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STENOSIS LENGTH AND ITS IMPACT ON FLOW OF BLOOD THROUGH RECTANGULAR STENOSED CORONARY ARTERY

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ABSTRACT

Impact of stenosis length on flow of blood through rectangular stenosed coronary artery has been investigated in case of non dimension stenosis lengths of 0.1, 0.5, 1.0 and 2.0 for the restriction of 50% (by diameter) and Reynolds number of 200. Effect on streamline contour and wall shear stress has been discussed and presented along with its possible physiological aspects. It is revealed that the wall shear stress is dependent upon the stenosis length but reattachment length is relatively independent of stenosis length.

Keywords: Stenosis, Wall pressure, Wall Shear Stress, Stenosis Length

1. INTRODUCTION

Atherosclerosis is a progressive disease characterized by localized plaques that form within the artery wall. One of the fundamental causes of the plaque development is believed to be the abnormal enlargement of the intima by infiltration and accumulation of macromolecules such as lipoproteins, and the associated cellular and synthetic reactions. As the disease progresses, these plaques enlarge and, either directly or indirectly, lead to impairment of blood flow. This in turn can have serious consequences, such as blockage of the coronary arteries. Both clinical reports and numerical simulations show that haemodynamics play an important role in the pathogenesis of atherosclerosis. It is widely believed that atherosclerosis development and progression are affected by many risk factors, such as static pressure, wall shear stress, blood viscosity, flow velocity etc.

Some attempts to study across rectangular stenoses have been done by different researchers numerically and experimentally. Among them, the numerical work have been done by Cheng [1, 2] for steady, oscillatory and pulsatile flow, Coder and Buckley[3] for unsteady flow, Wille [4] for steady, laminar, Newtonian flow conditions for several models of stenosed vessels with different constriction ratios of 50%, 75% and 90% with Reynolds numbers 10, 100 and 200, and Liou et al.[5] for steady flow with Reynolds numbers of 25 and 150. The experimental work have been done by Seeley and Young [6] for steady flow with Re varying from 0 to 1000 with 60%, 75%, 85% and 90% area reduction considering different stenosis lengths, Solzbach et al. [7] have investigated experimentally the influence of stenosis geometry in steady flow conditions for Re varying from 1 to 500 with percentage lumen area reduction of 77%, 87% and 94% for two stricture lengths. Liepsch et al. [8] have studied experimentally the flow behavior under steady flow condition in four models of cylindrical stenosis with rectangular cross section at Re varies from 150 to 920 and percentage of restriction of 21%, 45%, 49% and 73% approximately for a fixed stenosis length.

From brief review of literature, it is noted that a number of researchers have studied the effect of percentage of restriction, Reynolds number and stenosis length on pressure drop and flow separation for rectangular shaped constriction. Very few of them have discussed the flow separation with the variation of stenosis length. None of them has investigated the effect of stenosis length of a rectangular stenosis on the wall shear stress. Therefore, in this work, an attempt has been made to study systematically the effect of stenosis lengths of 0.1, 0.5, 1.0 and 2.0, on stream line contour and shear stress typically for Reynolds number of 200 and percentage of restriction of 50% respectively.

2. MATHEMATICAL FORMULATION

2.1 Governing Equations

A schematic diagram of the computational domain is illustrated in Fig. 1. The flow under consideration has been assumed to be steady, two-dimensional and laminar. In the study, the blood is considered as Newtonian and incompressible, and the arterial wall is rigid.

The following dimensionless variables are defined to obtain the governing conservation equations in the non-dimensional form;

Lengths: $x^* = x/D \qquad y^* = y/D \quad L_i^* = L_i/D$

$$L_{ex}^* = L_{ex}/D \qquad Ls^* = Ls/D$$

Velocities: $u^* = u/V_1 \qquad v^* = v/V_1$

Pressure $p^* = \left(p + \rho g y\right)/\rho V_1^2$

With the help of these variables, the mass and momentum conservation equations are written as follows,

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$\frac{\partial\left(u^* u^*\right)}{\partial x^*} + \frac{\partial\left(v^* u^*\right)}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial x^*}\left(\frac{\partial u^*}{\partial x^*}\right) + \frac{\partial}{\partial y^*}\left(\frac{\partial u^*}{\partial y^*}\right)\right] \quad (2)$$

$$\frac{\partial\left(u^* v^*\right)}{\partial x^*} + \frac{\partial\left(v^* v^*\right)}{\partial y^*} = -\frac{\partial p^*}{\partial y^*} + \frac{1}{\mathrm{Re}}\left[\frac{\partial}{\partial x^*}\left(\frac{\partial v^*}{\partial x^*}\right) + \frac{\partial}{\partial y^*}\left(\frac{\partial v^*}{\partial y^*}\right)\right] \quad (3)$$

Where, the flow Reynolds number, . $\mathrm{Re} = \rho V_1 D/\mu$

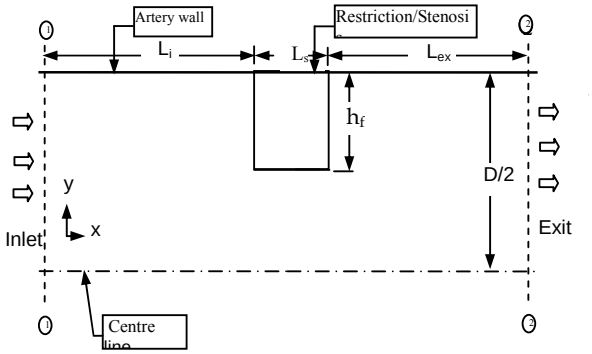

Fig 1. Computational Domain

2.2 Boundary Conditions

Four different types of boundary conditions have been applied to the present problem. They are as follows,

1. At the walls: $u^* = 0$, $v^* = 0$.
2. At the inlet: Fully developed flow condition has been specified at the inlet, i.e., . $u^* = 1.5\left[1 - \left(2y^*\right)^2\right]$
3. At the exit: $\partial u^*/\partial x^* = 0 \quad \partial v^*/\partial x^* = 0$
4. At the line of symmetry: $\partial u^*/\partial y^* = 0 \quad v^* = 0$

2.3 Numerical Procedure

The partial differential equations (1), (2) and (3) have been discretised by a control volume based finite difference method. Power law scheme has been used to discretise the convective terms as per Patankar[9]. The discretised equations have been solved iteratively by SIMPLE algorithm, using line-by-line ADI method. The convergence of the iterative scheme has been achieved when the normalized residuals for mass and momentum equations summed over the entire calculation domain will fall below 10^{-8}.

In the computation, flow has been assumed fully developed at the inlet and exit and therefore, the inlet and exit have been chosen far away from the restriction. For all calculations, the non-dimensional inlet and the exit lengths have been considered to be 50. The distributions of grid nodes have been considered non-uniform and staggered in both coordinate direction allowing higher grid node concentrations in the region close to the wall and restriction.

3. RESULTS AND DISCUSSION

3.1 Variation of Streamline Contour

A detailed knowledge associated with the flow pattern is essential for detection of stenosis. The recirculation zone in the post stenotic region is considered to be an important phenomenon for the formation and

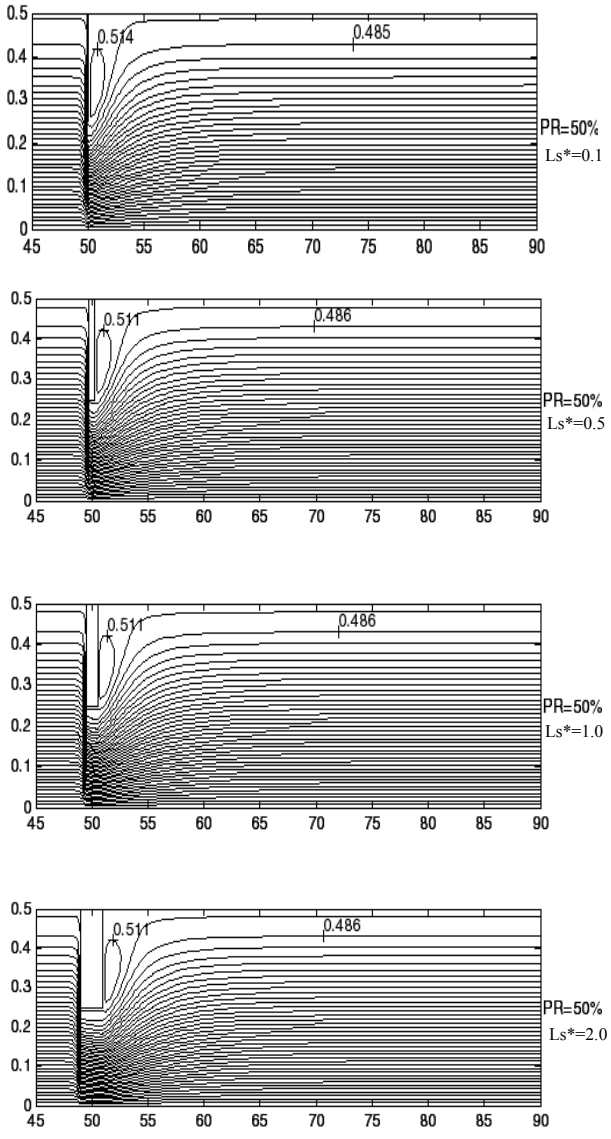

Fig 2. Effect of stenosis length on streamline contour at Reynolds number of 200 for PR of 50%.

atherosclerosis. The physiological significance of the recirculation zone is that the bloodstream stagnates locally in this area and allows platelets and fibrin to form a mesh at the inner wall in which lipid particles become trapped and eventually coalesce to form atheromatous

plaque, this may tend to accumulate to cause a more severe stenosis (Tu et al. [11]). Apart from that, the reattachment point is having also significance on the formation and propagation of atherosclerosis. The high cell turnover rate takes place near the reattachment point due to high cell division and low cell density near that region. For this, a leaky junction may develop which is considered to be the possible pathway for transport of low-density lipoprotein through the arterial wall, (Weinbaum and Chien [12]).

Figure 2 depicts the streamline contours for all the considered stenosis lengths of 0.1, 0.5, 1.0 and 2.0 for typical flow Reynolds number of 200 and 50% restriction. From the study of streamlines, it is noted that, at downstream to the throat section, separation phenomenon takes place and then it reattaches at some distal location. For considered PR and Re, no change in reattachment length is noted with the change in stenosis length. Solzbach et al. [7] have experimentally observed the same nature of flow stabilization with stenosis length. It indicates that once the stenosis is developed, there is no further disturbance of flow characteristics in terms of the length of the recirculation zone with the propagation of stenosis length for the considered geometry of stenosis. It is also observed that the reattachment point moves downstream with increase in stenosis length which indicates the movement of cell turnover point on the arterial wall with the progression of stenosis length.

3.2 Variation of Wall Shear Stress

Fry [13, 14] has first postulated that high wall shear stresses do mechanical damage to the arterial wall and consequently initiate the process for formation of atherosclerosis but Caro et al [15] have claimed that such lesions occur in regions of low wall shear stress. It has been found that initially blood cells are damaged or their surface changes in a high shear field, and then the particles stick to wall and form deposits at low shear stress fields, (Chakraborty [16]). Apart from that, RBC damage and thrombosis formulation are thought to occur due to combination of low and high shear stress. Therefore, both high and low wall shear stress regions have been considered to be important aspects in respect to atherogenesis.

The non-dimensional wall shear stress at any position is computed with the help of the following expression:

$$\tau_w^* = \frac{\tau_w}{\tau_{wref}}$$

Where, τ_{wref} is the reference wall shear stress, which is considered to be the wall shear stress far away from the stenosis.

Figure 3 represents the wall shear stress distributions for the considered stenosis lengths of 0.1, 0.5, 1.0 and 2.0 for typical flow Reynolds number of 200 and 50% restriction only. Curves of wall shear stress show that, at the restriction zone, rise of wall shear stress occurs due to high velocity gradient and then, at the downstream of restriction, it attains negative wall shear stress due to adverse pressure gradient.

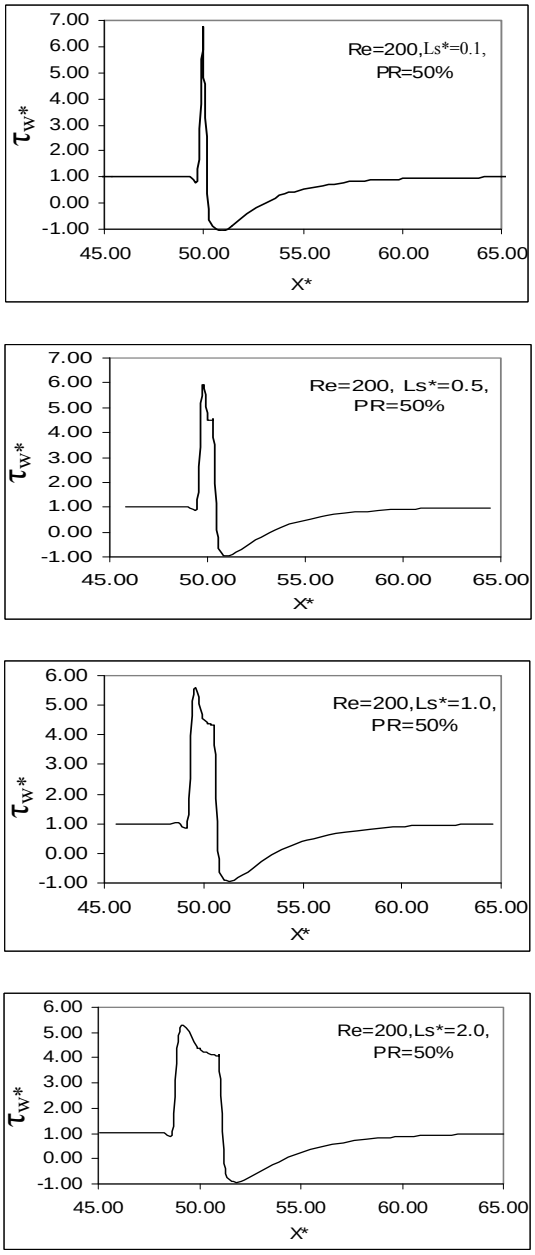

Fig 3. Effect of stenosis length on wall shear stress at Reynolds number of 200 for PR of 50%.

In this study, it is noted that the magnitude of non-dimensional peak wall shear stress is very high for the stenosis length of 0.1 compared to other stenosis lengths of 0.5, 1.0, and 2.0 for the considered Reynolds number and PR, and the magnitude of peak wall shear stress decreases with increase in stenosis length. It is also observed that the magnitude of maximum negative value of wall shear stress remains more or less same with the increase in stenosis length. From the observations, it can be stated that the combined effect of both the peak and low wall shear stress can be considered as the effect of peak wall shear stress only. Therefore, at the time of initiation, i.e. for low value of stenosis length, the chance of damaging to the arterial wall due to peak wall stress is high but it decreases when the stenosis length increases.

4. CONCLUSION

From the present numerical study, it is revealed that during initiation of stenosis length, peak wall shear stress increases appreciably and then its magnitude decreases with the progression of stenosis length. There is no change in the size of recirculation zone with stenosis length.

It is also noted that the size of recirculation zone is not depending on stenosis length. Therefore, it may be mentioned that during initiation of the stenosis length, there may be maximum possibility of lipid deposition on the wall, but the progression of this phenomenon may not take place with further increase in stenosis length. However, the maximum cell turnover point on the arterial wall, due to reattachment point, moves downstream with stenosis length.

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6. NOMENCLATURE

Symbol	Meaning	Unit
A	Area at any section	(m^2)
D	Diameter of the artery	(m)
h_f	Depth of the restriction	(m)
L_i	Inlet length (i.e., length between inlet section and middle of restriction)	(m)
L_{ex}	Exit length (i.e., length between middle of restriction and exit section)	(m)
Ls	Stenosis length	(m)
P	Static pressure	(Nm^{-2})
PR	Percentage of restriction or Percent stenosis = $\frac{2h_f}{D} \times 100\%$	
Pw	Wall pressure	(Nm^{-2})
u	Velocity in x-direction	(ms^{-1})
v	Velocity in y-direction	(ms^{-1})
V_1	Average velocity in x-direction at inlet	(ms^{-1})
τ_w	Wall shear stress	(Nm^{-2})
x, y	Cartesian co-ordinates	
ρ	Density	($kg\ m^{-3}$)
μ	Dynamic viscosity	($kg\ m^{-1}s^{-1}$)
Ψ	Stream function	
Subscripts*	Dimensionless terms	
1-1	Inlet	
2-2	Exit	

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