Proceedings of the
International Conference on Mechanical Engineering 2009
(ICME2009) 26- 28 December 2009, Dhaka, Bangladesh

ICME09-FM-31

EFFECTS OF RADIATION AND PRESSURE WORK ON MHD NATURAL CONVECTION FLOW AROUND A SPHERE

M. A. Alim[1] and Tahmina Akhter[2]

[1]Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh
[2]Department of Mathematics, The University of Asia Pacific, Dhaka, Bangladesh

ABSTRACT

The effects of radiation and pressure work on magnetohydrodynamic (MHD) natural convection flow on a sphere have been investigated in this paper. The governing boundary layer equations are first transformed into a non-dimensional form and the resulting nonlinear partial differential equations are then solved numerically using finite-difference method with Keller-box scheme. We have focused our attention on the evaluation of shear stress in terms of local skin friction and rate of heat transfer in terms of local Nusselt number, velocity as well as temperature profiles. Numerical results have been shown graphically and tabular form for some selected values of parameters set consisting of radiation parameter Rd, pressure work parameter Ge, Magnetohydrodynamic parameter M and the Prandtl number Pr.

Keywords: Thermal Radiation, Natural Convection, Pressure Work, Magnetohydrodynamics.

1. INTRODUCTION

Radiative energy passes perfectly through a vacuum thus radiation is significant mode of heat transfer when no medium is present. Radiation contributes substantially to energy transfer in furnaces, combustion chambers, fires and to the energy emission from a nuclear explosion. Radiation must be considered in calculating thermal effects in rocket nozzles, power plants, engines, and high temperature heat exchangers. Radiation can sometimes be important even though the temperature level is not elevated and other modes of heat transfer are present. Radiation has a great effect in the energy equation which leads to a highly non-linear partial differential equation.
Magnetohydrodynamic (MHD) is the science, which deals with the motion of conducting fluid in presence of a magnetic field. Study of the flow of electrically conducting fluid in presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researchers. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force which tends to oppose the fluid motion.

The pressure work effect plays an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotational speeds and also in strong gravitational field processes on large scales and in geological processes. It is established that pressure work effects are generally rather more important both for gases and liquids.

The problems of natural convection boundary layer flow have been studied by many researchers. Amongst them Nazar et al [1], Huang and Chen [2] considered the free convection boundary layer on an isothermal sphere and on an isothermal horizontal circular cylinder both in a micropolar fluid. Alim et al. [3-4] considered the pressure work effects along a circular cone and stress work effects on MHD natural convection flow along a sphere. Alam et al [5-7] considered the pressure work effects for flow along vertical permeable circular cone, vertical flat plate and along a sphere.

Soundalgekar et al. [8] have studied radiation effects on free convection flow of a gas past a semi-infinite flat plate using the Cogley-Vincenti-Giles equilibrium model Cogley et al. [9], later Hossain and Takhar [10] have analyzed the effects of radiation using the Rosseland diffusion approximation which leads to non-similar solutions for free convection flow past a heated vertical plate. Akhter and Alim [11] studied the effects of radiation on natural convection flow around a sphere with uniform surface heat flux.

In the present work, the effects of radiation and pressure work on MHD natural convection flow around a sphere have been investigated. The transformed boundary layer equations are solved numerically using implicit finite difference method together with Keller box scheme describe by Keller [12] and later by Cebeci and Bradshaw [13]. Numerical results have been shown in terms of local skin friction, rate of heat transfer, velocity profiles as well as temperature profiles for a selection of relevant physical parameters consisting of heat radiation parameter Rd, Prandtl number Pr, Magnetic parameter M and the pressure work parameter Ge have been shown graphically.

2. FORMULATION OF THE PROBLEM

It is assumed that the surface temperature of the sphere is Tw, where $T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, r (x) is the radial distance from the symmetrical axis to the surface of the sphere

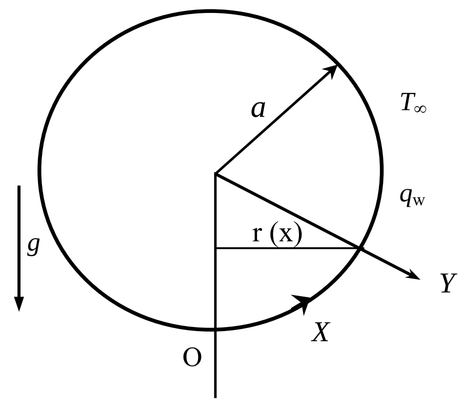

Fig 1. Physical model and coordinate system

and (u, v) are velocity components along the (x, y) axis.

Under the usual Bousinesq approximation, the equations those govern the flow are

$$\frac{\partial}{\partial X}(rU)+\frac{\partial}{\partial Y}(rV) \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\nu\frac{\partial^2 U}{\partial Y^2} +\rho g\beta(T-T_\infty)\sin\left(\frac{X}{a}\right)-\frac{\sigma_0\beta_0{}^2}{\rho}U \tag{2}$$

$$U\frac{\partial \mathrm{T}}{\partial X}+V\frac{\partial \mathrm{T}}{\partial Y}=\frac{\mathrm{k}}{\rho \mathrm{C_p}}\frac{\partial^2 \mathrm{T}}{\partial Y^2} -\frac{1}{\rho \mathrm{C_p}}\frac{\partial \mathrm{q_r}}{\partial Y}+\frac{T\beta}{\rho c_p}U\frac{\partial p}{\partial X} \tag{3}$$

We know for hydrostatic pressure, $\partial p/\partial X = \rho g$.

The boundary conditions of equation (1) to (3) are

$$\begin{aligned} &U=V=0,\quad T=T_w \quad \text{at} \quad Y=0 \\ &U\to 0,\ T\to T_\infty \quad \text{as} \quad Y\to\infty \end{aligned} \tag{4}$$

where r is the density, k is the thermal conductivity, b is the coefficient of thermal expansion, m is the viscosity of the fluid, Cp is the specific heat due to constant pressure and qr is the radiative heat flux in the y direction. In order to reduce the complexity of the problem and to provide a means of comparison with future studies that will employ a more detail representation for the radiative heat flux; we will consider the optically dense radiation limit. Thus the Rosseland diffusion approximation proposed by Siegel and Howell [15] and is given by simplified radiation heat flux term as:

$$q_r=-\frac{4\sigma}{3(\alpha_r+\sigma_s)}\frac{\partial T^4}{\partial Y} \tag{5}$$

We now introduce the following non-dimensional variables:

$$\begin{aligned} &\xi=\frac{\mathrm{X}}{\mathrm{a}},\ \eta=\mathrm{Gr}^{1/4}\left(\frac{\mathrm{Y}}{\mathrm{a}}\right), \\ &\mathrm{u}=\frac{\mathrm{a}}{\nu}\mathrm{Gr}^{-1/2}\mathrm{U},\quad \mathrm{v}=\frac{\mathrm{a}}{\nu}\mathrm{Gr}^{-1/4}\mathrm{V}, \\ &\theta=\frac{\mathrm{T}-\mathrm{T}_\infty}{\mathrm{T_w}-\mathrm{T}_\infty},\ \mathrm{Gr}=\mathrm{g}\beta\left(\mathrm{T_w}-\mathrm{T}_\infty\right)\frac{\mathrm{a}^3}{\upsilon^2} \end{aligned} \tag{6}$$

where ν (= μ/ρ) is the reference kinematic viscosity and Gr is the Grashof number, θ is the non-dimensional temperature function.

Substituting variable (6) into equations (1)-(3) leads to the following non-dimensional equations

$$\frac{\partial}{\partial\xi}(ru)+\frac{\partial}{\partial\eta}(rv)=0 \tag{7}$$

$$u\frac{\partial u}{\partial\xi}+v\frac{\partial u}{\partial\eta}=\frac{\partial^2 u}{\partial\eta^2}+\theta\sin\xi-Mu \tag{8}$$

$$u\frac{\partial\theta}{\partial\xi}+v\frac{\partial\theta}{\partial\eta}= \frac{1}{\Pr}\frac{\partial}{\partial\eta}\left[\left\{1+\frac{4}{3}Rd\left(1+\Delta\theta\right)^3\right\}\frac{\partial\theta}{\partial\eta}\right] +Ge\left(\theta+\frac{T_\infty}{T_w-T_\infty}\right)u \tag{9}$$

Where $M=\dfrac{\sigma_0\beta_0^2 a^2}{\rho\nu Gr^{1/2}}$ is the Magnetohydrodynamic parameter and $\Delta=\dfrac{T_w}{T_\infty}-1$

With the boundary conditions (4) as

$$\begin{aligned} &u=v=0,\ \theta=1\ \ at\ \ \eta=0 \\ &u\to 0,\ \ \theta\to 0\ \ \ as\ \ \eta\to\infty \end{aligned} \tag{10}$$

where Rd is the radiation-conduction parameter and Pr is the Prandtl number defined respectively as

$$Rd=\frac{4\sigma T_\infty{}^3}{k\left(\alpha_r+\sigma_s\right)}\ and\ \ \Pr=\frac{\mu C_p}{k} \tag{11}$$

To solve equations (8)-(9), subject to the boundary conditions (10), we assume the following variables

$$\psi=\xi\ r(\xi)\ f(\xi,\eta),\ \ \theta=\theta(\xi,\eta) \tag{12}$$

where ψ is the non-dimensional stream function defined in the usual way as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial\eta}\ \ \text{and}\ \ v=-\frac{1}{r}\frac{\partial\psi}{\partial\xi} \tag{13}$$

Substituting (13) into equations (8)-(9), after some algebra the transformed equations take the following form

$$\frac{\partial^3 f}{\partial \eta^3}+\left(1+\frac{\xi}{\sin\xi}\cos\xi\right)f\frac{\partial^2 f}{\partial \eta^2}-\left(\frac{\partial f}{\partial \eta}\right)^2$$
$$+\frac{\sin\xi}{\xi}\theta-M\frac{\partial f}{\partial \eta}=\xi\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial\xi\partial \eta}-\frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2}\right) \quad (14)$$

$$\frac{1}{\Pr}\frac{\partial}{\partial\eta}\left[\left\{1+\frac{4}{3}Rd\left(1+\Delta\theta\right)^3\right\}\frac{\partial\theta}{\partial\eta}\right]$$
$$+\left(1+\frac{\xi}{\sin\xi}\cos\xi\right)f\frac{\partial\theta}{\partial\eta}+Ge\left(\theta+\frac{T_\infty}{T_w-T_\infty}\right)\xi f' \quad (15)$$
$$=\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial\theta}{\partial\eta}\frac{\partial f}{\partial\xi}\right)$$

Along with boundary conditions

$$f=\frac{\partial f}{\partial\eta}=0,\ \theta=1\ \ at\ \ \eta=0$$
$$\frac{\partial f}{\partial\eta}\to 0,\ \ \theta\to 0\quad as\quad \eta\to\infty \quad (16)$$

The physical quantities of principal interest are the wall-shear-stress, the heat transfer rate in terms of the skin-friction coefficients C_f and Nusselt number Nu_x respectively, which can be written as

$$C_f=\frac{Gr^{-3/4}\,a^2}{\rho\nu}\tau_w \text{ and } Nu=\frac{a\,Gr^{-1/4}}{k\left(T_w-T_\infty\right)}q_w \quad (17)$$

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (18)$$

Here we have used a reference velocity $U=\frac{\nu Gr^{1/5}}{a}$

Using the variables (6) and (13) into (17)-(18), we get

$$C_f=\xi f''(\xi,0) \quad (19)$$

$$Nu=-\left(1+\frac{4}{3}Rd\theta_w^3\right)\theta'(\xi,0) \quad (20)$$

The values of the velocity and temperature distribution are calculated respectively from the following relations:

$$u=\frac{\partial f}{\partial\eta},\quad \theta=\theta\left(\xi,\eta\right) \quad (21)$$

3. RESULTS AND DISCUSSION

The present problem has been solved numerically for different values of relevant physical parameters and for a fixed value of $\Delta = 0.1$. Results have been obtained in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity as well as temperature profiles.

Velocity and temperature profiles are shown in figures 2 for different values of radiation parameter Rd while Pr = 7.0 , M=1.0 and Ge = 1.5. It is also observed that for higher values of radiation the velocity and temperature becomes higher and there is no significant change found in the boundary layer thickness.

Effects of the variation of pressure work on velocity and temperature profiles are shown in the figures 3. Significant changes have been found in maximum velocity and temperature due to the change of Ge. For Ge = 0.1 the maximum velocity is .19573 which occurs at η = 0.88811 and for Ge = 2.5 the maximum velocity is 0. .77787 which occurs at η = 0.78384. Thus we observe that due to the change of Ge from 0.1 to 2.5 the velocity rises up 297.4%. Again small value of Ge (= 0.1) gives the typical temperature profile which is maximum temperature at wall then it gradually decrease along η direction and finally approaches to the asymptotic value (zero). But larger values of Ge do not show the typical temperature profiles. In this case along η direction temperature gradually increased from the wall value to the peak and then decrease and approach to the asymptotic value.

Moreover, in the figure 4 it is observed that the velocity decrease with increasing M but the temperature increase along η direction up to the maximum value and then gradually decreases to zero.

Figure 5 show the skin friction and rate of heat transfer against ξ for different values of radiation parameter Rd. From this figure we observe that skin friction becomes lower for higher values of radiation, also along ξ direction skin friction always positively increasing and no separation occurs within the region of our consideration that is $0 \le \xi \le \pi/2$. There may be flow separation beyond this value of ξ which has not been investigated in this study due to numerical instability. Again the rate of heat transfer increase, between $\xi = 0.0$ to 0.125 intersect at $\xi = 0.125$ and then decrease for higher values of Rd within the region $\xi > 0.125$. In this figure we found both positive and negative Nusselt numbers. When the wall temperature is higher that the fluid the Nusselt number is positive but when the fluid temperature is higher than the temperature of wall the Nusselt number will be negative which may occur due to the imposed conditions on the problem.

Figure 6 shows the skin friction coefficient C_f for different values of pressure work parameter Ge. It is observed from the figure that the pressure work have great influence on skin friction as well as on the rate of heat transfer. Frictional force at the wall becomes much higher towards the downstream for higher values of Ge and the rate of heat transfer as shown in 6 gradually decreased for higher values of pressure work parameter.

From figure 7 it is observed that the rise of magnetohydrodynamic parameter, M leads to decrease the skin friction and the rate of heat transfer. Which also reduce the gradient of heat transfer and the rate of heat transfer at the leading edge, as a result the rate of heat transfer becomes same at the point $\xi = 1.0$. That is, the rate of heat transfer increases between $\xi = 0.0$ to 1.0 intersect at $\xi = 1.0$ and then decrease for increasing M while $\xi > 1.0$.

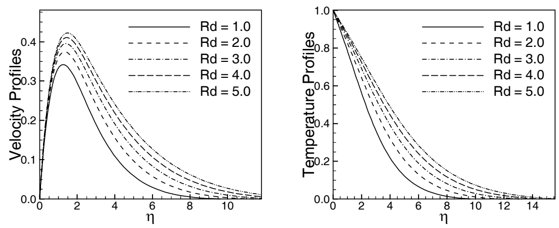

Fig 2. Velocity and temperature profiles for different values of Rd when Pr = 7.0, M = 1.0 and Ge=0.1

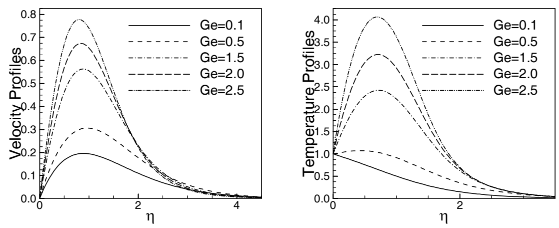

Fig 3. Velocity and temperature profiles for different values of Ge when Pr = 7.0, M = 1.0 and Rd =1.0

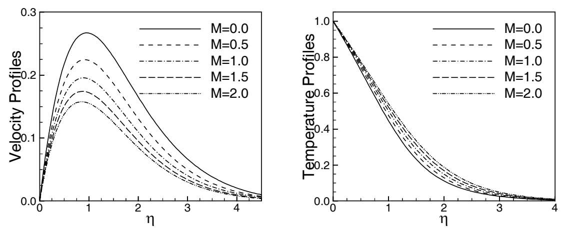

Fig 4. Velocity and temperature profiles for different values of M when Pr = 7.0, Rd = 1.0 and Ge=0.1

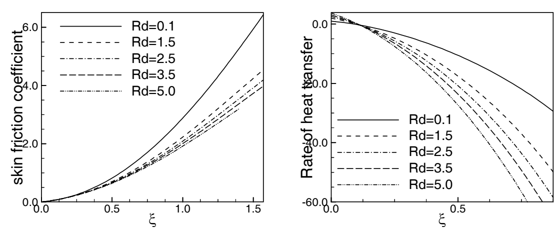

Fig 5. Skin friction and Rate of heat transfer for different values of Rd when Pr = 7.0, M = 1.0 and Ge=0.1

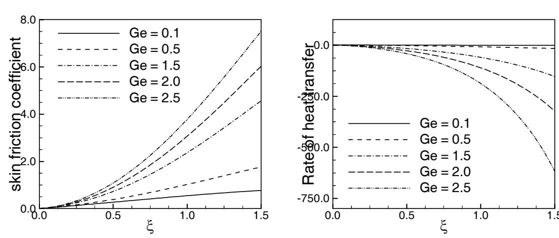

Fig 6. Skin friction and Rate of heat transfer for different values of Ge when Pr = 7.0, M = 1.0 and Rd=1.0

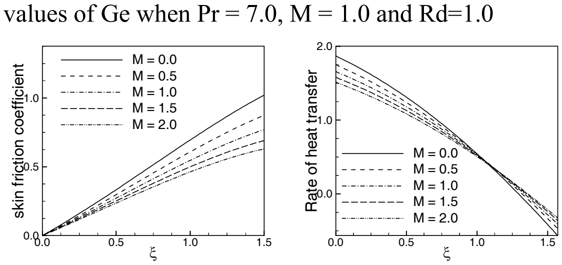

Fig 7. Skin friction and Rate of heat transfer for different values of M when Pr = 7.0, Ge=0.1 and Rd=1.0

4. CONCLUSION

MHD Natural convection flow around a sphere has been studied with the effects of pressure work and radiation. From the present investigation the following conclusions may be drawn:

The velocity as well as the temperature becomes higher for higher values of radiation, which also leads to increase the boundary layer thickness. Also the skin friction becomes lower for higher values of radiation and rate of heat transfer increase near the wall and then decrease for higher values of Rd.

Due to the increase of pressure work parameter Ge the velocity rises up. For larger values of Ge the temperature profiles change its typical nature, such as along η direction temperature gradually increase from the wall value to the peak and then decrease and approach to the asymptotic value. Also the frictional force at the wall becomes much higher towards the downstream for higher values of Ge and the rate of heat transfer gradually decreased for higher values of pressure work parameter.

The rise of magnetohydrodynamic parameter, M leads to decrease the velocity, increase the temperature and also reduce the skin friction. The rate of heat transfer shows regular profile before they cross at the point $\xi = 1.0$ and then show an irregular form.

5. REFERENCES

1. Nazar, R. Amin, N. Grosan, T. and Pop, I., 2002, "Free convection boundary layer on an isothermal sphere in a micropolar fluid", Int. Comm. Heat Mass Transfer, 29 (3): 377-386.
2. Huang, M.J. and Chen, C.K., 1987, "Laminar free convection from a sphere with blowing and suction", J. Heat Transfer, 109:529-532.
3. Alim, M. A., Alam, Md. M. and Chowdhury, Md. M. K., 2006, "Pressure work effect on natural convection flow from a vertical circular cone with suction and non-uniform surface temperature", Journal of Mechanical Engineering, The Institution of Engineers, Bangladesh, 36:6-11.
4. Alim, M. A., Alam, Md. M. and Chowdhury, Md. M. K., 2008, "Work Stress Effects on MHD Natural Convection Flow Along a Sphere", Thammasat International Journal of Science and Technology, 13(1):1-10.
5. Alam, Md. M., Alim, M. A. and Chowdhury, Md. M. K., 2007, "Free convection from a vertical permeable circular cone with pressure work and non-uniform surface temperature", Nonlinear Analysis: Modelling and Control, 12(1):21-32.
6. Alam, Md. M., Alim, M. A. and Chowdhury, Md. M. K., 2006, "Effect of pressure stress work and viscous dissipation in natural convection flow along a vertical flat plate with heat conduction", Journal of Naval Architecture and Marine Engineering, 3(2):69-76.
7. Alam, Md. M., Alim, M. A. and Chowdhury, Md. M. K., 2007, "Viscous dissipation effects with MHD natural convection flow on a sphere in presence of

heat generation, Nonlinear Analysis: Modelling and Control", 12(4):447-459.

8. Soundalgekar, V. M., Takhar, H. S. and Vighnesam, N.V., 1960, "The combined free and forced convection flow past a semi-infinite vertical plate with variable surface temperature", Nuclear Engineering and Design , 110:95-98.
9. Cogley, A.C., Vincenti, W.G. and Giles, S.E.,1968, "Differential approximation for radiation transfer in a nongray near equilibrium", AIAA Journal, 6:551–553.
10. Hossain, M. A. and Takhar, H. S, 1996, "Radiation effect on mixed convection along a vertical plate with uniform surface temperature", Heat and Mass Transfer, 31:243-248.
11. Akhter, Tahmina and Alim, M. A., 2008, "Effects of Radiation on Natural Convection Flow around a Sphere with Uniform Surface Heat flux", Journal of Mechanical Engineering, The Institution of Engineers, Bangladesh, 39(1):50-56.
12. Keller, H. B., 1978, "Numerical methods in boundary layer theory", Annual Rev. Fluid Mechanics, 10:417-433.
13. Cebeci, T. and Bradshaw, P., 1984, "Physical and Computational Aspects of Convective Heat Transfer", Springer, New York.
14. Siegel, R. and Howell, J. R., 1972, "Thermal Radiation Heat Transfer", McGraw-Hill, New York.

6. NOMENCLATURE

Symbol	Meaning	Unit
a	Radius of the sphere	(m)
C_f	Skin-friction coefficient	
C_p	Specific heat at constant pressure	($Jkg^{-1}k^{-1}$)
f	Dimensionless stream function	
g	Acceleration due to gravity	(ms^{-2})
Gr	Grashof number	
k	Thermal conductivity	($wm^{-1}k^{-1}$)
M	Magnetohydrodynamic parameter	Dimensionless
Nu	Nusselt number	Dimensionless
Pr	Prandtl number	Dimensionless
q_r	Radiative heat flux	(W/m^2)
R_d	Radiation parameter	Dimensionless
r	Distance from the symmetric axis to the surface	(m)
T	Temperature of the fluid in the boundary layer	(K)
T_∞	Temperature of the ambient fluid	(K)
T_w	Temperature at the surface	(K)
U	Velocity component along the surface	(ms^{-1})
V	Velocity component normal to the surface	(ms^{-1})
u	Dimensionless velocity along the surface	
v	Dimensionless velocity normal to the surface	
X	Coordinate along the surface	(m)
Y	Coordinate normal to the surface	(m)
α_r	Rosseland mean absorption co-efficient	(cm^3/s)
β	Volumetric coefficient of thermal expansion	(K^{-1})
η	Dimensionless Coordinates	
θ	Dimensionless temperature	
μ	Dynamic viscosity of the fluid	($kgm^{-1}s^{-1}$)
ν	Kinematic viscosity	(m^2/s)
ξ	Dimensionless co-ordinates	()
ρ	Density of the fluid	(kgm^{-3})
σ	Stephan Boltzmann constant	($js^{-1}m^{-2}k^{-4}$)
σ_s	Scattering coefficient	(m^{-1})
τ_w	Wall-shear-stress	(N/m^2)
ψ	Stream function	(m^2s^{-1})

7. MAILING ADDRESS

M. A. Alim
Department of Mathematics,
Bangladesh University of Engineering and Technology,
Dhaka-1000, Bangladesh.
E-mail: maalim@math.buet.ac.bd