**Proceedings of the**
**International Conference on Mechanical Engineering 2009**
**(ICME2009) 26- 28 December 2009, Dhaka, Bangladesh**

**ICME09-RT-28**

# RESPONSE SURFACE MODELING OF FRACTAL DIMENSION IN CNC TURNING

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## ABSTRACT

In the present study, a second order response surface model is developed in CNC turning of mild steel materials. Work-piece speed, feed rate and depth of cut are considered as the process parameters and corresponding fractal dimension (D) is considered as the response. For the experimentation, a rotatable central composite design is selected. The developed model is also checked for adequacy. ANOVA table shows that work speed, feed rate, interaction of depth of cut with work-piece speed and feed rate with work-piece speed are significant at 95% confidence level. It is also seen that with increase in work speed, fractal dimension (D) increases but with increase in feed rate, fractal dimension decreases in mild steel turning.

**Keywords:** Fractal Dimension (D), CNC Turning, RSM.

## 1. INTRODUCTION

In a metal removal process, the generated surface consists of inherent irregularities which are commonly termed as surface roughness whic1h is generally, expressed by some statistical parameters such as centre line average value ($R_a$), root mean square value ($R_q$), mean line peak spacing ($R_{sm}$) etc. The surface roughness is widely used as an index of product quality which has an impact on the mechanical properties like fatigue behavior, corrosion resistance, creep life etc. It also affects other functional attributes of parts like friction, wear, light reflection, heat transmission, lubrication, electrical conductivity etc. Thus the quality of the generated surfaces in machining is very important in manufacturing science. In order to get prescribed surface finish, selection of proper combination of process parameters is mandatory.

As the surface topography is a non-stationary random process, the variances of slope and curvature depend strongly on the resolution of the roughness-measuring instrument or any other form of filter and these conventional roughness parameters are strongly depend on the resolution and filter processing of the instrument. In this context to express surface roughness the concept of fractal is applied. The concept is based on the self-affinity and self-similarity of surfaces at different scales. Roughness measurements on a variety of surfaces show that the power spectra of the surface profiles follow power laws. This suggests that when a surface is magnified appropriately, the magnified image looks very similar to the original surface. Fractals may retain all the structural information and are characterized by single descriptor, the fractal dimension, *D*. This fractal dimension, which forms the essence of fractal geometry, is both scale-invariant and is closely linked to the concepts of self-similarity and self-affinity [1]. It is therefore essential to use fractal dimension to characterize rough surfaces and provide the geometric structure at all length scales [2]. In a material removal process, mechanical intervention happens over length scales, which extend from atomic dimensions to centimeters. The machine vibration, clearances and tolerances affect the outcome of the process at the largest of length scales (above $10^{-3}$ m). The tool form, feed rate, tool radius in case of single point cutting and grit size in multiple point cutting affect the process outcome at the intermediate length scales ($10^{-6}$ to $10^{-3}$ m). The roughness of the tool or details of the grit surfaces influence the final topography of the generated surface at the lowest length scales ($10^{-9}$ to $10^{-6}$ m). It has been shown that surfaces formed by electric discharge machining, milling, cutting or grinding [3, 4, 5, 6, 7], and worn surfaces have fractal structures, and fractal parameters can reflect the intrinsic properties of surfaces to overcome the disadvantages of conventional roughness parameters.

In the literature review, some relevant literatures in connection with surface roughness modeling in turning are presented here. Grzesik [8] presents how tribological interactions at the interface between the chip and the tool control the surface roughness generation in finish turning with a single point tool. Yang & Tarng [9] have developed a model to find the optimal cutting parameters for turning operations to minimize the surface roughness using Taguchi method. Abouelatta & Madl [10] have found a correlation between surface roughness and cutting vibrations in turning and mathematical models are developed. They have considered three conventional roughness parameters (center line average roughness value, maximum height of the profile and skewness). Davim [11] presents a study of the influence of cutting velocity, feed and depth of cut on the surface roughness obtained in turning using Taguchi design. Conventional roughness parameters, center line average roughness value ($R_a$) and maximum height of the profile ($R_t$) have been considered for the analysis. It is shown that the cutting velocity has a greater influence on the roughness followed by the feed rate. Lin et al. [12] used regression analysis to predict surface roughness and cutting force in turning. Suresh et al. [13] have developed a surface roughness prediction model in turning of mild steel using response surface methodology. They have also attempted to optimize the surface roughness prediction model using genetic algorithm (GA). Arbizu and Perez [14] have developed models, which permit to determine surface quality of parts obtained through turning processes. They have also shown from the developed model that the depth of cut and feed rate have negative influence on surface roughness. Feng and Wang [15] have presented a nonlinear multiple regression analysis and neural network algorithm to predict surface roughness in finish turning. Sahin and Motorcu [16] have developed a surface roughness model for turning of mild steel with coated carbide tools using response surface methodology. Kirby et al. [17] have applied Taguchi design to optimizing the surface finish in a turning operation. Palanikumar et al. [18] have focused on the parametric influence of machining parameters on the surface roughness in turning of glass fiber reinforced polymer. Nalbant et al. [19] (2007) have tried to optimize cutting parameters (insert radius, feed rate and depth of cut) in turning using Taguchi approach. Ramesh et al. [20] have tried to minimize the surface roughness in machining titanium alloy using RSM while Palanikumar [21] have used both RSM and Taguchi techniques to minimize the surface roughness in machining glass fiber reinforced. Sahoo et al. [22] have applied Taguchi method to determine optimum fractal dimension in CNC turning. But the most of the available literatures deal with conventional roughness parameters.

In this study, a second order response surface equation has been developed to predict fractal dimension using Minitab software in turning of AISI 1040 materials. A rotatble central composite design of experiments is used for carrying out the experiments. Work-piece speed (N, rpm), feed rate (f, mm/rev) and depth of cut (d, mm) are the three cutting parameters considered in the study. The developed model has been tested for adequacy by carrying out ANOVA test. The effects of cutting parameters on surface roughness are also analyzed

## 2. FRACTAL CHARACTERIZATION

Multi-scale property of the rough surfaces including machining surfaces is characterized as self similarity and self affinity in fractal geometry implying that when the surface or the profile is magnified more and more details emerge and the magnified image is statistically similar to the original topography. Statistical self-similarity means that the probability distribution of a small part of a profile will be congruent with the probability distribution of the whole profile if the small part is magnified equally in all directions. On the other hand, self-affinity implies unequal scaling in different directions. The qualitative description of statistical self-affinity for a surface profile is shown in Fig. 1.

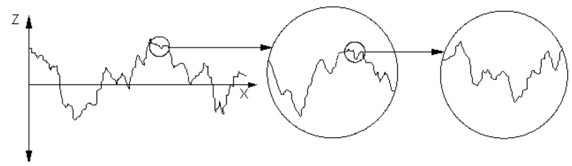



Fig 1. Qualitative description of self-affinity of a surface profile

The property of self-affinity can be characterized by the profile fractal dimension D (1< D <2). Isotropic and homogeneous rough engineering surface of dimension Ds (2 < Ds < 3) is considered in this study. The property of isotropy means that the probability distribution of heights is invariant when the coordinate axes are rotated and the surface is reflected on any plane. The property of homogeneity of a surface indicates that the probability distribution of the height is independent of the location on the surface. Therefore, the profile, z(x), of such a surface along a straight line and in any arbitrary direction is of dimension D = Ds -1 and is a statistically valid representation of the surface. Thus the profile fractal dimension D is adopted to characterize the fractal nature of the surface in this paper. The profile z(x) in Fig. 1 has the mathematical properties of being continuous everywhere but non-differentiable and is self-affine in roughness structure. These properties are satisfied by the Weierstrass–Mandelbrot (W-M) fractal function, which can be used to characterize the roughness of surface profile and is given as

$$z(x) = G^{(D-1)} \sum_{n=n_1}^{\infty} \frac{\cos 2\pi\gamma^n x}{\gamma^{(2-D)n}}; \; 1 < D < 2; \; \gamma > 1 \tag{1}$$

where G is a characteristic length scale, $\gamma$ where L is the sampling length. $\gamma^n = \omega$, where frequency $\omega$ is the reciprocal of wave length and n is called wave number. To provide both the phase randomization and high spectral density $\gamma$ is selected to be 1.5. The parameters G and D form the set to characterize profile z(x). The methods for calculating profile fractal dimension mainly include the yard-stick, the box counting, the variation, the structure function and the power spectrum methods. Out of these, the power spectrum and structure function methods are most popular. Sahoo et al. [22] have presented the procedure to calculate fractal dimension using power spectrum and structure function methods.

## 3. EXPERIMENTAL DETAILS

The design of experiments technique is a very powerful tool, which is used for modeling and analysis of the influence of process variables on the response variables. There are a large number of factors that can be considered for machining of a particular material in turning. However, the review of literature shows that the following three machining parameters are the most widespread among the researchers and machinists to control the turning process: depth of cut (d, mm), spindle speed (N, rpm) and feed rate (f, mm/rev). In the present study these are selected as design factors while other parameters have been assumed to be constant over the experimental domain. A rotatable central composite design (CCD) is selected for the experimentation. The basic central composite design for k variables consists of a 2k factorial design with each factor at two coded levels: -1, +1. For a given number of variables, the $\alpha$ required to achieve rotatability is computed as $\alpha = (n_f)^{1/4}$, where $n_f$ is the number of points in the $2^k$ factorial design (k is the number of factors). In rotatable designs, all points at the same radial distance (r) from the centre point have the same magnitude of prediction error. A rotatable CCD consists of $2^k$ fractional factorial points (usually coded as $\pm 1$), augmented by 2k axial points [($\pm\alpha$, 0,…0),(0, $\pm\alpha$,…0), (0, 0, … $\pm\alpha$)] and $n_c$ centre points (0, 0, 0, 0…,0). The centre points vary from three to six. With proper choice of $n_c$ the CCD can be made orthogonal or it can be made uniform precision design. It means that the variance of response at origin is equal to the variance of response at a unit distance from the origin. Hence, a CCD with uniform precision has been selected for investigation. For three factor experimentation, eight ($2^3$) factorial points, six axial points (2 × 3) and six centre runs, a total of 20 experimental runs have been considered. The value of $\alpha$ is chosen as 1.682. The process variables with their values on different levels are listed in Table 1. The experimental design matrix is shown in Table 2. The CNC turning operations on AISI 1040 materials have been carried out using a JOBBERXL CNC lathe having the control system FANUC Series Oi Mate-Tc and equipped with maximum spindle speed of 3500 rpm, feed rate 15-20 m/rev and KVA rating 16 KVA. For generating the turned surfaces, CNC part programs for tool paths are created with specific commands. Commercially available CVD coated carbide tools are used in this investigation. The tool holder used is PTGNR-25-25 M16 050, WIDIA and insert used is TNMG 160404 –FL, WIDIA. The tool is coated with titanium nitride coating having hardness, density and transverse rupture strength as 1570 HV, 14.5 g/cc and 3800 N/mm$^2$. The compressed coolant WS 50-50 with a ratio of 1:20 with water is used as cutting environment. The response variables used to accomplish the present study on surface topography characterization is the profile fractal dimension D. Roughness profile measurement is done using a portable stylus-type profilometer, Talysurf (Taylor Hobson, Surtronic 3+). The profilometer is set to a cut-off length of 0.8 mm, filter 2CR, traverse speed 1 mm/sec and 4 mm traverse length. The measured profile is digitized and processed through the dedicated advanced surface finish analysis software Talyprofile.

Table 1: Process parameters levels used

| Parameters | Levels | | | | | |
|---|---|---|---|---|---|---|
| | -1.682 | -1 | 0 | 1 | 1.682 | -1.682 |
| Depth of cut | 0.032 | 0.1 | 0.2 | 0.3 | 0.368 | 0.032 |
| w/p speed | 528 | 800 | 1200 | 1600 | 1872 | 528 |
| Feed rate | 0.0224 | 0.07 | 0.14 | 0.21 | 0.2576 | 0.0224 |

## 4. RESPONSE SURFACE METHOD

Response surface methodology (RSM) adopts both mathematical and statistical techniques, which are useful for the modeling, and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize the response [23]. RSM helps in analyzing the influence of the independent variables on a specific dependent variable (response) by quantifying the relationships amongst one or more measured responses and the vital input factors. The mathematical models thus developed relating the machining responses and their factors facilitate the optimization of the machining process. In most of the RSM problems, the form of the relationship between the response and the independent variables is unknown. Thus the first step in RSM is to find a suitable approximation for the true functional relationship between response of interest 'y' and a set of controllable variables {x1, x2, ……xn}. Usually when the response function is not known or non-linear, a second order model is utilized [24] in the form:

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} b_{ii} x_i^2 + \sum \sum_{i<j} b_{ij} x_i x_j + \varepsilon \tag{2}$$

where, $\varepsilon$ represents the noise or error observed in the response y such that the expected response is (y - $\varepsilon$) and b's are the regression coefficients to be estimated. The least square technique is being used to fit a model equation containing the input variables by minimizing the residual error measured by the sum of square deviations between the actual and estimated responses. To check the adequacy of the model Analysis of Variance (ANOVA) test for the regression model is carried out. ANOVA calculates the F-ratio, which is the ratio between the regression mean square and the mean square error. The F-ratio, also called the variance ratio, is the ratio of variance due to the effect of a factor (the model) and variance due to the error term. This ratio is used to measure the significance of the model under investigation with respect to the variance of all the terms included in the error term at the desired significance level.

## 5. RESULTS AND DISCUSSION

In the present study, the second order response model is developed using Minitab in terms of coded values of the independent machining parameters, viz., work-piece speed, feed rate and depth of cut. The response model for the AISI 1040 material is given in the following equation.

$$D_{MS} = 1.40674 + 0.00453\, d + 0.02446\, N - 0.00883\, f + 0.005745\, dN + 0.002873\, df + 0.006850\, Nf - 0.00697\, d^2 - 0.00510\, N^2 - 0.00947\, f^2 \tag{3}$$

The developed model is also checked for adequacy. Table 3 represents the ANOVA table for the second order response model developed for D. It is clear that the developed model is significant at 95% confidence level. The calculated value of F ratio is greater than the tabulated value of F ratio and it can be concluded that the model is adequate at 95% confidence level. ANOVA table for mild steel (Table 4) shows that work speed, feed rate, interaction of depth of cut with work-piece speed are significant factors at 95% confidence level. The $R^2$ value for the model is 0.9385 and it is a clear indication of good correlation. The main effects plots for fractal dimension are shown in Fig. 2. From the main effect plots, it is concluded that with increase in work speed, D increases but with increase in feed rate, D decreases in mild steel turning. Response surface plots are also generated using Minitab. Figure 3 shows the surface plots for fractal dimension as functions of two independent machining parameters while the third machining parameter is held constant at central value. All these figures clearly depict the variation of fractal dimension with controlling variables within the experimental regime. The normal probability plot of the residuals for D is shown in Fig. 4. It is seen that the residuals generally fall on a straight line implying that the errors are distributed normally.

Table 2: Experimental results

| Std. order | Coded Values | | | |
|---|---|---|---|---|
| | *d* | *N* | *f* | *D* |
| 1 | -1 | -1 | -1 | 1.370 |
| 2 | 1 | -1 | -1 | 1.300 |
| 3 | -1 | 1 | -1 | 1.362 |
| 4 | 1 | 1 | -1 | 1.410 |
| 5 | -1 | -1 | 1 | 1.267 |
| 6 | 1 | -1 | 1 | 1.282 |
| 7 | -1 | 1 | 1 | 1.390 |
| 8 | 1 | 1 | 1 | 1.417 |
| 9 | -1.682 | 0 | 0 | 1.320 |
| 10 | 1.682 | 0 | 0 | 1.370 |
| 11 | 0 | -1.682 | 0 | 1.300 |
| 12 | 0 | 1.682 | 0 | 1.420 |
| 13 | 0 | 0 | -1.682 | 1.360 |
| 14 | 0 | 0 | 1.682 | 1.290 |
| 15 | 0 | 0 | 0 | 1.397 |
| 16 | 0 | 0 | 0 | 1.400 |
| 17 | 0 | 0 | 0 | 1.415 |
| 18 | 0 | 0 | 0 | 1.415 |
| 19 | 0 | 0 | 0 | 1.402 |
| 20 | 0 | 0 | 0 | 1.412 |

Table 3: ANOVA for second order model for D

| Source | DF | SS | MS | F | $F_{0.05}$ | P |
|---|---|---|---|---|---|---|
| Regre-ssion | 9 | 0.049 | 0.005409 | 16.96 | 3.02 | 0 |
| Residu al Error | 10 | 0.0032 | 0.000319 | | | |
| Total | 19 | 0.052 | | | | |

Table 4: Full ANOVA table for the model

| Source | Sum of Squares | df | Mean Square | F Value | P value |
|---|---|---|---|---|---|
| Model | 0.049 | 9 | 0.005409 | 16.96 | 0.0001 |
| *A-d* | 0.0007933 | 1 | 0.0007933 | 2.49 | 0.1458 |
| *B-N* | 0.023 | 1 | 0.023 | 72.48 | 0.0001 |
| *C-f* | 0.003009 | 1 | 0.003009 | 9.44 | 0.0118 |
| *AB* | 0.00211 | 1 | 0.00211 | 6.62 | 0.0277 |
| *AC* | 0.0005281 | 1 | 0.0005281 | 1.66 | 0.2271 |
| *BC* | 0.003003 | 1 | 0.003003 | 9.42 | 0.1190 |
| $A^2$ | 0.005608 | 1 | 0.005608 | 17.59 | 0.0018 |
| $B^2$ | 0.002998 | 1 | 0.002998 | 9.40 | 0.0119 |
| $C^2$ | 0.010 | 1 | 0.010 | 32.46 | 0.0002 |
| Residual | 0.00318 | 10 | 0.0003189 | | |
| Lack-of-fit | 0.002871 | 5 | 0.0005742 | 9.04 | 0.0152 |
| Pure Error | 0.0003177 | 5 | 0.00006354 | | |
| Cor Total | 0.05186 | 19 | | | |

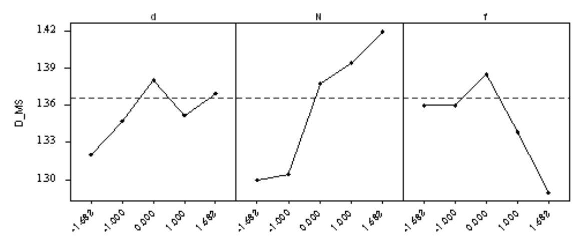
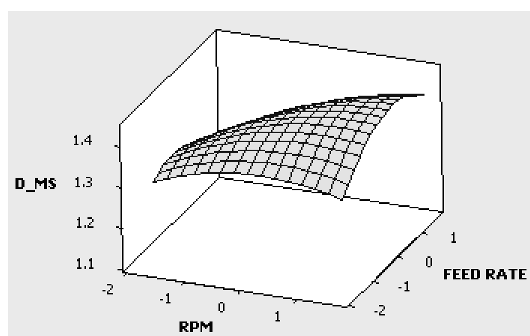
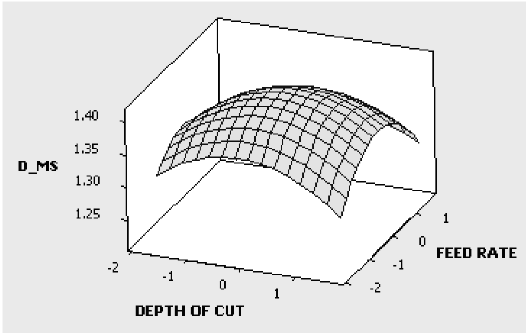


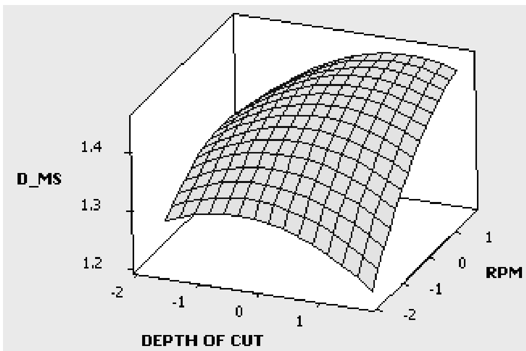

Fig 2. Main effect plots for mild steel




(a)




(b)




(c)

Fig 3. Surface plots for *D* in CNC turning of mild steel

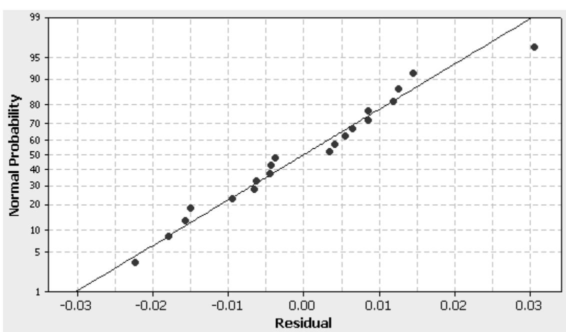



Fig 4. Normal probability plot of the residuals for D

## 6. CONCLUSION

This study presents the development of a second order response surface model for fractal dimension in CNC turning of AISI 1040 material. The second order model is also checked for adequacy using ANOVA. It is seen that the developed model is adequate at 95% confidence level. The normal probability plot of the residuals shows that the errors are distributed normally. From the analysis, it is also seen that the spindle speed is the most significant factor affecting the fractal dimension. It can be concluded from the analysis that with increase in work-piece speed, D increases but with increase in feed rate, D decreases in mild steel turning. With increase in spindle speed, fractal dimension also increases.

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