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ANALYSIS OF TRANSOM STERN FLOWS BY MODIFIED RANKINE SOURCE PANEL METHOD

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ABSTRACT

The paper presents a numerical method for calculating a potential flow around a ship of transom stern with respect to the double-body potential. The method of solution is based on the distribution of Rankine sources on the hull as well as its image and on the free surface. An iterative algorithm is used for determining the free surface and wave resistance using Dawson's upstream finite difference operator. A verification of numerical modeling is made using R V Athena hull and the validity of the computer program is examined by comparing the wave making resistance with NPL-100A hull.

Keywords: Wave Resistance, Transom Hull, Rankine Source Panel Method, R V Athena, NPL-100A

1. INTRODUCTION

A large number of vessels currently being used in Bangladesh in passenger service share the characteristics of possessing a cut-off or transom stern. This feature of the vessel defies simple hydrodynamic analysis because the extent and detailed shape of the flow behind the transom is unknown and must be a part of the mathematical solution to the problem. In principle, the pressure acting on the free surface behind the transom must be atmospheric and the flow must separate from the transom tangentially.

High speed displacement vessels are frequently used as patrol boats because of their speed capability combined with good seakeeping characteristics. These vessels have a high length-beam ratio and a transom stern. A practical computational method for such transom stern vessels is of special interest to ship hydrodynamicists due to the peculiar property of the flow pattern. If the ship speed is high enough, (Froude numbers, $F_n > 0.4$ approximately), the transom clears the surrounding water and the entire transom area is exposed to the air. The transom flow detaches smoothly from the undesired of the transom and a depression is created on the free surfaced behind the transom.

Tulin and Hsu [1] developed a theory for high-speed displacement ships with transom sterns. The flow was assumed to be smooth at the aft waterline and to have a trailing wake. The trailing wake resulted in substantial residuary resistance at high speeds for normal waterline ships. Cheng [2] presented a practical computational method for 3-dimensional transom stern flows. The boundary condition for dry transom stern was derived within the framework of a free surface potential flow. The transom was treated as an inflow boundary and the transom boundary condition was used to specify the starting values of linearized free surface conditions.

Telste and Reed [3] presented a method of calculating the flow near a transom stern ship moving forwarded at a moderate to high steady speed into otherwise undisturbed water. Modified free-stream linearization was used to obtain a Neumann-Kelvin boundary value problem in which the usual linearization about the man free surface level was replaced in the area behind the transom stern by linearization about a surface originating at the hull-transom intersection. A Rankine singularity integral equation was presented for obtaining the solution of the resulting mathematical boundary value problem.

Wang et al. [4] calculated the wave resistance of a fast displacement hull with a transom stern using several different theoretical methods. The results of the calculations were compared with experimental data for a hull from the National Physical Laboratory (NPL) round-bilge hull series.

Sireli and Insel [5] investigated the effects of transom stern on the wave resistance of high speed marine craft by using a series of five monohull forms by both experimental and numerical methods. The wave resistance of the hull was calculated by potential flow theory based on Dawson's algorithm and the viscous resistance by boundary layer calculations.

Doctors and Day [6] used the linearized near field solution for the flow past a vessel with a transom stern. The hollow in the water behind the stern was represented by an extension to the usual centre-plane source distribution employed to model the hull itself. The resistance, sinkage and trim were computed by means of an integration of the resulting pressure distribution over the wetted surface of the vessel.

Millward et al. [7] developed a numerical method to calculate the flow past light displacement ships hulls with a transom stern for Froude numbers ranging from 0.4 to approximately 1.0. The hull was allowed to trim and heave in calculations and the results of the calculations were compared with experimental data available for the Model 100A of NPL round bilge hull series.

The objective of the present research is to develop a numerical model for calculating the flow around the transom stern of a high speed vessel by the modified Rankine source panel method. The special boundary condition is applied to the transom and to the portion of the free surface downstream of the stern. The physical constraints imposed by this transom boundary condition require that the static pressure be atmospheric and that the flow leave tangentially at the transom. The computational method has been applied to R V Athena and NPL 100A hulls and then compared the results with the existing results available in the literature.

2. MATHEMATICAL FORMULATION

Let us consider a ship moving in an infinite depth of water with a constant speed U in the direction of the negative x-axis as shown in Figure 1. The z-axis is vertically upwards and the y-axis extends to starboard. The origin of the co-ordinate system is located in an undisturbed free surface at amidship. The total velocity potential ϕ is the sum of the double body velocity potential Φ and the perturbed velocity potential φ representing the effect of free surface wave.

$$\phi = \Phi + \varphi \qquad (1)$$

Now the problem for a ship can be constructed by specifying the Laplace equation

$$\nabla^2 (\Phi + \varphi) = 0 \qquad (2)$$

with the following boundary conditions:

(a) Hull boundary conditions: The normal velocity component on the hull surface must be zero.

$$\nabla(\Phi + \varphi) \cdot n = 0 \qquad (3)$$

where $n = n_x i + n_y j + n_z k$ represents a normal to the hull surface in the outward direction.

(b) Free surface condition: The velocity potential needs to satisfy the dynamic and the kinematic conditions on the free surface

$$g\zeta + \frac{1}{2}\nabla\phi \cdot \nabla\phi = \frac{1}{2}U^2 \ \text{ on } \ z = \zeta(x, y) \qquad (4)$$

$$\phi_x \zeta_x + \phi_y \zeta_y - \phi_z = 0 \ \text{ on } \ z = \zeta(x, y) \qquad (5)$$

Eliminating ζ from equations (4) and (5)

$$\frac{1}{2}\phi_x \left(\nabla\phi \cdot \nabla\phi\right)_x + \frac{1}{2}\phi_y \left(\nabla\phi \cdot \nabla\phi\right)_y + g\phi_z = 0 \ \text{ on } \ z = \zeta(x, y) \qquad (6)$$

The free surface condition equation (6) is nonlinear in $\nabla\phi$ and should be satisfied on the free surface at $z = \zeta(x, y)$, which is unknown and can be linearized about the double body solution Φ by neglecting the non-linear terms of φ. After linearization the free surface boundary condition (6) can finally be expressed as

$$\Phi_l{}^2 \varphi_{ll} + 2\Phi_l \Phi_{ll} \varphi_l + g\varphi_z = -\Phi_l{}^2 \Phi_{ll} \ \text{ on } \ z = 0 \qquad (7)$$

where the subscript l denotes the differentiation along a streamline of double body potential Φ on the symmetry panel z = 0. The free surface boundary condition given in equation (7) involves the gradient of the velocity potential along a streamwise direction designated by l and differentiation is carried out along the corresponding double body streamlines. Finally it is necessary to impose a radiation condition to ensure that the free surface waves vanish upstream of the disturbance. The solution of the Laplace equation in connection with the boundary conditions (3), (7) and the radiation condtion for the flow around the cruiser stern is given in Tarafder and Suzuki [8].

3. SOLUTION FOR DRY TRANSOM STERN

For the case of a transom stern, another constraint to be satisfied at the transom is the exit flow that must be tangential to the hull surface. The free surface of the transom stern is considered to be consisted of two parts. The main section of the free surface is handled in the same way as for cruiser sterns. The fluid domain behind the transom is treated as an inflow boundary and the starting values of a free surface calculation are specified at the transom. The transom stern solution begins with a double model computation. After getting cruiser stern solution, the flow direction for the transom geometry is specified by a local tangential unit vector:

$$\Gamma_x = \frac{\Phi_x}{\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_z^2}} \qquad (8)$$

$$\Gamma_y = \frac{\Phi_y}{\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_z^2}} \qquad (9)$$

$$\Gamma_z = \frac{\Phi_z}{\sqrt{\Phi_x^2 + \Phi_y^2 + \Phi_z^2}} \qquad (10)$$

where the vector Φ_x, Φ_y and Φ_z represents the velocity for a hull panel whose centroid lies just forward of the sharp corner at the transom. Ensuring the flow tangential to the hull surface, the velocity components at the transom are approximated as (see Cheng, 1989):

$$u_T = 2U_\infty \sqrt{1 - \frac{2g}{U_\infty^2} z_T} \ \ \Gamma_x \qquad (11)$$

$$v_T = 2U_\infty \sqrt{1 - \frac{2g}{U_\infty^2} z_T} \ \ \Gamma_y \qquad (12)$$

$$w_T = 2U_\infty \sqrt{1-\frac{2g}{U_\infty^2} z_T}\ \ \Gamma_z \tag{13}$$

The streamwise velocity on the free surface is computed by

$$\phi_{1l} = \frac{\Phi_x}{\sqrt{\Phi_x^2+\Phi_y^2}}\varphi_x + \frac{\Phi_y}{\sqrt{\Phi_x^2+\Phi_y^2}}\varphi_y \tag{14}$$

Note that the differentiation scheme approximates the free surface flow direction by the double model flow direction. In the numerical calculation of the convective term, finite differencing is used to calculate derivatives between adjacent panels along a double model streamline. A three point upstream finite differencing scheme is used to eliminate upstream propagating disturbances as recommended by Dawson [9]. For the foremost upstream point, a two point upstream finite difference operation is used.

4. RESULTS AND DISCUSSIONS

To investigate the effect of the transom stern on the wave resistance and wave pattern, the method has been applied first for NPL-100A hull. Since the body is symmetric one-half of the computational domain is used for numerical treatment. The extent of free surface domain is 3.0 ship-lengths upstream to 8.0 ship-lengths downstream. The transverse extension of the free surface domain is about 2.0 ship-lengths. The NPL-100A hull is discretized by 2×20×10 and is shown in Fig. 1. The free surface is discretized by 2×1985 panels (main domain: 2×90×19, domain behind stern: 2×55×5) and is given in Fig 2.

Figure 3 gives a side-view of the computed wave profiles behind the transom for RV Athena at various Froude numbers. The vertical and horizontal axes are normalized by L and are plotted to the same scale. The figure indicates that the transom clears the surrounding water and the assumption of a dry transom stern is valid.

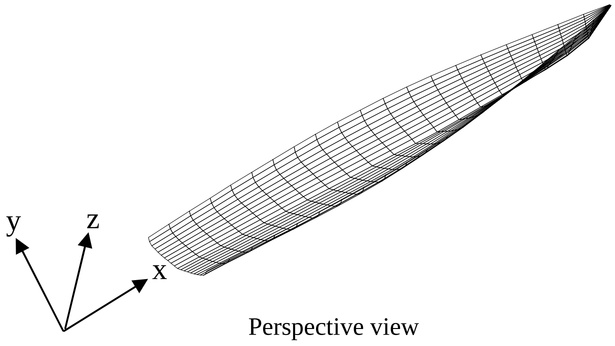


Fig 1. Discretization of NPL-100A by 2×20×10 panels

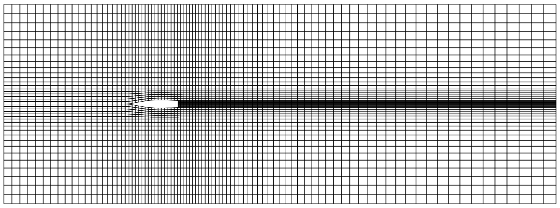


Fig 2. Discretization of free surface for NPL-100A by 2×1985 panels (main domain: 2×90×19, domain behind stern: 2×55×5)

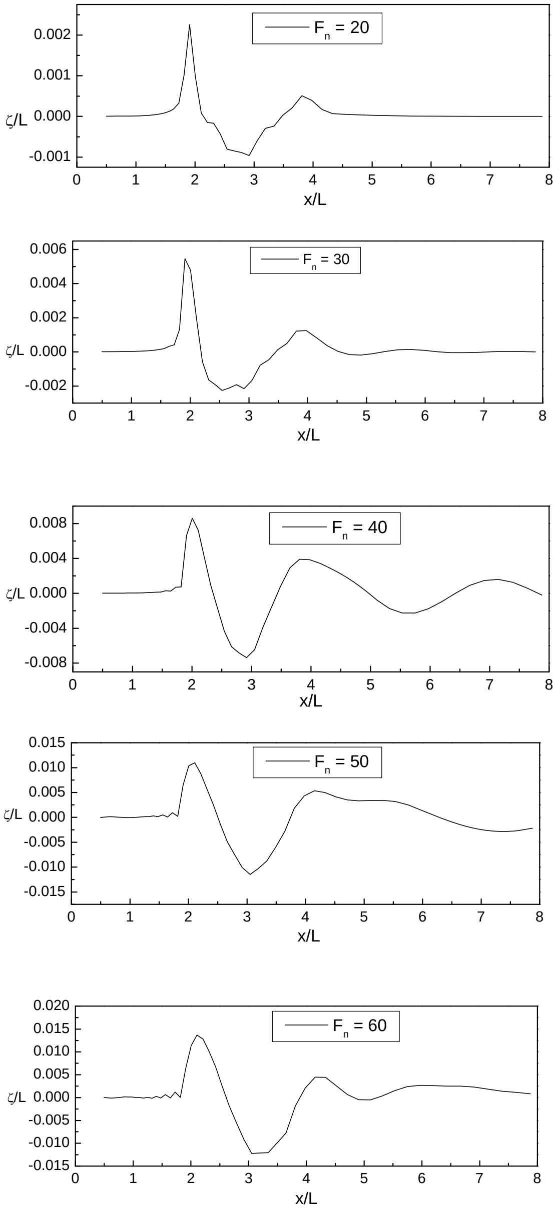


Fig 3. Wave profile at the central line behind the transom stern of NPL-100A hull at various Froude numbers

The comparison of computed wave resistance and measured wave pattern resistance is presented in Fig 4. The computations were performed for Froude numbers from 0.2 to 1.0 at increment of 0.05. The hull form was held in fixed at the even keel position for these computations. The calculated wave resistance and measured residual wave resistance show general agreement in respect of hump and hollow but the significant difference is found between the present numerical result and Millward's calculated results.

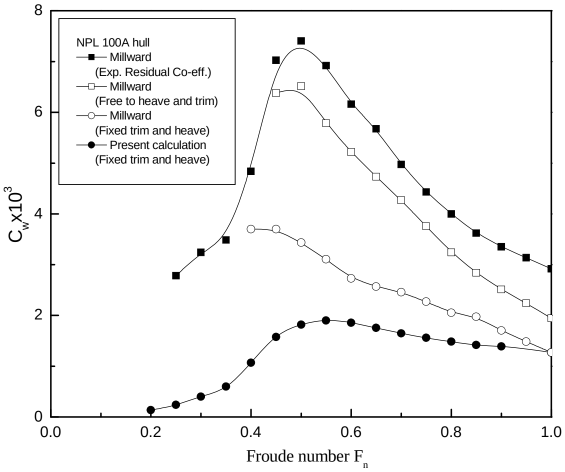
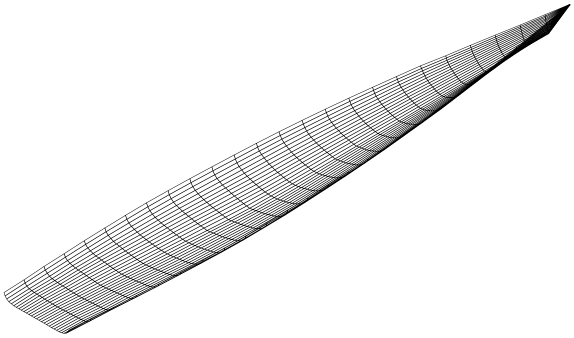


Fig 4. Comparison of calculated wave resistance with Millward's results for NPL-100A hull

The discretization of R V Athena hull and its corresponding free surface are given in Fig 5 and 6 respectively. The calculated wave-making resistance of Athena hull is shown in Fig 7.



Perspective view

Fig 5. Discretization of R V Athena hull by 2×25×14 panels

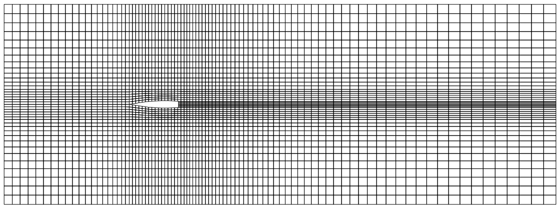


Fig 6. Discretization of free surface for R V Athena hull (main domain: 2×90×19, domain behind stern: 2×55×5)

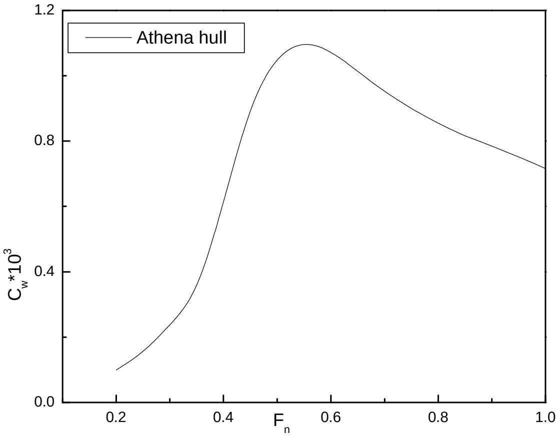


Fig 7. Calculated wave-making resistance of Athena hull

5. CONCLUSIONS

The paper presents the modified Rankine source panel method for calculating the flow of transom stern ship using double body linearization of the free surface boundary condition. The following conclusions can be drawn from the present numerical analysis:

1. The present method could be an efficient tool for evaluating the flow field, wave pattern and wave resistance for practical ship forms.

2. The trend of the calculated and measured wave making resistance curve for NPL-100A hull is quite satisfactory but a significant difference is found in magnitude.

3. The calculated results depend to a certain extent on the discretization of the hull and the free surface. Similar panel arrangement should therefore be used if relative merits of different competing ship designs are to be judged.

6. REFERENCES

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7. NOMENCLATURE

ϕ	Total velocity potential
ϕ_x	velocity of the fluid in the x-direction
ϕ_y	velocity of the fluid in the y-direction
ϕ_z	velocity of the fluid in the z-direction
Φ	Double body velocity potential
φ	Perturbation velocity potential
n	Unit normal vector
ζ	Wave elevation
Γ	Tangential unit vector
u_T	Velocity component at the transom along x-direction
v_T	Velocity component at the transom along y-direction
w_T	Velocity component at the transom along z-direction