

## ANALYSIS OF A NEGATIVELY BUOYANT JET

Chandan Kumar, D. Das and M.M. Razzaque

Department of Mechanical Engineering, Bangladesh University of Engineering and Technology (BUET),  
Dhaka, Bangladesh.

### ABSTRACT

The behavior of plane fountains, resulting from the injection of dense fluid (water) upwards into a large container of homogeneous fluid of lower density (air), is investigated. In this study we experimentally examine the behavior of fountains for different Froude and Reynolds numbers. The fountain inlet flow rate and nozzle diameter of the inlet fluid is varied to cover a wide range of Reynolds and Froude numbers. Fountain behavior is observed by changing the inclination angle of the fountain for different nozzle diameter and flow rate. We found that the penetration height greatly depends on Froude number. We develop an empirical correlation of the non-dimensional fountain height with Froude number ( $\frac{H}{D} = 1.452Fr^{1.94}$ ). But well defined by both Reynolds and Froude number. Finally we also develop an empirical correlation of the non-dimensional fountain height Reynolds and Froude number ( $\frac{H}{D} = 0.0001Re^{-0.72}Fr^{2.26}$ ) by numerical regression. The result are compared with previous numerical and experimental results and found consistent.

**Keywords:** Buoyant Jet, Negatively Buoyant Jet, Fountain, Positively Buoyant Jet, Reynolds Number.

### 1. INTRODUCTION

Negatively buoyant jets occur when a fluid is injected into another fluid of differing density where the buoyancy force opposes the momentum of the flow. In our case water (denser) is injected upward into air (lighter).

The injected fluid penetrates a distance into the environment before stagnating and falling back around itself. The characteristics of the fountain flow, such as the penetration height and the amount of mixing, depend on the Froude number and Reynolds number, which for round fountains are defined as

$$Fr = \frac{v}{\sqrt{Dg}}$$

and

$$Re = \frac{\rho v D}{\mu}$$

respectively, where 'D' is the characteristics dimension or hydraulic diameter of the source and. The characteristic velocity 'v' is defined here as  $v = Q_0/\pi r^2$ , where  $Q_0$  is the volume flow rate at the fountain source;  $\mu$  is the dynamic viscosity of the fluid at the fountain source, and  $\rho$  is the density of fluid of fountain source .

In this study we experimentally examine the behavior of a negatively buoyant jet for wide range of Froude and Reynolds numbers.

There have been numerous studies investigating fountain behavior. Significant contributions have been made by Turner (1966), Abraham (1967), Mizushima et al. (1982),

Baines et al. (1990) and Bloomfield & Kerr (2000). The consensus in these works, on both dimensional (Turner 1966), analytical (Abraham 1967) and experimental grounds (Turner 1966; Mizushima et al. 1982; Baines et al. 1990.

Some of them correlate non-dimensional fountain height with Froude number and both with Reynolds and Froude number. Most of the previous work was done in the context of Boussinesq approximation where density differences are sufficiently small to be neglected. For example they study the fountain created by injecting salt water up into fresh water (n.williamson, n.srinarayana, s.armfield, g,d Mcbain), flow of hot gas downward into air. In this case densimetric Froude number is used.

Our objectives are to study the fountain created by non-Boussinesq approximation injecting water up into air. In our study density difference is not small. We aimed to investigate the behavior of the jet with different Reynolds number and Froude number. Our also aimed to see the effect of inclination angle on the fountain behavior.

Finally the effect Reynolds number and Froude number on penetration height of jet is investigated. By numerical regression analysis we develop an empirical correlation of penetration height with Reynolds and Froude number.

## 2. EXPERIMENTAL SETUP

Experiments have been performed by injecting water into air up in an observation tank. The observation tank is 12inch deep with 12 inch square base. The water is fed from a header tank to the base of the observation tank as shown in the arrangement in figure 1. The water is injected from a sudden start and maintained at a constant flow rate throughout the experiment. A ball valve is used to control the flow.

The observation is made of transparent plastic glass to observe the jet behavior. In the base of the observation tank there are two holes. One is to collect the dispersed water from nozzle and another is to supply water to nozzle.

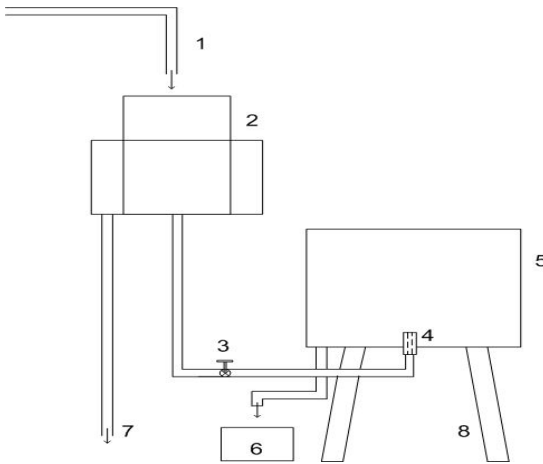


Fig 1. Schematic representation of experimental setup with 1: supply line, 2: header tank, 3: control valve, 4: nozzle, 5: observation tank, 6: collection bucket, 7: drain line, 8: stand.

A protractor is attached at the lower part of the observation tank inline with base. Inclination angle is measured by this protractor by holding a set square vertically and take the reading.

Header tank consist of two tank one act as a supply tank and another is act as an overflow tank. In the supply tank the head of water remains always constant thus supplying constant flow throughout the experiment. The excess water of the supply tank flow to overflow tank. The water in overflow tank is drained by a pipe. In the supply tank water is supplied at a rate higher than the flow rate need to make the fountain. The excess water spill over from the supply tank to overflow tank thus the head of water remains same in the supply tank.

Water flows from header tank to observation tank through a pipe. In the middle of this pipe a ball valve is connected to control the flow rate. A solid ball with hollow section is rotated with to adjust the flow. Here ball valve is used to control the flow rate for to vary Reynolds and Froude number.

The nozzle in our experiment was a straight nozzle there is no variation of cross sectional area. In the source side of the nozzle is stepped as Christmas tree to prevent leakage. Diameter of middle section of the nozzle is higher than two sections to fix with the observation tank. We use five nozzle of different diameter. Nozzle inner diameter of 2.50mm, 3.00mm, 3.50mm, 4.00mm and 4.70mm were used. The outer side of the nozzle is same for all nozzles. Five nozzles are manufactured at BUET machine shop and are made of mild steel.

Platform scales are instruments used for measuring weight. It may be various types. In our experiment portable beam scale type platform scale was used.

## 3. OBSERVATIONS

The experiments were performed over the ranges  $3.00 < Fr < 13.00$  and  $1500 < Re < 8500$ , and the penetration height and the fountain behavior was observed carefully. It is observed that when the nozzle is purely vertical i.e. no inclination in jet there is rapid fluctuation in fountain height but in case of inclined jet height is constant for a Reynolds and Froude number. In case of vertical jet fountain front continually rises until its kinetic energy becomes zero, then stagnates here kinetic energy is zero but potential energy is maximum, and then collapses around the next rising front, so the next rising front can not reach its maximum height. As a result fluctuation occurs in a vertical jet. On the other hand height of inclined jet remains almost constant in all time for a Reynolds and Froude number.

But a peculiar and very interesting phenomenon occurs in the low Reynolds number for every nozzle. We observed that inclined jet also fluctuate for a Reynolds and Froude number. This is due to cohesion between rising and falling water. As the height decrease the distance between the falling and rising column decreases for some certain angle. It is observed that at some state the rising and falling column mix at the top of the jet due to cohesion. Due to this interaction and mixing between rising and falling column, decreased in vertical momentum of the injected fluid occur, the fountain height decrease. The height of the fluctuation depends upon inclination angle and nozzle diameter. This Fluctuation height increases with diameter decrease. And fluctuation height increases with inclination angle increases. This behavior is due a combination of the decreased interaction between the up-flow and the down-flow when inclination angle increase.

The rising column of every fountain can be categorized of three segments as rising or initial stage, intermediate or wobbling stage and stagnating stage. At rising stage fluid column is pure stream of jet no variation in diameter of the jet. But in intermediate stage fluid column starts wobbling. In this stage diameter of jet increases gradually as height increases. In the final stage fluid particle starts to reverse its direction and they disperse disorderly. In the wobbling stage it is observed that the fluid column twisted as it rises. This is due to Coriolis effect of earth. In the north hemisphere fluid column twisted counterclockwise, but in the south hemisphere fluid column twisted clockwise.

#### 4. ANALYSIS

We tried to find whether the height is dependent on Reynolds number or Froude number or both in Reynolds and Froude number. For this reason we observe the variation of non-dimensional parameter Height /radius (H/r) with respect to Reynolds (Re) and Froude (Fr) number.

We also correlate the non-dimensional fountain height (Height/radius) with Froude number. And for both Reynolds and Froude number.

The non-dimensional fountain height (Height/radius) and Reynolds number is plotted for different diameter (in figure 2). From this graph it is observed that the fountain height increases as the Reynolds number increases for a diameter of nozzle. But as the diameter increases fountain height decreases for a Reynolds number. Because if the diameter increases then flow area increases as a result velocity decreases. So fountain height decreases.

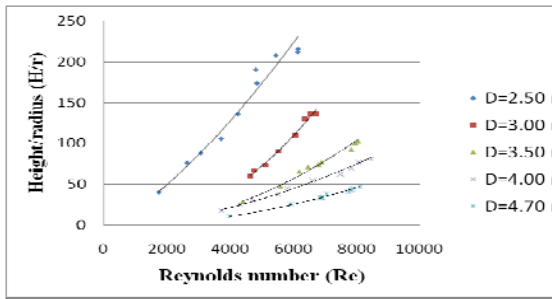


Fig 2. Non-dimensional penetration Height/radius (H/r) as a function of Reynolds number (Re).

The variation of non-dimensional fountain height (H/r) for all the experiment in this study is plotted (figure 3) against Froude number. It is observed that a relationship can be obtained between the non-dimensional fountain height (H/d) and Froude number. By numerical regression analysis we find a nonlinear relationship of non-dimensional fountain height (H/d) with Froude number. 
$$\frac{H}{r} = 1.452 Fr^{1.915}$$

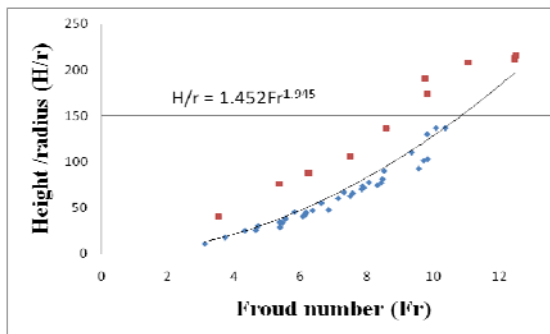


Fig 3. Non-dimensional fountain height (H/r) as a function of Froude number (Fr).

In the above figure it is observed that some data are scattered from the regression line. So we find another

correlation of non-dimensional penetration height including both Reynolds and Froude number. By numerical regression analysis we found that the non-dimensional penetration height is well described by the following relationship,

$$\frac{H}{r} = 0.374 Re^{-0.072} Fr^{2.26}$$

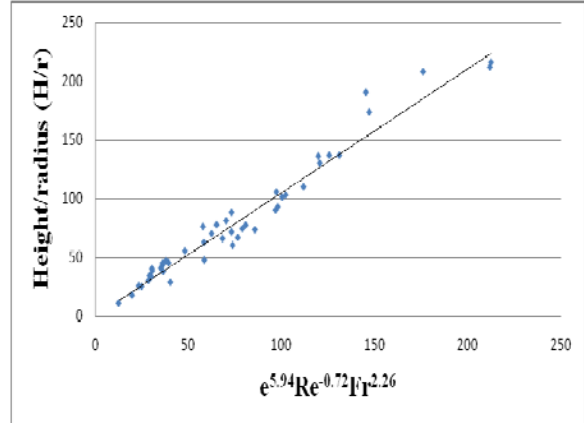


Fig 4. Comparison of experimental non-dimensional fountain height with correlation obtained.

#### 5. DISCUSSION AND RESULT

Turner(1966) performed the first experimental work on turbulent round fountains using salt-water solution injected upwards into a fresh-water tank and found maximum penetration depth  $Z_m$  scales as  $Z_m/R_0 = CFr$ , where  $C=2.46$  over the range  $2 \leq Fr \leq 30$ . Other experimental studies followed which confirmed the linear relationship, albeit with some small variation in the magnitude of the coefficient.

Mizushima et al. (1982) obtained  $C = 2.35$  for  $5 < Fr < 260$  and  $1130 < Re < 2710$ ; ampbell & Turner (1989) found  $C = 2.46$ ; Baines et al. (1990) found  $C = 2.46$  for  $10 < Fr < 300$  and  $Re = 2000$ ; and Pantzlaflf & Lueptow (1999) found  $C = 2.1$  for  $15 \leq Fr \leq 78$  and  $1250 \leq Re \leq 7500$ .

Lin & Armfield (2000a) numerically simulated fountains with  $0.2 \leq Fr \leq 1.0$  and  $Re = 200$ . At these very low Froude numbers the fountain flow is very weak and penetrates only a very small distance before reversing and flowing radially out from the source. In this study the authors found that a linear Froude number scaling for the penetration height, i.e.  $Z_m/R_0 = CFr$ , is still valid. The authors also simulated fountain flow for  $1.0 \leq Fr \leq 2.0$  and found the linear scaling to be invalid.

Kaye & Hunt (2006) later re-plotted these results and found them to be well fitted by  $Z_m/R_0 = CFr^{2/3}$  for  $Fr < 1$  and  $Z_m/R_0 = CFr^2$  for  $Fr > 1$ .

But recently baines et al obtained an analytical scaling  $H/r = C Fr^{1.915}$ . Campbell and turner obtained  $C=1.64-1.97$  from their experiment on turbulent round fountain.

Our investigation shows that the non-dimensional fountain height varies as the following relationship 
$$\frac{H}{r} =$$

1.452  $Fr^{1.843}$ . This is very similar to above studies.

Recently Lin and Armfield investigated the effect of the Reynolds number on the height of plane fountains. They found that for  $Re \leq 200$  the non-dimensional fountain height was dependent on the Reynolds number with the following scaling:  $\frac{H}{r} = C Fr^{0.8}$ .

We conduct our experiment in the following range  $3.00 < Fr < 13.00$  and  $1500 < Re < 8500$ . The relationship in this range is  $\frac{H}{r} = 0.264 Re^{-0.72} Fr^{0.85}$ .

From the above relation we can predict the fountain height for a Reynolds and Froude number for any diameter of the nozzle.

## 6. CONCLUSION

The behavior of negatively buoyantly jet (fountain) has been investigated experimentally. It has been found that the pure vertical jet of water always fluctuates because falling front falls upon the nest rising front.

Inclined jet remains at a constant height up to a minimum height for a fixed flow and nozzle diameter. But at that minimum height interaction between falling and rising front occur and fluctuation of inclined jet starts.

For high Reynolds number the jet stream can be divided into three segments. Rising stage here the flow is streamlined no variation in the diameter of the jet, Intermediate stage with increasing diameter and wobbling of fountain occur and at final stage jet stagnates and reverse its direction.

Non-dimensional fountain height ( $H/r$ ) is greatly depend on the Froude number ( $Fr$ ) by the correlation  $\frac{H}{r} =$

$$1.452 Fr^{1.843}$$

But it is relatively well fitted by  $\frac{H}{r} = 0.264 Re^{-0.72} Fr^{0.85}$ ,

## 7. REFERENCE

1. n.williamson, n.srinarayana, s.armfield, g.d McBain (2008) Low-Reynolds-number fountain behavior
2. Philippe, C. Raufaste, P. P Kurowski, and P. Petitjeans (2005) Penetration of a negatively buoyant jet in a miscible liquid.
3. T. Mizushima, F. Ogino, H. Takeuchi, and H. Ikawa, Kyoto, Japan(1982) An Experimental Study of Vertical Turbulent Jet with Negative Buoyancy.
4. N. Srinarayana, S. W. Armfield and W. X. Lin (2007) Numerical Simulation of Free-fountains in a Homogeneous Fluid (AUS JMF).
5. Williamson, N.1, Srinarayana, N.1, Armeld, S.W.1, McBain, G.1 and Lin,(2007) Characterization of Low Reynolds Number Fountain Behavior.
6. Abraham, G. 1967 Jets with negative buoyancy in homogeneous fluid. J. Hydraul Res. 5, 235-248.
7. Lin, W. and Armeld, S. W., Direct simulation of fountains with intermediate Froude and Reynolds numbers, ANZIAM J., 45(E), 2004, C66.C77.
8. Baines, W. D., Turner, J. S. & Campbell, I. H. 1990 Turbulent fountains in an open chamber. J. Fluid Mech. 212, 557-592.
9. Bloomfield, L. J. & Kerr, R. C. 2000 A theoretical model of a turbulent fountain. J. Fluid Mech. 424,197-216.
10. Zhang, H. & Baddour, R. E1998 Maximum penetration of vertical round dense jets at small and large Froude numbers. J. Hydraul. Engg.
11. Friedman, P. D. 2006 ,Oscillation height of a negatively buoyant jet. Trans. ASME J:J.FluidsEngng 128,880-882.

## 8. MAILING ADDRESS

Chandan Kumar  
Department of Mechanical Engineering, Bangladesh University of Engineering and Technology (BUET), Dhaka-1000,Bangladesh.