Proceedings of the
International Conference on Mechanical Engineering 2009
(ICME2009) 26- 28 December 2009, Dhaka, Bangladesh

ICME09-RT-25

HYDRODYNAMIC INTERACTION FOR A LONG ARRAY OF FREELY FLOATING 3-D RECTANGULAR BOXES

Mir Tareque Ali and Tasnuva Anan

Department of Naval Architecture and Marine Engineering, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh.

ABSTRACT

This paper investigates the first order wave exciting forces and moments and motion responses due to hydrodynamic interactions between an array of twenty-one freely floating three dimensional rectangular bodies in regular waves. The rectangular bodies are identical, and equally spaced along the array. In the present study, the three dimensional source-sink method has been adopted to carry out the numerical investigation. The validation of the computer code developed for this purpose has been justified by comparing the present results with that of the published ones for simple geometrical shaped floating bodies. The numerical computations have been carried out for different wave heading angles and separation distances between the floating rectangular bodies and the hydrodynamic interactions on different members in the array have been studied by calculating the first order wave exciting forces and moments as well as motion responses. Finally some conclusions have been drawn on the basis of the present analysis.

Keywords: Hydrodynamic Interaction, Three Dimensional Source-Sink Method, Wave-Exciting Forces, Hydrodynamic Coefficients.

1. INTRODUCTION

In recent years, there is a significant increase in the number of activities that involves applications of offshore structures, where two or, more bodies are floating in sufficiently close proximity, experiencing significant interactions. The hydrodynamic interactions occur as a result of the wave motion between neighbors in the array of floating bodies. The number of individual structures as well as their geometry, within such arrangement can vary greatly. Hydrodynamic analysis of multiple floating bodies is different from that of a single floating body, because one body is situated in the diffracted wave field of others. The bodies will experience incident as well as scattered waves impinges upon them. Now if these waves arrive in phase then there will be a considerable escalation in the magnitude of the wave exciting forces on the floating bodies compared to a body in isolation. On the other hand, if these waves arrive out of phase then there will be a significant reduction of wave force. Moreover, each body will also experience radiated waves due to the motion of other bodies. The actual importance of interaction effect depends on the configuration of the multi-body system, which means the size and shape of the floating bodies and the separation distances between them.

Understanding hydrodynamic interaction is particularly important while designing floating bridges. Floating bridges have various advantages comparing to land based structures. They are minimally influenced by water level fluctuations due to tide and storm surge. These structures are not influenced by soil/seafloor condition, so they do not suffer from differential settlement and can easily be relocated. The deck of a floating bridge is usually supported on a number of rectangular floating boxes. To assist the proper design of a floating bridge, it is quite important to study the wave exciting forces and moments and motion responses on an array of freely floating rectangular boxes.

There are many investigations related to the hydrodynamic interaction between multiple floating bodies in waves. Ohkusu [1] extended the classical solution for a single heaving circular cylinder to the case of two cylinders in a catamaran configuration. Faltinsen & Michelsen [2] used panel method for direct numerical solution of wave effects on 3D floating bodies. The panel method was further extended for two independent bodies by van Oortmerssen [3]. Lee & Newman [4] and Kim *et al.* [5] used panel method for more complex structure like MOB (Mobile Offshore bases). Maniar & Newman [6] presented results for the diffraction past an array of 100 vertical cylinders. Chakrabarti [7] used multiple-scattering method in combination with panel method, while Choi & Hong [8] used higher-order boundary element method to solve the interactions problem between multiple floating bodies.

In this paper, a frequency domain analysis has been carried out to calculate the first order wave exciting forces and moments and motion responses to study hydrodynamic interactions between an array of twenty-one identical freely floating 3D rectangular bodies in regular waves. Using 3D source sink method a computer code has been developed to investigate the hydrodynamic interaction phenomena. In order to justify the validity of the code, some published results have been verified for two freely floating vertical cylindrical bodies. First order wave exciting forces and moments and motion responses for each member of the array comprised of twenty-one rectangular boxes have been computed for different wave heading angles as well as for various separation distances between the members. It is observed from the present study that hydrodynamic interactions vary with the position of a member in the array and with the separation distance between the members of this long array of rectangular boxes.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a group of N 3-dimensional bodies of arbitrary shape, oscillating in water of uniform depth. The amplitudes of the motions of the bodies and waves are assumed to be small, whereas the fluid is supposed to be ideal and irrotational. Two right-handed Cartesian coordinate systems are considered: one fixed to the bodies and another fixed to the space. In regular waves a linear potential Φ, which is a function of space and of time, can be written as a product of space-dependent term and a harmonic time-dependent term as follows:

$$\Phi(x,y,z;t) = \phi(x,y,z)\,.\,e^{-i\omega t} \tag{1}$$

The wave circular frequency ω can be written

$$\omega = \frac{2\pi}{T} \tag{2}$$

where T is the wave period. The potential function ϕ can be separated into contributions from all modes of motion of the bodies and from the incident and diffracted wave fields as follows:

$$\phi = -i\omega\left[(\phi_0+\phi_7)\zeta_a + \sum_{m=1}^{N}\sum_{j=1}^{6}(X_j^m\phi_j^m)\right] \tag{3}$$

where ϕ_0 is the incident wave potential, ϕ_7 is the diffracted wave potential, ϕ_j^m represent potentials due to motion of body 'm' in j-th mode i.e., radiation wave potentials, X_j^m is the motion of body 'm' in j-th mode and ζ_a is the incident wave amplitude. The incident wave potential can be expressed as

$$\phi_0 = \frac{g}{\omega^2}\frac{\cosh[k(z+h)]}{\cosh kh}e^{ik(x\cos\chi + y\sin\chi)} \tag{4}$$

where χ is the wave heading angle measured from $+X$-axis, h is the depth of water, g is the acceleration due to gravity and k is the wave number. The individual potentials are all solutions of the Laplace equation, which satisfy the linearized free surface condition and the boundary conditions on the sea floor, on the body's surface and at infinity.

2.1 Source Density and Velocity Potentials

The potential function at some point (x,y,z) in the fluid region in terms of surface distribution of sources can be written as:

$$\phi_j^m(x,y,z) = \frac{1}{4\pi}\sum_{n=1}^{N}\iint_{S^n}\sigma_j^m(\xi,\eta,\zeta)\,G(x,y,z;\xi,\eta,\zeta)\,dS \tag{5}$$

where (ξ,η,ζ) is a point on surface S and $\sigma(\xi,\eta,\zeta)$ is the unknown source density. The solution to the boundary value problem is given by Eq. (5), which satisfies all the boundary conditions. And since Green's function (G) satisfies these conditions, applying the kinematics boundary condition on the immersed surface yields the following integral equation:

$$\frac{\partial\phi_j^m(x,y,z)}{\partial n} = -\frac{1}{2}\sigma_j^m(x,y,z) + \frac{1}{4\pi}\sum_{n=1}^{N}\iint_{S^n}\sigma_j^m(\xi,\eta,\zeta)\,\frac{\partial}{\partial n}G(x,y,z;\xi,\eta,\zeta)\,dS \tag{6}$$

2.2 Numerical Evaluation of Velocity Potentials

A numerical approach is required to solve the integral Eq. (6), as the kernel $\frac{\partial G}{\partial n}$ is complex and it does not permit any solution in closed form. The wetted surface of body is divided into l number of quadrilateral panels of area $\Delta s_l^m (l = 1........E_n)$ and the node points are considered at the centroid of each panel. The continuous formulation of the solution indicates that Eq. (6) is to be satisfied at all points (x,y,z) on the immersed surface but in order to obtain a dicretized numerical solution it is necessary to relax this requirement and to apply the condition at only N control points and the location of the control points are chosen at the centroids of the panels. Consequently, discretization process allows Eq. (6) to be replaced as

$$-\frac{1}{2}(\sigma_j^m)_l + \frac{1}{4\pi}\sum_{n=1}^{N}\sum_{k=1}^{E_n}\iint_{\Delta S_k^n}(\sigma_j^m)_k \frac{\partial G}{\partial n}(l,k)dS$$
$$= \begin{cases} \left(-\frac{\partial \phi_0}{\partial n}\right)_l & , \quad j=7 \\ \left(n_j^m\right)_l & , \quad j=1,2....6 \\ \text{(if panel 'l' belongs to body 'm')} & \\ 0 & , \quad j=1,2....6 \\ \text{(if panel 'l' does not belong to body 'm')} & \end{cases} \tag{7}$$

2.3 Hydrodynamic Coefficients and Wave Exciting Forces and Moments

Once the velocity potentials have been determined, then the added-mass coefficients (a_{kj}^{mn}), fluid damping coefficients (b_{kj}^{mn}) and first order wave-exciting forces and moments (F_k^m) can be calculated as follows:

$$a_{kj}^{mn} = -\Re e\left[\rho\iint_{S^m}\phi_j^n n_k^m dS\right] \tag{8}$$

$$b_{kj}^{mn} = -\Im m\left[\rho\omega\iint_{S^m}\phi_j^n n_k^m dS\right] \tag{9}$$

$$F_k^m = -\rho\zeta_a\omega^2 e^{-i\omega t}\iint_{S^m}(\phi_0+\phi_7)n_k^m dS \tag{10}$$

For $m=n$, the added mass and damping coefficients are due to body's own motion, on the other hand for $m \neq n$, the coefficients are due to the motion of other bodies.

Using 'Haskind Relation', first order wave-exciting forces and moments (F_k^m) can also be calculated as follows:

$$F_k^m = -\rho\zeta_a\omega^2 e^{-i\omega t}\iint_{S^m}(\phi_0\frac{\partial\phi_j^m}{\partial n} - \phi_j^m\frac{\partial\phi_0}{\partial n})dS \tag{11}$$

2.4 Equations of Motions in Frequency Domain

The equations of motion can be expressed by using the following matrix relationship:

$$(M+a)\ddot{X} + b\dot{X} + cX = F \tag{12}$$

where M is the inertia matrices, a is the added mass matrices, b is the fluid damping matrices, C is the hydrostatic stiffness matrices, F is the wave exciting force vector and X is the motion response vector. The above equations of motion are established at the centers of gravity of each body of the multi-body floating system. Since each body is assumed rigid and has six degrees of freedom, each matrix on the left-hand side of Eq. (12) has a dimension of $(6N \times 6N)$ and X and F are $(6N \times 1)$ column vectors for N floating body system. The elements of matrices and vectors in Eq. (12) are discussed in details by Inoue & Seif [9].

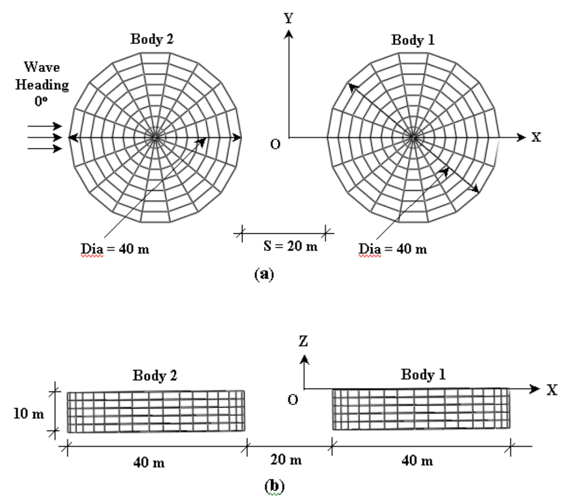
3. RESULTS AND DISCUSSIONS

On the basis of above formulations, a computer code written in F77 has been developed considering double precision variables. To justify the validity of the computer code, the numerical results are checked for two freely floating vertical cylinders. To study the interaction effects, a detailed computation is then conducted for twenty-one identical freely floating rectangular boxes in regular waves.

3.1 Two Freely Floating Vertical Cylinders

The diameter and draft of each cylinder is 40.0 m and 10.0 m respectively and the separation distance or, gap between them is 20.0 m. The water depth is considered as 200.0 m. The wetted surface of each cylinder is divided into 234 panels as shown in Fig 1. For 0^o wave-heading angle, body 1 and body 2 represent the lee side and weather side cylinder respectively.

The non-dimensional surge wave exciting forces on body 1 and body 2 are shown in Fig 2. To verify the accuracy of the present computations, the wave exciting forces are computed using diffraction potential as well as Haskind relationship. Fig 3 presents the surge motions of body 1 and body 2. The results are plotted against ka, where k and a denote the wave number and radius of each cylinder respectively. Fig 2 and 3 also depict comparisons between the present results with Goo and Yoshida [10] results and the agreement between both the results are found quite satisfactory.



(c)

Fig 1. Mesh arrangements of the wetted surfaces of two floating vertical cylinders (a) Top view, (b) Side view and (c) 3-D view

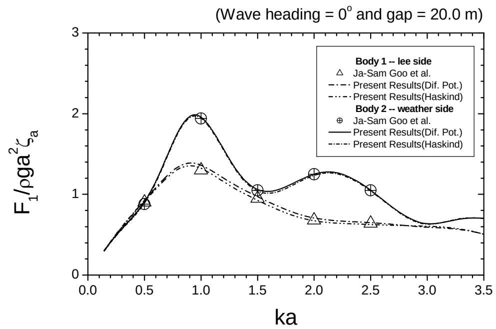


Fig 2. Surge wave exciting forces on floating cylinders

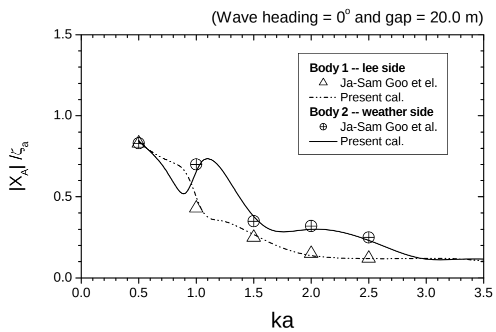


Fig 3. Surge motions of vertical floating cylinders

3.2 Twenty-One Identical Freely Floating Rectangular Boxes

Twenty-one identical freely floating rectangular boxes have been considered to study the first order wave exciting forces and moments on individual members for a long array of floating bodies. A frequency domain analysis has been carried out to compute the motion responses of the floating boxes in regular waves. The length, breadth and draft of each box are 109.7 m, 101.4 m and 30.0 m respectively. The C.G. of each body is considered 0.2 m below still water level. The water depth is taken as 100.0 m. The wetted surface of each body is divided into 188 panels. The numbering and arrangement of rectangular boxes in the array is shown in Fig 4. Body 1, Body 11 and Body 21 represent the first, middle and last body in the array for a wave heading of 180^o.

Fig 5 shows the variation of non-dimensional surge wave exciting forces with wave circular frequency for Body1 at 180^o wave heading angle. The separation distances between the members in the array are varied as 25m, 50m and 75m. Strong and complicated hydrodynamic interaction is observed for these three separation distances. To demonstrate the interaction effect, surge wave exciting forces for an isolated rectangular body is also plotted in these figures. It is observed that the magnitude of wave exciting forces in the frequency range is higher or, close to that of an isolated body. Fig 6 shows the variation of non-dimensional surge wave exciting forces with wave circular frequency for Body21 at 180^o wave heading angle. Hydrodynamic interaction is also evident in this figure. However, the magnitudes of surge wave exciting forces show sharp and significant reduction in amplitude for Body21. Since the number of shielding bodies is more for Body 21 compared to other bodies in the array, so, the magnitude of surge exciting force rapidly approaches to zero. Sharp peaks appear in the figures, which may be due to locally resonated waves in confined fluid domain.

Fig 7 show the variation of non-dimensional heave wave exciting forces with wave circular frequency for Body1 and Body21 at 180^o wave heading angle. Similar to previous figures, the results are plotted for the separation distances of 25m, 50m and 75m. Hydrodynamic interaction for Body1 for the three separation distance is quite weak as revealed in these figures. The magnitude of heave exciting force for Body1 is nearly close to that of an isolated body. Hydrodynamic interaction is also weak for Body21. However, for Body21 the magnitude of heave wave exciting forces show gradual decrease in magnitude as the number of shielding body is maximum for this body.

Fig 8 show the variation of non-dimensional pitch wave exciting moments with wave circular frequency for Body1 and Body21 at 180^o wave heading angle. Hydrodynamic interaction for the three separation distances is also prominent as revealed in these figures. The magnitude of pitch exciting moment for Body1 is also nearly close to that of an isolated body. Similar to surge and heave forces, pitch wave exciting moment for Body21 show sharp decline in magnitude as the number of shielding body is increased for this body.

Fig 9 show the variation of non-dimensional surge motions with wave circular frequency for Body1 and Body21 at 180^o wave heading angle. Hydrodynamic interaction for the three separation distance is quite strong as revealed in these figures. The magnitude of surge motion for Body1 is nearly close to that of an isolated body, whereas for Body21 the magnitude of surge motion shows gradual decrease in magnitude.

Finally, Figs 10 and 11 show the variation of non-dimensional heave and pitch motions with wave circular frequency for Body11 at 180^o wave heading angle. The figures clearly show that the hydrodynamic interaction for the three separation distance is quite weak. Resonance is quite prominent for the pitch motion that occurs in between the frequency range of 0.35 rad/s and 0.40 rad/s. The probable reason may be the presence of pitch natural frequency of motion of the floating box within this range. Due to the dominance of resonance, interaction effect is almost absent for pitch motion.

4. CONCLUSIONS

3D source sink method has been adopted to compute the first order wave exciting forces and moments and motion responses by taking into account the effect of hydrodynamic interactions among the different floating bodies for an array of twenty-one freely floating rectangular boxes in regular waves. The hydrodynamic interaction effect is strong and complicated for surge exciting forces, pitch exciting moments and surge motions, while it is very weak for heave exciting forces and heave and pitch motions. The variation of the separation distance between the boxes in the array shows interaction effect near lower frequency region. Sharp peaks appear in the results, which may be due to locally resonated waves in confined fluid domain. In general, the magnitude of wave exciting forces and moments and motions show gradual decline in magnitude as the number of shielding body is regularly increased.

5. REFERENCES

1. Ohkusu, M., 1969, "On the heaving motion of two circular cylinders on the surface of a fluid", *Reports of Research Institute for Applied Mechanics*, XVII, No. 58, pp. 167-185.
2. Faltinsen, O. and Michelsen, F., 1974, "Motions of large structures in waves at zero Froude number", *Proc. Intl. Symp. on the Dynamics of Marine Vehicles and Structures in Waves*, London, pp. 91-106.
3. van Oortmerssen, G., 1979, "Hydrodynamic interaction between two structures, floating in waves", *Proc. 2nd. Intl. Conf. on Behaviour of Offshore Structures* (BOSS'79).
4. Lee, C. H. and Newman, J. N., 2000, "An assessment of hydroelasticity for very large hinged vessels", *Proc. 2nd Intl. Conf. on Hydroelasticity in Marine Technology*, Kyushu, pp. 27-36.
5. Kim, D., Chen, L. and Blaskowski, Z., 1999, "Linear frequency domain hydroelastic analysis for McDermott's mobile offshore base using WAMIT", *VLFS'99*, Honolulu.
6. Maniar, H. D. and Newman, J. N., 1997, "Wave diffraction by a long array of cylinders", *J. Fluid Mec.*, Vol. 339, pp. 309-330.
7. Chakrabarti, S., 2000, "Hydrodynamic interaction forces on multimoduled structures", *Ocean Engineering*, Vol. 27, pp. 1037-1063.
8. Choi, Y. R. and Hong, S. Y., 2002, "An analysis of hydrodynamic interaction of floating multi-body using higher-order boundary element method", *ISOPE'2002*, Kitakyushu, Japan.
9. Inoue, Y. and Seif, M. S., 1997, "Nonlinear responses of multiple floating systems", *OMAE'97*, Yokohama.
10. Goo, J. and Yoshida, K., 1989, "Hydrodynamic interaction between multiple three dimensional bodies of arbitrary shape in waves", *Journal of the Society of Naval Architects Japan*, Vol. 165, pp. 193-202.

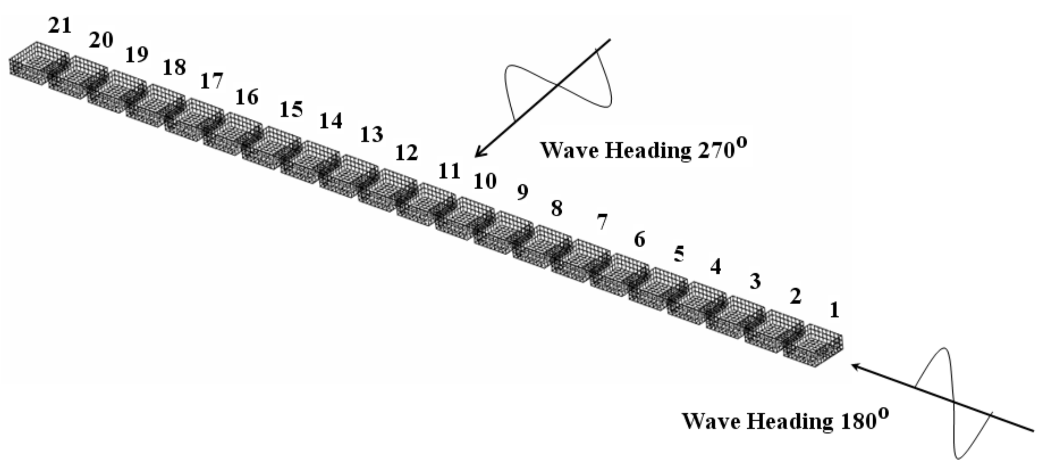


Fig 4. Mesh arrangements and numbering of a long array of 21 freely floating identical rectangular boxes

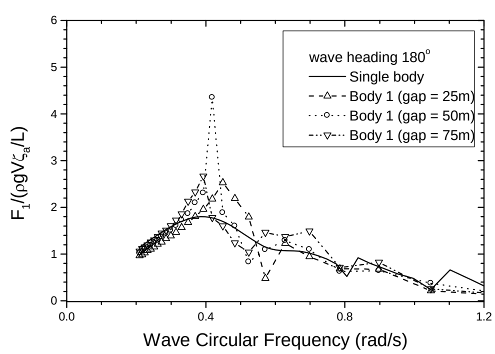


Fig 5. Surge wave exciting forces on Body 1 for a long array of 21 identical freely floating rectangular bodies

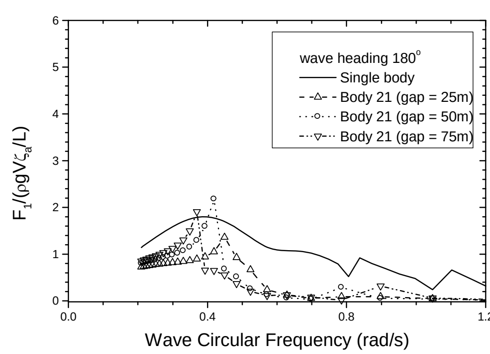


Fig 6. Surge wave exciting forces on Body 21 for a long array of 21 identical freely floating rectangular bodies

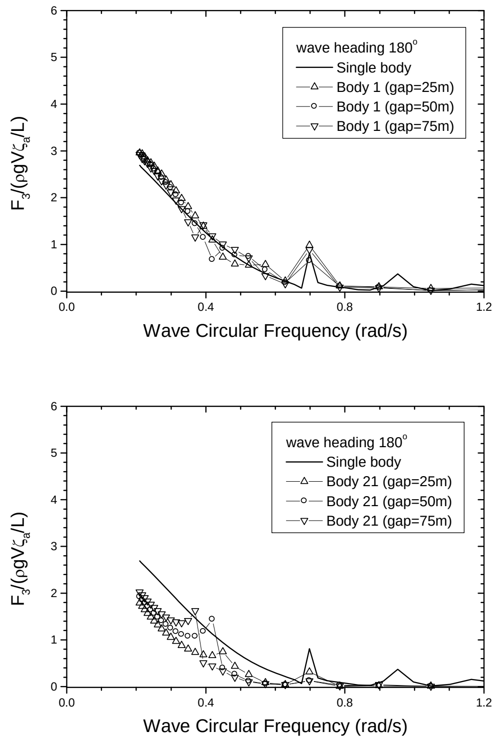


Fig 7. Heave wave exciting forces on Body 1 and Body21 for a long array of 21 identical freely floating rectangular boxes

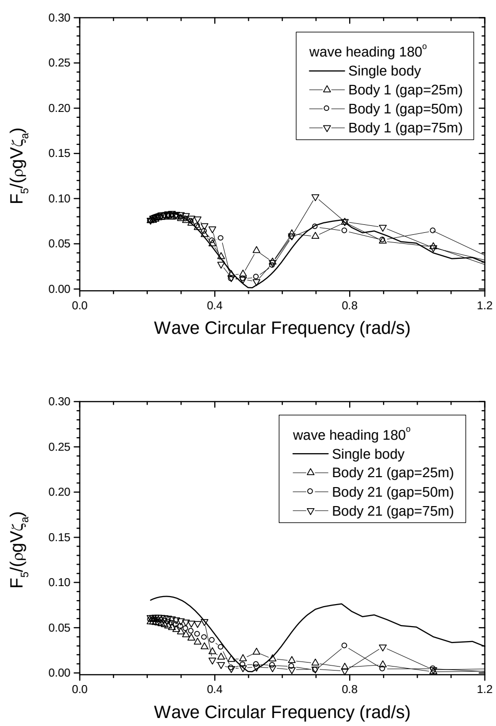


Fig 8. Pitch wave exciting moments on Body 1 and Body21 for a long array of 21 identical freely floating rectangular boxes

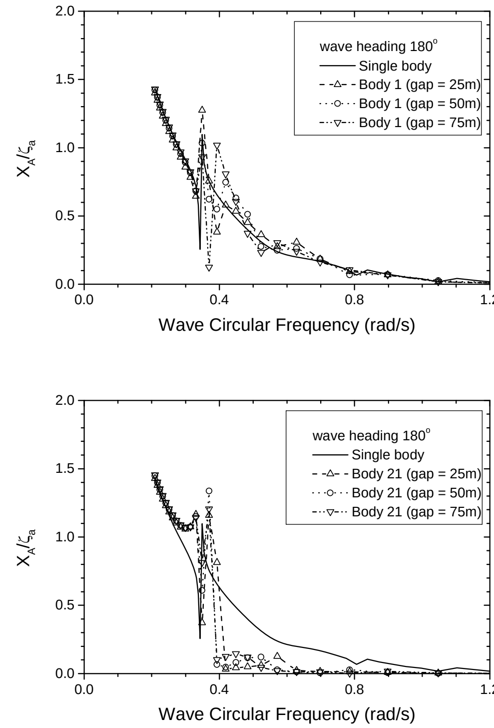


Fig 9. Surge motions for Body 1 and Body 21 for a long array of 21 identical freely floating rectangular bodies

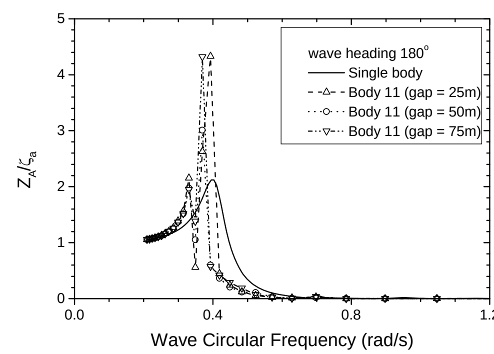


Fig 10. Heave motion for Body 11 for a long array of 21 identical freely floating rectangular boxes

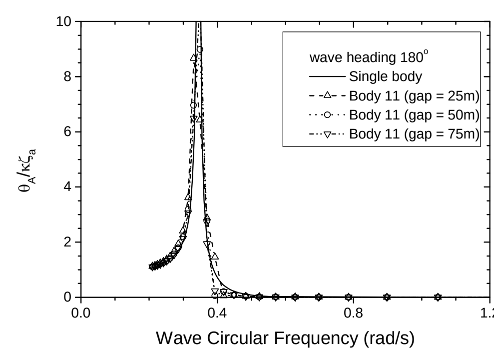


Fig 11. Pitch motion for Body 11 for a long array of 21 identical freely floating rectangular boxes