flows through a rotating curved duct with square cross section. Flow characteristics are studied over a wide of the Taylor number for the Grashof number Gr = 500. Studying the effects of rotation on the flow characteristics, caused by the buoyancy forces, is an important objective of the present study.

2. BASIC EQUATIONS

Consider a hydro-dynamically and thermally fully developed two-dimensional flow of viscous incompressible fluid through a rotating curved duct with square cross section, whose height and wide are $2h$ and $2l$, respectively. In the present case, we consider $h = l$. The coordinate system with the relevant notation is shown in Fig. 1, where x' and y' axes are taken to be in the horizontal and vertical directions respectively, and z' is the axial direction. The system rotates at a constant angular velocity Ω_T around the y' axis. It is assumed that the outer wall of the duct is heated while the inner wall cooled. The temperature of the outer wall is $T_0 + \Delta T$ and that of the inner wall is $T_0 - \Delta T$, where $\Delta T > 0$. u, v and w be the velocity components in the x', y' and z' directions respectively. All the variables are non-dimensionalzed.

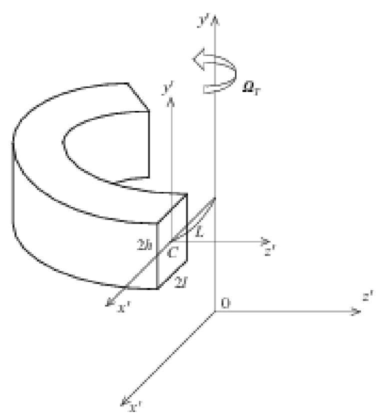


Fig 1. Coordinate system of the rotating curved square duct

The sectional stream function ψ is introduced as

$$u = \frac{1}{1+\delta x}\frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{1+\delta x}\frac{\partial \psi}{\partial x} \qquad (1)$$

Then, the basic equations for the axial velocity w, the stream function ψ and temperature T are expressed in terms of non-dimensional variables as:
:

$$(1+\delta x)\frac{\partial w}{\partial t} + \frac{\partial (w,\psi)}{\partial (x,y)} - Dn + \frac{\delta^2 w}{1+\delta x} = (1+\delta x)\Delta_2 w - \frac{\partial}{1+\delta x}\frac{\partial \psi}{\partial y} w + \delta \frac{\partial w}{\partial x} - \delta Tr \frac{\partial \psi}{\partial y} \qquad (2)$$

$$\left(\Delta_2 - \frac{\delta}{1+\delta x}\frac{\partial}{\partial x}\right)\frac{\partial \psi}{\partial t} = -\frac{1}{(1+\delta x)}\frac{\partial(\Delta_2\psi,\psi)}{\partial(x,y)} + \frac{\delta}{(1+\delta x)^2}\left[\frac{\partial \psi}{\partial y}\left(2\Delta_2\psi - \frac{3\delta}{1+\delta x}\frac{\partial \psi}{\partial x} + \frac{\partial^2 \psi}{\partial x^2}\right) - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial x \partial y}\right] + \frac{\delta}{(1+\delta x)^2}\left[3\delta \frac{\partial^2 \psi}{\partial x^2} - \frac{3\delta^2}{1+\delta x}\frac{\partial \psi}{\partial x}\right] - \frac{2\delta}{1+\delta x}\frac{\partial}{\partial x}\Delta_2\psi + w\frac{\partial w}{\partial y} + \Delta_2{}^2\psi - G_r(1+\delta x)\frac{\partial T}{\partial x} + \frac{1}{2}Tr\frac{\partial w}{\partial y}, \qquad (3)$$

$$\frac{\partial T}{\partial t} + \frac{1}{(1+\delta x)}\frac{\partial (T,\psi)}{\partial (x,y)} = \frac{1}{\Pr}\left(\Delta_2 T + \frac{\delta}{1+\delta x}\frac{\partial T}{\partial x}\right) \qquad (4)$$

where

$$\Delta_2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \frac{\partial (T,\psi)}{\partial (x,y)} \equiv \frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x} \qquad (5)$$

The non-dimensional parameters Dn, the Dean number, Gr, the Grashof number, Tr, the Taylor number and $\Pr$, the Prandtl number, which appear in equation (2) to (4) are defined as:

$$Dn = \frac{Gl^3}{\mu\upsilon}\sqrt{\frac{2l}{L}}, \quad Gr = \frac{\beta g \Delta T\, l^3}{\upsilon^2}, \quad Tr = \frac{2\sqrt{2\delta}\,\Omega_T l^3}{\upsilon\delta}, \quad \Pr = \frac{\upsilon}{\kappa} \qquad (6)$$

where the parameters denote their usual meaning.

The rigid boundary conditions for w and ψ are used as

$$w(\pm 1, y) = w(x, \pm 1) = \psi(\pm 1, y) = \psi(x, \pm 1) = \frac{\partial \psi}{\partial x}(\pm 1, y) = \frac{\partial \psi}{\partial y}(x, \pm 1) = 0 \qquad (7)$$

and the temperature T is assumed to be constant on the walls as:

$$T(1,y) = 1, \; T(-1,y) = -1, T(x,\pm 1) = x \qquad (8)$$

3. NUMERICAL METHODS

In order to solve the Equations (2) to (4) numerically, the spectral method is used. By this method the expansion functions $\phi_n(x)$ and $\psi_n(x)$ are expressed as

$$\left.\begin{aligned}\phi_n(x) &= (1-x^2)\,C_n(x),\\ \psi_n(x) &= (1-x^2)^2 C_n(x)\end{aligned}\right\} \qquad (9)$$

Where $C_n(x) = \cos(n\cos^{-1}(x))$ is the n^{th} order Chebyshev polynomial. $w(x,y,t)$, $\psi(x,y,t)$ and $T(x,y,t)$ are expanded in terms of the expansion functions $\phi_n(x)$ and $\psi_n(x)$ as

$$\left.\begin{aligned} w(x,y,t) &= \sum_{m=0}^{M}\sum_{n=0}^{N} w_{mn}(t)\,\varphi_m(x)\,\varphi_n(y)\\ \psi(x,y,t) &= \sum_{m=0}^{M}\sum_{n=0}^{N} \psi_{mn}(t)\,\psi_m(x)\,\psi_n(y).\\ T(x,,y,t) &= \sum_{m=0}^{M}\sum_{n=0}^{N} T_{mn}\,\varphi_m(x)\,\varphi_n(y) + x \end{aligned}\right\} \qquad (10)$$

where M and N are the truncation numbers in the x and y directions respectively.

First, steady solutions are obtained by the Newton-Rapshon iteration method and then linear stability of the steady solutions is investigated against only two-dimensional ($z-$ independent perturbations. Finally in order to calculate the unsteady solutions, the Crank-Nicolson and Adams-Bashforth methods together with the function expansion (10) and the collocation methods are applied to Eqs. (2) to (4).

4. FLUX THROUGH THE DUCT

The dimensional total flux Q' through the duct in the rotating coordinate system is calculated by:

$$Q' = \int_{-d}^{d}\int_{-d}^{d} w'\,dx'\,dy' = v\,dQ \qquad (11)$$

Where

$$Q = \int_{-1}^{1}\int_{-1}^{1} w\,dx\,dy \qquad (12)$$

is the dimensionless total flux. The mean axial velocity $\overline{w}'$ is expressed as

$$\overline{w}' = \frac{Qv}{4d}.$$

In the present study, Q is used to denote the steady solution branches and to pursue the time evolution pf the unsteady solutions.

5. RESULTS AND DISCUSSION

We take a curved duct with square cross section and rotate it around the center of curvature with an angular velocity Ω_T. In the present study, we investigate the flow characteristics and discuss the flow phenomena for two cases of the Dean numbers, *Case 1*: $Dn = 1000$ and *Case I1*: $Dn = 2000$, over a wide range of the Taylor number $0 \le Tr \le 3000$, for the Grashof number $Gr = 500$. Thus an interesting and complicated flow behavior will be expected if the duct rotation is involved for these two cases.

5.1 Case *I*: $Dn = 1000$

5.1.1 Steady solutions and their linear stability analysis

With the present numerical calculation, we obtain two branches of steady solutions for $Dn = 1000$ over the Taylor number $0 \le Tr \le 3000$. The two steady solution branches are named the *first steady solution branch* (first branch, thin solid line) and the *second steady solution branch* (second branch, dashed line), respectively. Figure 2 shows the flux Q through the duct versus the Taylor number Tr for the Dean number $Dn = 1000$. It is found that the steady solution branches are independent and there exists no bifurcating relationship between the two branches in the parameter range investigated in this study. It is found that the first branch is composed of two-vortex solutions only, while the second branch consists of two- and four-vortex solutions.

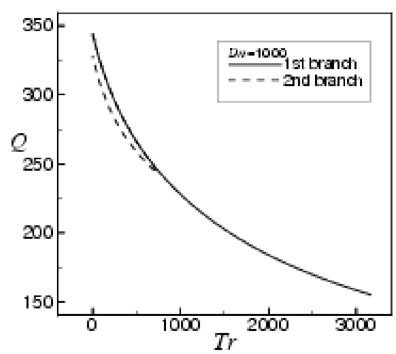


Fig 2. Steady solution branches for $0 \le Tr \le 3000$ and *Gr* = 500.

Linear stability of the steady solutions shows that only the first steady solution branch is partly unstable for small Tr ($Tr \le 15.3$). However, as Tr increases, the steady solution becomes stable and remains stable onwards for larger *Tr*. Thus we find that the steady solution is linearly stable for $15.4 \le Tr \le 3000$. The second steady solution branch is linearly unstable everywhere.

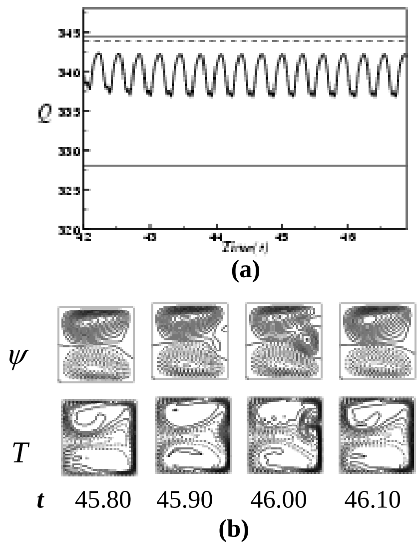


Fig 3. Unsteady solutions for $Dn = 1000$ and $Tr = 0$. (a) Time evolution of Q and the values of Q for the steady solutions. (b) Contours of secondary flow (top) and temperature profile (bottom) for one period of oscillation at $45.80 \le t \le 46.10$.

5.1.2 Unsteady solutions

In order to study the non-linear behavior of the unsteady solutions, time-evolution calculations are performed for $Dn = 1000$. Time evolution of Q for $Tr = 0$ is shown in Fig. 3(a). It is found that the flow is time periodic. In the same figure, the relationship between the periodic solution and the steady states, the values of Q for the steady solution branches at $Tr = 0$, are also shown by straight lines using the same kind of lines as were used in the bifurcation diagram in Fig. 2. The periodic solution at $Tr = 0$ oscillates in the region between the upper and the lower parts of the second steady solution branch. Then, in order to see the change of the flow characteristics, as time proceeds, contours of typical secondary flow and temperature distribution are shown in Fig. 3(b), where it is seen that the periodic solution at $Tr = 0$ oscillates between asymmetric two- and four-vortex solutions.

5.2 Case *II*: $Dn = 2000$

5.2.1 Steady solutions and their linear stability analysis

We obtain four branches of steady solutions for $Dn = 2000$ over a wide range of Tr for $0 \le Tr \le 3000$. The bifurcation diagram of steady solutions is shown in Figure 4. The four steady solution branches are named the *first steady solution branch* (first branch, thick solid line), the *second steady solution branch* (second branch, dashed line), the *third steady solution branch* (third branch, thin solid line) and the *fourth steady solution branch* (fourth branch, dashed dotted line), respectively.

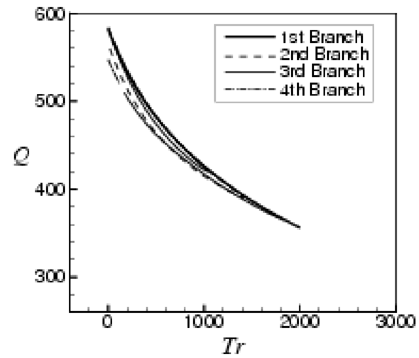


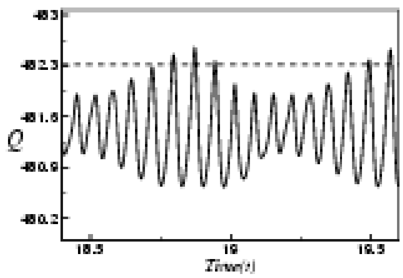
Fig 4. Steady solution branches for $Dn = 2000$.

The steady solution branches are obtained by the path continuation technique with various initial guesses as discussed by Mondal [4] and are distinguished by the nature and number of secondary flow vortices appearing in the cross section of the duct. The first branch is composed of only two-vortex solutions. The second branch consists of asymmetric two-and nearly symmetric four-vortex solutions. The third branch is comprised of two- and four- vortex solutions while the forth branch is composed of four-vortex solutions. It is found that, at the same value of *Tr* we obtain both two- and four-vortex solutions. In the case of temperature transmission from inner wall to the fluid, it is found that the convection becomes more frequent with the increase of rotation.

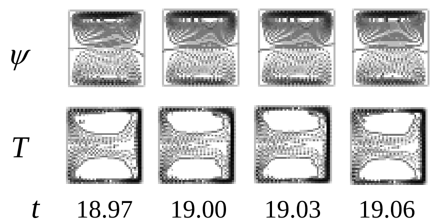
Linear stability of the steady solutions shows that only the first branch is linearly stable in a couple of interval of *Tr*, while the other branches are linearly unstable at any value of *Tr*. The first branch is linearly stable for $0 \le Tr < 279.1$ and $922.8 < Tr \le 3000$ and unstable for $279.1 \le Tr \le 922.80$.

5.2.2 Unsteady solutions

We perform time-evolutions of the unsteady solutions for $Dn = 2000$ and $0 \le Tr \le 3000$. The time evolutions of Q, together with the values of Q for the steady solution branches, are shown in Fig. 5(a) for $Tr = 500$. It is found that the flow oscillates multi-periodically. The associated secondary flow patterns and temperature profiles are shown in 5(b) for $18.97 \le Tr \le 19.06$. As seen in Figs. 5(a) and 5(b), the unsteady flow at $Tr = 500$ oscillates between the asymmetric two-vortex solutions. Next, the time evolution of Q is shown in Fig. 6(a) for $Tr = 900$. It is found that the flow oscillates periodically. The associated secondary flow patterns and temperature profiles are shown, for one period of oscillation, in Fig. 6(b) at $7.81 \le Tr \le 7.89$. It is found that the unsteady flow at $Tr = 900$ also oscillates between the asymmetric two-vortex solutions.

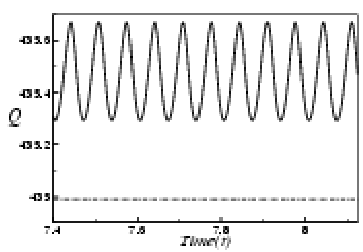


(a)

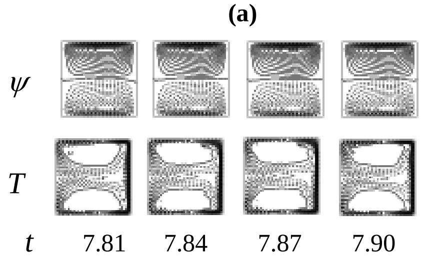


(b)

Fig 5. Unsteady solutions for $Dn = 2000$ and $Tr = 500$. (a) Time evolution of Q and the values of Q for the steady solutions. (b) Contours of secondary flow (top) and temperature profile (bottom) for $Dn = 2000$ and $Tr = 500$ at Gr = 500.



(a)



(b)

Fig 6. Unsteady solutions for $Dn = 2000$ and $Tr = 900$. (a) Time evolution of Q and the values of Q for the steady solutions. (b) Contours of secondary flow (top) and temperature profile (bottom) for one period of oscillation at $7.81 \le t \le 7.90$.

6. CONCLUSIONS

In this study, a numerical result is presented for the fully developed two-dimensional flow of viscous incompressible fluid through a rotating curved square duct over a wide range of the Taylor number, $0 \le Tr \le 2000$ and the Dean number, $0 \le Dn \le 2000$ for the curvature $\delta = 0.1$. Spectral method is used as a basic tool to solve the non-linear system of equations. In this study, a detail discussion on $Dn = 1000$ and $Dn = 2000$ are presented with a temperature difference between the vertical sidewalls for the Grashof number $Gr = 500$, where the outer wall is heated and the inner wall cooled.

We obtain two and four branches of asymmetric steady solutions for $Dn = 1000$ and $Dn = 2000$ respectively. It is found that there exist two-and four-vortex solutions on various branches. These vortices are generated due to the combined action of the centrifugal force and Coriolis force. It is found that as Dn increases the number of steady solutions also increases. Linear stability of the steady solutions shows that only the first branch is linearly stable while the other branches are linearly unstable. In the unstable region, we perform time evolution calculations of the unsteady solutions and it is found that for $Dn = 1000$ flow becomes periodic before turning to steady state. For $Dn = 2000$, however, the unsteady flow becomes steady-state first, then periodic or multi-periodic and finally steady state once again, if Tr is increased.

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