

## EFFECT OF GAS WEBER NUMBER ON LIQUID SHEET BREAKUP

Mohammad Ali, M. Quamrul Islam and R. Mahamud

Department of Mechanical Engineering, Bangladesh University of Engineering and Technology  
Dhaka, Bangladesh

### ABSTRACT

In this study dynamic behavior of liquid sheet of thermoplastic with co flowing air is discussed and numerically simulated. The Navier-Stokes systems associated with surface tension forces are solved by the Volume of Fluid (VOF) technique with a Continuum Surface Force (CSF) manner. The velocity of liquid is kept constant throughout the study whereas the velocity of air is varies which eventually varies the gas Weber number. Sulphur hexa fluoride ( $SF_6$ ) and high density polyethylene (HDPE) are considered as liquid to investigate the physics of breakup process. The effects of gas Weber number on liquid sheet breakup process are discussed to reveal the underlying physics of liquid disintegration. It is found that under any flow conditions a range of gas Weber number controls the instability for the breakup of liquid sheet. The pressure as well as velocity distribution of the flow field are also discussed to study the breakup processes in details.

**Keywords:** Weber Number, VOF, CSF, Sulphur Hexa Fluoride, HDPE.

### 1. INTRODUCTION

A thermoplastic is a polymer that turns to a liquid when heated and freezes to a very glassy state when cooled sufficiently. Most thermoplastics are high-molecular-weight polymers whose chains are associated through weak Vander Waals forces (polyethylene); stronger dipole-dipole interactions and hydrogen bonding (nylon); or even stacking of aromatic rings (polystyrene). One of the advantages of thermoplastic polymers is that they can be re-melted and re-moulded. Many thermoplastic materials are addition polymers; e.g., vinyl chain-growth polymers such as polyethylene and polypropylene. Thermoplastic is widely used for injection molding, sheet forming, thermal coating, foam processing etc. In previous many experiments and numerical simulations were done on thermoplastic processing. Pham et al [1] performed an experiment to understand the formation of Polyethylene Terephthalate (PET) bottles. He also conducted numerical simulations to study the effects of temperature and strain rate on the strain-hardening properties of the polymer during the stretch blow molding process.

Yijie Wang [2] studied the gas assisted injection molding technique. The displacement of a low viscosity fluid in higher viscosity material causes the formation of a long bubble penetrating through the high viscosity fluid. This left a thin coating on the wall, which had important application in gas-assisted injection molding. The effect of fluid rheological properties on the fractional coverage left by the bubble was studied under isothermal conditions. The polymer solutions of different shear thinning properties were used as high viscosity fluids,

and silicon oil was used as the displacing fluid in his experiment. The result showed that as the absolute viscosity increased, the thickness of the high viscosity fluid on the wall also increased. Gas-assist injection molding is a process that utilizes an inert gas (normally nitrogen) to create one or more hollow channels within an injection-molded plastic part. At the end of the filling stage, the gas ( $N_2$ ) is injected into the still liquid core of the molding. The gas then follows the path of the least resistance and replaces the thick molten sections with gas-filled channels. The gas pressure packs the plastic against the mold cavity surface, compensating for volumetric shrinkage until the part solidifies. Finally, the gas is vented to atmosphere or recycled. Gas-assist injection molding has been around for well over two decades. Now; gas-assist injection molding is widely practiced. Design engineers and processors alike are discovering that this technology is an attractive option for certain applications and offers many benefits.

Early studies of droplets and spray formation from liquid jets issuing into ambient gases can be traced back to the late nineteenth century. Rayleigh [3] showed that liquid jet break-up in still gases is a consequence of hydrodynamic instability. Research on the break-up of liquid jets was continued with much of the theoretical work based on linear instability analysis of absolute and convective instabilities of liquid jets issuing into gaseous environments by Tomotika [4], Chandrasekar [5], Reitz and Bracco [6], and laterly Teng et al. [7], Gordillo and Perez-Saborid [8], Lozano et al. [9] and Lin [10]. Recently, Mehring and Sirignano [11, 12] presented a nonlinear analysis of the spatial temporal development of

axisymmetric capillary waves in a thin annular liquid sheet sheared by fast co-flowing gas streams. Their study showed that the break-up of the annular liquid sheet is initiated by sheet modulation in sinusoidal and dilatational modes. Numerical simulation of the problem showed that there was a nonlinear coupling between the dominant sinusoidal and dilatational mode.

Ali [13] found that the disturbances occurred by the gas Weber number controls the instability process for the liquid sheet breakup. Two modes of forces for liquid stretching were found; one was shear force causing the stretching of liquid by shear velocity and the other was normal force causing the stretching of liquid by gas velocity ahead the tip of the liquid sheet. Stretching of liquid by shear force caused the protrusion of liquid from the tip of liquid sheet at initial stage. The surface tension force made the tip of the sheet to become round and gradually the aerodynamic normal force at the tip of the sheet played an important role to continue the stretching of sheet and controls the formation of droplet and occurrence of sheet breakup.

In this work the dynamical behavior of SF<sub>6</sub> and High Density Polyethylene (HDPE) sheet at high temperature and pressure is discussed in context of capillary phenomena. A linear polymer, HDPE is prepared from ethylene by a catalytic process. High density polyethylene lends well to blow molding, e.g. for bottles, cutting boards, dipping baskets, dippers, trays and containers. HDPE is also somewhat harder and more opaque and it can withstand rather higher temperatures (120° Celsius for short periods, 110° Celsius for continuous). Almost same dynamical behavior of HDPE can be found as SF<sub>6</sub>.

## 2. MATHEMATICAL MODELING

The flow field is governed by time dependent three-dimensional Navier-Stokes equations with surface tension force. Body forces are neglected. These equations can be expressed as

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{P}}{\partial x} + \frac{\partial \vec{Q}}{\partial y} + \frac{\partial \vec{R}}{\partial z} = \frac{\partial \vec{P}_v}{\partial x} + \frac{\partial \vec{Q}_v}{\partial y} + \frac{\partial \vec{R}_v}{\partial z} + \vec{F}_{sv} \quad (1)$$

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \end{pmatrix}, \quad P = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \end{pmatrix}, \quad Q = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \end{pmatrix},$$

$$R = \begin{pmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \end{pmatrix}$$

$$P_v = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{zx} \end{pmatrix}, \quad Q_v = \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \end{pmatrix}, \quad R_v = \begin{pmatrix} 0 \\ \tau_{zx} \\ \tau_{yz} \\ \tau_{zz} \end{pmatrix}$$

$$F_{sv} = \begin{pmatrix} 0 \\ \sigma \kappa f n_x \\ \sigma \kappa f n_y \\ \sigma \kappa f n_z \end{pmatrix}$$

Where,  $F_{sv}$  is the surface tension force,  $\sigma$  the surface tension,  $\kappa$  the curvature of surface,  $n$  the unit normal to the surface and  $f$  is a function for continuous change of the color variable (here density) across the thickness of fluid interface.

## 3. NUMERICAL SIMULATION

A numerical analysis is performed on moving liquid sheet in a gaseous medium (Nitrogen) of much higher velocity compared with the liquid. The effects of co-flowing gas with different Gas Weber number on the dynamical behavior of the liquid sheet of thermoplastic are included in the analysis. For numerical simulation, volume of fluid (VOF) method based on a simplified treatment of the Navier-Stokes equation with a fixed, regular, uniform grid is used to solve the problem. Piecewise Linear Interface Calculation (PLIC) is implemented for the advection of the liquid interface. The treatment of surface tension consists of artificially smoothing the discontinuity present at the interface by Continuum Surface Force (CSF) manner. In the present work the surface tension force is estimated by a volume force which gives the correct surface tension. The volume force is calculated with the area integral over the portion of the interface lying within the small volume of integration. The construction of algorithm for calculation of normal vector is based on the idea that a normal vector together with the fractional volume of one fluid contained by the cell determines a unique planar surface cutting the cell into two parts as shown in Fig.1. Each part of the cell contains proper volume of one of the two fluids. Figure 1 shows a planar surface ABCD, under which the liquid lies in the cell.

To determine the fractional volume and interface position, a parameter is searched which is related to the smallest distance between the planar surface of liquid and the origin of the cell. Therefore this parameter represents the distance along the normal and also defines the planar surface of liquid in the cell. Utilizing this parameter we can determine the area of different sides of the cell occupied by the liquid. A comprehensive description of this calculation can be found in Gueyffier et al. [14], where the description is started with two-dimensions and later it is generalized to three

dimensions for calculation of area as well as volume of the fluid. Both area and volume calculations are continuous, one-to-one, and have a functional relationship with volume inside the cell lying below the planar surface and the parameter which characterizes the plane. For easy calculation of area and volume of the liquid, the cells are identified as three categories: (i) cells with zero value of one component of normal to the planar surface, (ii) cells with zero values of two components of normal to the planar surface, and (iii) cells with non-zero normal components. Algorithm is constructed for these categories of cells with the implementation of all possible logics.

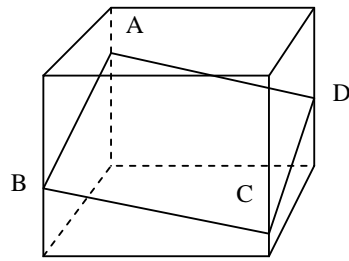


Fig 1. Liquid planar surface, ABCD cuts the cubic cell into two parts

After the construction of liquid interface in cell, its motion in the underlying flow field must be introduced in the algorithm by a suitable advection method. Since the atomization process is very fine and rapid, the simulating code should be capable of capturing all the phenomena behind the atomization process. It can be pointed out that the proper and precise computation of this type of problem is a complex one even when it is two-dimensional case. More complexity arises when the problem is three dimensional. Therefore, calculation of fluid flux should be precise for accuracy of the result. By the motion of the fluid, flux of fluid from one cell to its neighboring cells is calculated. Using the expression described in Gueyffier et al. [14], we can determine the “wetted” area occupied by the liquid in the cell through which the flux of fluid occurs. But this area does not remain constant during one time step. To incorporate this change of flux area, first we need to search the cells in which the “wetted” area is changing during the time step and then calculate the changing area as well as flux of the liquid. It can be pointed out that during calculation, different critical shapes of the cut cube can occur as shown in Fig. 2 where six critical shapes of the cut cubes are presented. Special care must be taken for the advection of liquid from one computational cell to neighboring cells during this occurrence. In present calculation after finding out the cells, the change of flux is calculated with the conjunction of normal to liquid planar surface and geometry of the interface position

The interface where the fluid changes from one fluid to the other discontinuously is replaced by a continuous transition. It is not appropriate to apply a pressure jump induced by surface tension at the interface. Rather, the surface tension should act everywhere within the transition region. In fact, the surface tension contributes a

surface pressure which is the normal force per unit interfacial area. In present work the surface tension force is estimated by a volume force which gives the correct surface tension. The volume force is then calculated with the area integral over the portion of the interface lying within the small volume of integration. A suitable color function (density for present investigation) is chosen for smooth variation over the thickness across the interface. A detail description and the formulation for numerical simulation can be found in Gueyffier et al. [14]. The model is implemented in the algorithm for present investigation.

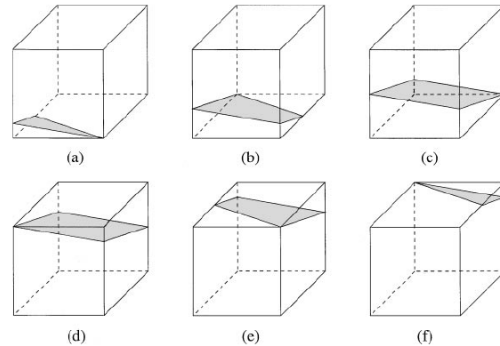
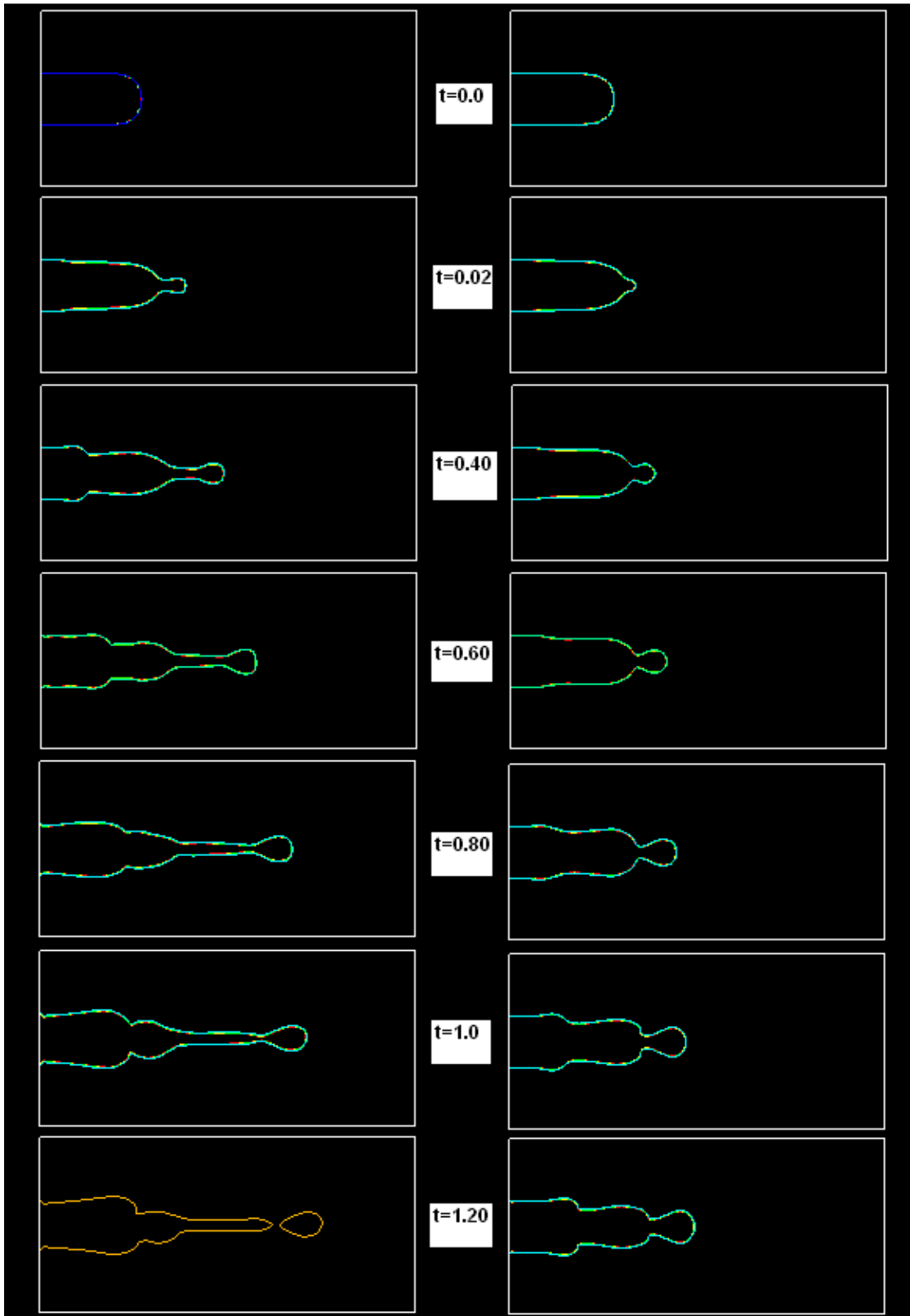


Fig 2. The different critical shapes of the cut cube (Gueyffier et al. [14]). The liquid in the cube changes shape with time and the plane crosses vertices.

## 4. RESULTS AND DISCUSSION

### 4.1 Effect of gas Weber Number

To study the effects of gas Weber number, a numerical study is carried out on water sheet with co-flowing air. It is seen that water sheet does not break without co-flowing air or gas flow. There is a minimum value of gas Weber number for a fixed liquid Weber number below which liquid sheet does not break. This critical value of gas Weber number is 14 for  $SF_6$ . Numerical calculation on various liquid also indicates the similar value of gas Weber number. Over this value of gas Weber number after certain time liquid sheet of HDPE breaks up. It is also found that if velocity of surrounding gas is increased which consequences to the increase of the Weber number of gas tends to precede the breakup processes of liquid sheet. Also if the gas Weber number is increased liquid sheet becomes thinner as the sheet is stretched by the surrounding gas. Throughout the calculation, at the inlet boundary the uniform gas velocity is imposed on top and bottom of the liquid sheet and constant gas pressure is considered for all boundaries of calculation domain. For liquid sheet, at inlet boundary the constant pressure is used. To investigate the effect of Gas Weber number on the liquid sheet breakup mechanism the Weber number of gas is varied from 10 to 100 by keeping constant liquid ( $SF_6$ ) Weber number,  $W_{el}=1$ . Figure 3 shows the breakup process for Gas Weber number of 14 at different time and the velocity profile of the flow field. On the other hand, below the critical gas Weber number only the shape of a drop is forming but no droplet is occurred.



(a) Gas Weber Number 14

(b) Gas Weber Number 10

Fig 3. (a) Weber Number 14 (b) Weber Number 10, Surface contour for SF6 with co-flowing Nitrogen gas (t=dimensionless time)

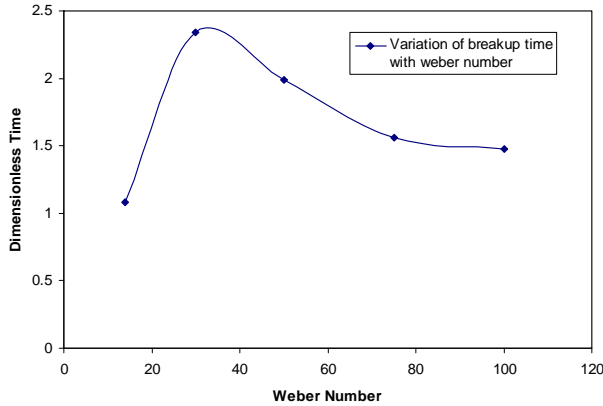


Fig 4. Variation of breakup time with gas Weber Number (SF<sub>6</sub>).

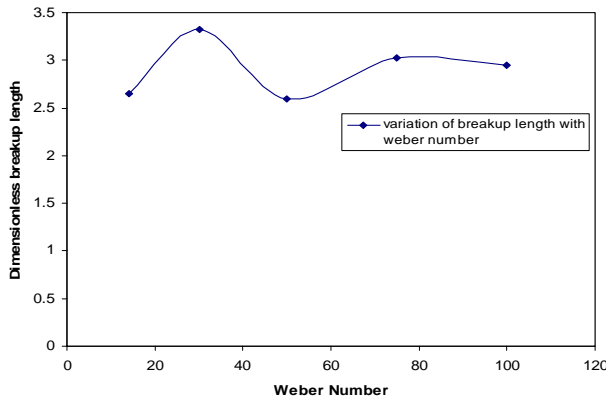


Fig 5. Variation of breakup length with gas Weber number.

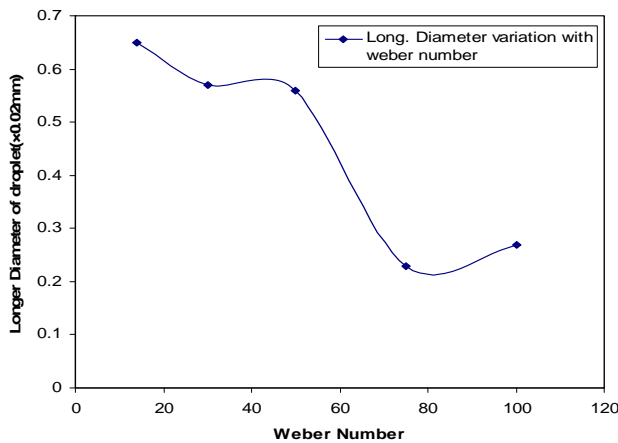


Fig.6 Variation of droplet diameter (along longitudinal direction) with gas Weber Number

#### 4.2 Effect Of Gas Weber Number On Breakup Time

If the gas Weber number increases gradually, the time for breakup decreases as shown in Fig. 4. The increasing of gas Weber number means the gas velocity is increased.

The aerodynamic force, which is the effect of gas velocity, would be dominant if gas Weber number increases. For low gas Weber number the breakup occurs earlier caused by the capillary instability. For much higher gas Weber number the aerodynamic force is dominant which acts on the surface of the liquid sheet and also on tip of the liquid sheet. The interaction between liquid and air also increases. It causes the breakup to be held earlier.

#### 4.3 Effect of gas weber number on breakup length

The length at which breakup occur increases if the gas Weber number increases as shown in Fig.5. Since liquid sheet is stretched at high gas Weber number, it causes the liquid sheet elongated at high gas Weber number.

#### 4.4 Effect of gas weber number on droplet diameter

The droplet diameter decreases at high gas Weber number as shown in Fig.6 caused by the aerodynamic force. The shape changes from round to rectangular at high gas Weber number.

### 5. CONCLUSIONS

A two dimensional study has been performed to discuss the pattern of liquid SF<sub>6</sub> as well as thermoplastic sheet for the study of injection molding and coating of thermoplastic and some other uses. The Navier-Stokes equations with surface tension force are used for numerical simulation and volume-of-fluid (VOF) techniques are applied for the advection of liquid from one computational cell to the neighboring cell. For smoothing the discontinuity present at the interface of the cells, a Continuum Surface Force (CSF) manner is adopted for the treatment of surface tension. The velocity of liquid and gas are determined from liquid and gas Weber number, respectively. Keeping constant the liquid Weber number, the gas Weber number is varied to investigate the range of gas Weber number under which breakup and droplet are possible. Two effects are activated on the flow field: aerodynamic effect and surface tension effect. The aerodynamic effect is caused by the interaction between the gas and liquid surface of the sheet and plays an important role for the formation and breakup of liquid droplet. When aerodynamic effect prevails over surface tension effect, the disturbances grow on liquid surface and eventually break up and droplet occurs. In that case surface tension force helps to form a droplet and make it bigger in diameter, and keeps it in a round shape. But when surface tension force is prevailing no breakup is occurred.

The aerodynamic force acts on both surfaces of the liquid sheet as well as on the tip of the liquid sheet. Due to interaction of aerodynamic force with the liquid on the surface of the sheet, liquid sheet is stretched and a protrusion on the tip of the sheet occurs. If the aerodynamic force is insufficient to keep the drop as its regular shape, the drop gradually losses its shape and eventually the drop are merged into the liquid sheet by surface tension force. The gas velocity ahead the liquid sheet, which is the consequences of aerodynamic effect,

varies with time. If the gas velocity is low enough, no droplet and breakup of liquid occur. If the gas velocity is high enough, the liquid protrusion length increases so much which causes the reduction of droplet diameter and losses the usual breakup procedure. This causes the stretching the liquid sheet so much and create an uneven pattern of sheet. Hence, a range of velocity can produce droplet and make a breakup of the liquid.

## 6. REFERENCES

1. Pham, X.-T. ; Thibault, F.; Lim, L.-T., 2004, "Modeling and simulation of stretch blow molding of polyethylene terephthalate", *Polymer Engg. and Science*, Vol.44, No.8, pp.1460-1472.
2. Yijie Wang, 2003, "The Effect of Non-Newtonian Rheology on Gas assisted Injection Molding Process", Ph. D. Thesis, Ohio State University, USA.
3. Rayleigh, L., 1879, "On the capillary phenomenon of jets", *Proc Royal Soc. London, Math* 29, pp.71–97.
4. Tomotika, S., 1935, "On the instability of a cylindrical thread of a viscous liquid surrounded by another viscous fluid", *Proc. Royal Soc. London, Series A* 150, pp.322–337.
5. Chandrasekar, S., 1961, "The capillary instability of a liquid jet with Hydrodynamic and Hydromagnetic stability", Oxford University Press, pp 537–542.
6. Reitz R. D, and Bracco, F. V., 1982, "Mechanisms of Breakup of Round Liquid Jets", *Phys Fluids*, Vol. 25, pp.1730–1742.
7. Teng, C. H., Lin, S. P., and Chen, J. N., 1997, "Absolute and convective instability of a viscous liquid curtain in a viscous gas", *J. Fluid Mech* Vol. 332, pp.105–120.
8. Gordillo, J. M., and Perez-Saborid, M., 2005, "Aerodynamic effects in the break-up of liquid jets: on the first wind-induced break-up regime"; *J. Fluid Mech.*, Vol. 541, pp.1–20.
9. Lozano, A., Barreras, F., Hauke, G., Dopazo, C., 2001, "Longitudinal instabilities in an air-blasted liquid sheet", *J. Fluid Mech.*, Vol. 437, pp.143– 173.
10. Lin, S. and Reitz, R., 1998, "Drop and spray formation from a liquid jet", *Annu. Rev. Fluid Mech.* Vol. 30, pp.85–105.
11. Mehring, C., and Sirignano, W. A., 1998, "Nonlinear capillary wave distortion and disintegration of thin planar liquid sheets", *J. Fluid Mech.*, Vol. 388, pp.69–113.
12. Mehring, C., and Sirignano, W. A., 2000, "Axisymmetric capillary waves on thin annular liquid sheets I: temporal stability", *Phys. Fluids*, Vol. 12(6), pp.1417–1439.
13. Mohammad Ali and Akira Umemura, 2008, "Dynamics and Breakup of Liquid Sheet", Research report, Department of Aerospace Engineering, Graduate School of Engineering, Nagoya University, Japan.
14. Gueyffier, D., Li, J., Nadim, A., Scardovelli, R. and Zaleski, S., 1999, "Volume-of-Fluid Interface Tracking with Smoothed Surface Stress Methods for Three-Dimensional Flows", *J. Computational Physics*, Vol. 152, pp.423-456.

## 7. MAILING ADDRESS

Mohammad Ali  
 Department of Mechanical Engineering,  
 Bangladesh University of Engineering and Technology  
 Dhaka – 1000, Bangladesh,  
 Email: mali@me.buet.ac.bd.