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FRACTAL DIMENSION MODELING IN CNC MILLING USING TAGUCHI METHOD

T. K. Barman, **P. Sahoo**

Department of Mechanical Engineering, Jadavpur University, Kolkata, India

ABSTRACT

This paper presents an experimental study of fractal dimension characteristics of surface profile produced in CNC milling and optimization of machining parameters based on Taguchi method. To express the surface roughness, fractal dimension (D) is used which is advantageous over the conventional roughness parameters like centre line roughness value (R_a), root mean square roughness (R_q), skewness, kurtosis etc as fractal dimension is scale independent. Experiments on AISI 1040 steels are carried out by utilizing the combination of machining parameters using L_{27} Taguchi orthogonal design with three machining parameters, viz., spindle speed, depth of cut and feed rate. From Taguchi analysis, it is observed that spindle speed has got the most significant influence in controlling fractal dimension characteristics of surface profile. It is also observed with increase in spindle speed the fractal dimension increases. Taguchi analysis is also employed to identify optimum machining parameter combination that yields maximum fractal dimension.

Keywords: Fractal Dimension (D), CNC Milling, Taguchi Method.

1. INTRODUCTION

CNC end milling is the most popular metal removing machines in industries which includes aerospace and automotive sectors, where quality is an important factor in the production of slots, pockets, precision moulds and dies. In end milling, multi point cutting tool removes material at a faster rate from the surface of the work-piece with a reasonably good surface quality. Several factors influence the final surface roughness in a CNC milling operation. In end milling, the parameters such as the tool geometry including the tool nose radius and flank width, run-out error and various cutting conditions including feed rate, depth of cut and cutting speed are influencing factors for the surface roughness.

Generally, the surface roughness is expressed by conventional roughness parameters such as centre line average value (R_a), root mean square value (R_q), mean line peak spacing (R_{sm}) etc. As the surface topography is a non-stationary random process, the variances of slope and curvature depend strongly on the resolution of the roughness-measuring instrument or any other form of filter and these conventional roughness parameters are strongly depend on the resolution and filter processing of the instrument. In this context to express surface roughness the concept of fractal is applied. The concept is based on the self-affinity and self-similarity of surfaces at different scales. Roughness measurements on a variety of surfaces show that the power spectra of the surface profiles follow power laws. This suggests that when a surface is magnified appropriately, the magnified image looks very similar to the original surface. Fractals may retain all the structural information and are characterized by single descriptor, the fractal dimension, *D*. This fractal dimension, which forms the essence of fractal geometry, is both scale-invariant and is closely linked to the concepts of self-similarity and self-affinity [1]. It is therefore essential to use fractal dimension to characterize rough surfaces and provide the geometric structure at all length scales [2]. In a material removal process, mechanical intervention happens over length scales, which extend from atomic dimensions to centimeters. The machine vibration, clearances and tolerances affect the outcome of the process at the largest of length scales (above 10^{-3} m). The tool form, feed rate, tool radius in case of single point cutting and grit size in multiple point cutting affect the process outcome at the intermediate length scales (10^{-6} to 10^{-3} m). The roughness of the tool or details of the grit surfaces influence the final topography of the generated surface at the lowest length scales (10^{-9} to 10^{-6} m). It has been shown that surfaces formed by electric discharge machining, milling, cutting or grinding [3, 4, 5, 6, 7], and worn surfaces have fractal structures, and fractal parameters can reflect the intrinsic properties of surfaces to overcome the disadvantages of conventional roughness parameters.

There are a number of researchers who have analyzed a lot relating to the surface roughness of the fine products and developed surface roughness prediction models. Fuh and Wu [8] developed a model for prediction of surface quality in end milling process using RSM and Takushi method. Lou et al. [9] have used multiple regression models to develop a surface roughness model to predict Ra in CNC end milling. Tsai et al. [10] have developed an artificial neural networks (ANN) model using spindle speed, feed rate, depth of cut, and the vibration average per revolution as four input neurons, to predict the output neuron-surface roughness R_a values for end milling operation. Yang and Chen [11] have found out optimum cutting parameters for milling of Al 6061 material using Taguchi design. Benardos and Vosniakos [12] have developed a neural network model to predict the surface roughness (R_a) in CNC face milling operation. Mansour and Abdalla [13] have developed a process of a surface roughness for end milling using RSM. Ghani et al. [14] have outlined the Taguchi optimization methodology, which is applied to optimize cutting parameters in end milling when machining hardened steel AISI H13 with TiN coated P10 carbide insert tool. Oktem et al. [15] have developed an effective methodology to determine the optimum cutting conditions leading to minimum roughness in milling using RSM. Reddy and Rao [16] have developed a mathematical model in terms of machining parameters for surface roughness prediction using RSM and optimization is done using GA. Sahoo et al. [17] have carried out fractal dimension modeling in CNC end milling using RSM. El-Sonabaty et al. [18] have used fractal geometry approach for predicting surface roughness using neural network. Routara et al. [19] have developed a prediction model of surface roughness in CNC end milling using response surface methodology. Samanta [20] has also applied different soft computing technique to model surface roughness. But most of these papers deal with the conventional roughness parameters and there are a few literatures which deal with fractal dimension.

In this study, a model is developed using Taguchi's orthogonal array with three cutting parameters, viz. depth of cut (A, mm), spindle speed (B, rpm) and feed rate (C, mm/min) to determine optimum fractal dimension, D in CNC end milling of AISI 1040 material. Confirmation test is conducted to verify the optimal machining parameter combination as predicted by Taguchi analysis. ANOVA test is also carried out to find out the significant factor affecting fractal dimension.

2. FRACTAL CHARACTERIZATION

Multi-scale property of the rough surfaces including machining surfaces is characterized as self similarity and self affinity in fractal geometry implying that when the surface or the profile is magnified more and more details emerge and the magnified image is statistically similar to the original topography. Statistical self-similarity means that the probability distribution of a small part of a profile will be congruent with the probability distribution of the whole profile if the small part is magnified equally in all directions. On the other hand, self-affinity implies unequal scaling in different directions. The qualitative description of statistical self-affinity for a surface profile is shown in Fig. 1.

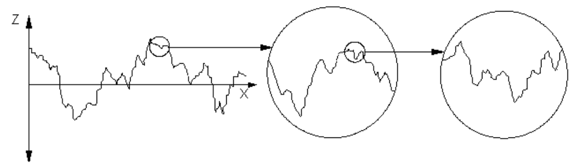


Fig 1. Qualitative description of self-affinity of a surface profile

The property of self-affinity can be characterized by the profile fractal dimension D (1< D <2). Isotropic and homogeneous rough engineering surface of dimension Ds (2 < Ds < 3) is considered in this study. The property of isotropy means that the probability distribution of heights is invariant when the coordinate axes are rotated and the surface is reflected on any plane. The property of homogeneity of a surface indicates that the probability distribution of the height is independent of the location on the surface. Therefore, the profile, z(x), of such a surface along a straight line and in any arbitrary direction is of dimension D = Ds -1 and is a statistically valid representation of the surface. Thus the profile fractal dimension D is adopted to characterize the fractal nature of the surface in this paper. The profile z(x) in Fig. 1 has the mathematical properties of being continuous everywhere but non-differentiable and is self-affine in roughness structure. These properties are satisfied by the Weierstrass–Mandelbrot (W-M) fractal function, which can be used to characterize the roughness of surface profile and is given as

$$z(x) = G^{(D-1)} \sum_{n=n_1}^{\infty} \frac{\cos 2\pi\gamma^n x}{\gamma^{(2-D)n}}; \; 1 < D < 2; \; \gamma > 1 \tag{1}$$

where G is a characteristic length scale, γ where L is the sampling length. $\gamma^n = \omega$, where frequency ω is the reciprocal of wave length and n is called wave number. To provide both the phase randomization and high spectral density γ is selected to be 1.5. The parameters G and D form the set to characterize profile z(x). The methods for calculating profile fractal dimension mainly include the yard-stick, the box counting, the variation, the structure function and the power spectrum methods. Out of these, the power spectrum and structure function methods are most popular. Sahoo et al. [18] have presented the procedure to calculate fractal dimension using power spectrum and structure function methods.

3. EXPERIMENTAL DETAILS

There are a large number of factors that can be considered for machining of a particular material in end milling. However, the review of literature shows that the following three machining parameters are the most widespread among the researchers and machinists to control the milling process: depth of cut (A, mm), spindle speed (B, rpm) and feed rate (C, mm/min). In the present study these are selected as design factors while other parameters have been assumed to be constant over the experimental domain. Considering L_{27} orthogonal array, total 27 experiments are carried out on AISI 1040 material. The process variables / design factors with their values on different levels are listed in Table 1. The selection of the values of the variables is limited by the capacity of the machine used in the experimentation as well as the recommended specifications for different work - tool material combinations. The machine used for the milling tests is a 'DYNA V4.5' CNC end milling machine having the control system SINUMERIK 802 D with a vertical milling head and equipped with maximum spindle speed of 9000 rpm, feed rate 10 m/min and 10kW driver motor. For generating the milled surfaces, CNC part programs for tool paths are created with specific commands. Commercially available CVD coated carbide tools are used in this investigation. The tools used are flat end mill cutters (8 mm diameter, 300 helix angle, TiAlN coated solid carbide, parallel shank, 4 flutes) produced by WIDIA (EM-TiAlN). The tools are coated with TiAlN coating having hardness, density and transverse rupture strength as 1570 HV, 14.5 g/cc and 3800 N/mm^2. The compressed coolant servo-cut is used as cutting environment. All the specimens are in the form of 100 mm x 75 mm x 25 mm blocks. The response variables used to accomplish the present study on surface topography characterization is the profile fractal dimension D. Roughness profile measurement is done using a portable stylus-type profilometer, Talysurf (Taylor Hobson, Surtronic 3+). The profilometer is set to a cut-off length of 0.8 mm, filter 2CR, traverse speed 1 mm/sec and 4 mm traverse length. The measured profile is digitized and processed through the dedicated advanced surface finish analysis software Talyprofile.

Table 1: Cutting parameters and their levels

Levels	Depth of cut (A, mm)	Spindle speed (B, rpm)	Feed rate (C, mm/min)
1	0.15	2500	300
2	0.20	3000	400
3	0.25	3500	500

4. TAGUCHI METHOD

Taguchi technique [21, 22] is a powerful tool for design of high quality systems based on orthogonal array experiments that provide much reduced variance for the experiments with an optimum setting of process control parameters. It introduces an integrated approach that is simple and efficient to find the best range of designs for quality, performance and computational cost. This method achieves the integration of design of experiments (DOE) with the parametric optimization of the process yielding the desired results. The orthogonal array (OA) provides a set of well balanced (minimum experimental runs) experiments. Taguchi's method uses a statistical measure of performance called signal–to-noise ratios (S/N), which are logarithmic functions of desired output to serve as objective functions for optimization. The S/N ratio takes both the mean and the variability into account and is defined as the ratio of the mean (signal) to the standard deviation (noise). The ratio depends on the quality characteristics of the product/process to be optimized. The three categories of S/N ratios are used: lower-the-better (LB), higher-the-better (HB) and nominal-the-best (NB). The parameter level combination that maximizes the appropriate S/N ratio is the optimal setting. Furthermore, a statistical analysis, ANOVA [23] is performed to find which process parameters are statistically significant. With the S/N ratio and ANOVA analyses, the optimal combination of the process parameters can be predicted. Finally, a confirmation experiment is to be conducted to verify the optimal process parameters obtained from the parameter design.

5. RESULTS AND DISCUSSION

Based on the Taguchi and ANOVA analysis, the results are listed below. The signal to noise analysis is carried out with the fractal dimension (D) as the performance index. The S/N ratio for D is calculated using Higher the Better (HB) criterion as higher the fractal dimension D gives a smoother surface topography and for this S/N ratio is given by

$$S/N = -10\log\left(\frac{1}{n}\sum\frac{1}{y^2}\right) \qquad (2)$$

where y is the observed data and n is the number of observations. Experimental results for fractal dimension of machined surfaces (D) and the corresponding S/N ratios are provided in Table 2. The mean S/N ratio for each level of three factors (A, B and C) and the total mean S/N ratio for the 27 experiments are summarized in Table 3. The main effect plot for mean S/N ratio for AISI 1040 material is also presented in Fig. 2. From the main effect plot, it is observed that the parameters A and B are the significant factors because in the main effect plot, lines for these two parameters have the highest inclination. Whereas parameter C is less significant as the line of C parameter in the main effect plot is near horizontal. It is clear from the figure that the optimum machining parameters combination for maximum fractal dimension (D) is A2B3C3.For estimating the interaction among the parameters, interaction plots are drawn. When the lines on the interaction plots are non-parallel, interaction occurs and when the lines cross, strong interaction occurs. It is seen from the figure that there are interactions between A and C and between B and C.

To check the accuracy of analysis, a verification test needs to be carried out. The estimated S/N ratio, $\hat{\gamma}$, using the optimal level of the machining parameters can be calculated as:

$$\hat{\gamma} = \gamma_m + \sum_{i=1}^{o}\left(\bar{\gamma}_i - \gamma_m\right) \qquad (3)$$

where γ_m is the total mean S/N ratio, $\bar{\gamma}_i$ is the mean S/N ratio at the optimal level, and o is the number of the main design parameters that significantly affect the fractal dimension. Table 4 shows the comparison of the estimated S/N ratio with the actual S/N ratio using the optimal parameters. It is seen that there is good agreement between the estimated and actual S/N ratios observed. The increase of the S/N ratio from the initial machining parameters to the optimal machining parameters is 0.50367dB, which means the fractal dimension is improved by about 20%. In other words, the experimental results confirm the prior design and analysis for optimizing the machining process parameters.

Table 2: Experimental results and corresponding S/N ratio

A	B	C	D	S/N ratio (dB)
1	1	1	1.31	2.34543
1	1	2	1.28	2.14420
1	1	3	1.32	2.41148
1	2	1	1.32	2.41148
1	2	2	1.37	2.73441
1	2	3	1.38	2.79758
1	3	1	1.39	2.86030
1	3	2	1.41	2.98438
1	3	3	1.37	2.73441
2	1	1	1.37	2.73441
2	1	2	1.43	3.10672
2	1	3	1.40	2.92256
2	2	1	1.37	2.73441
2	2	2	1.34	2.54210
2	2	3	1.37	2.73441
2	3	1	1.43	3.10672
2	3	2	1.40	2.92256
2	3	3	1.42	3.04577
3	1	1	1.30	2.27887
3	1	2	1.38	2.79758
3	1	3	1.39	2.86030
3	2	1	1.38	2.79758
3	2	2	1.34	2.54210
3	2	3	1.40	2.92256
3	3	1	1.40	2.92256
3	3	2	1.40	2.92256
3	3	3	1.36	2.67078

Table 5 shows that ANOVA results for fractal dimension. ANOVA calculates the F ratio, which is the ratio of the regression mean square and the mean square error. In general, when the F value increases the significant of the parameters also increases. From the table it is clear that the parameter B is the most significant factor at 95% confidence level within the specific test range whereas other terms are not so significant at 95% confidence level.

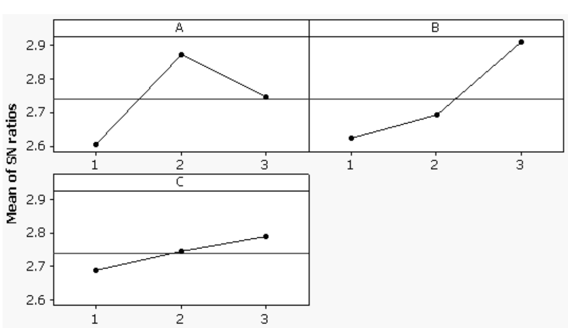
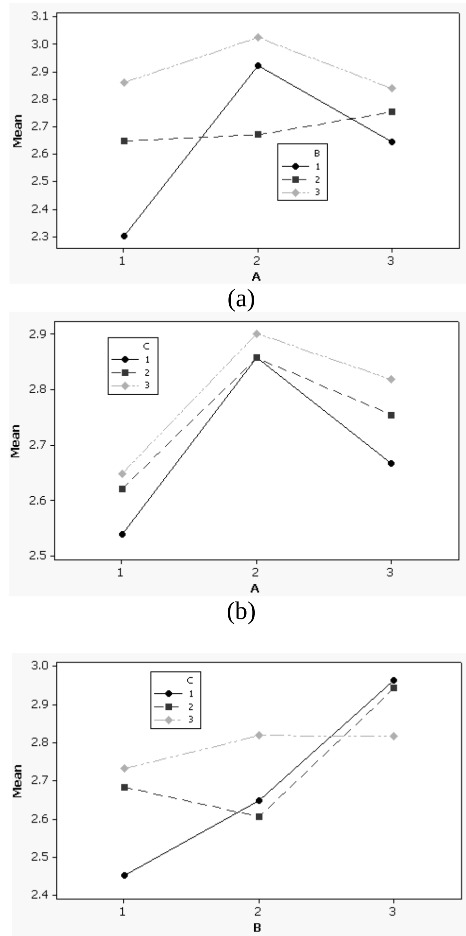


Fig 2. Main effect plots for mean S/N ratio



(a)

(b)

(c)

Fig 3. Interaction effect plots for mean S/N ratio: (a) A vs. B, (b) A vs. C and (c B vs. C.

Table 3: S/N Ratio response table for mild steel

Level	A	B	C
1	2.603	2.622	2.688
2	2.872	2.691	2.744
3	2.746	2.908	2.789
Delta	0.270	0.285	0.101
Rank	2	1	3

The total mean S/N ratio= 2.7403

Table 4: Result of confirmation experiment for fractal dimension of mild steel material

	Initial machining parameters	Optimal machining parameters	
		Prediction	Experiment
Level	A2B2C2	A2B3C3	A2B3C3
Fractal Dimension (D)	1.34		1.42
S/N ratio (dB)	2.54210	3.08840	3.04577

Improvement of S/N ratio = 0.50367 dB
% improvement= 19.81%

Table 5: ANOVA table for AISI 1040 material

Sou-rce	DF	SS	MS	F	P
A	2	0.008030	0.004015	4.00	0.062
B	2	0.009785	0.004893	4.87	0.041
C	2	0.001096	0.000548	0.55	0.599
AXB	4	0.008126	0.002031	2.02	0.184
AXC	4	0.000281	0.000070	0.07	0.989
BXC	4	0.004993	0.001248	1.24	0.366
Error	8	0.008030	0.001004		
Total	26	0.040341			

6. CONCLUSION

In this paper, a has been carried out to optimize the machining parameters with respect to fractal dimension using Taguchi method in CNC milling of AISI1040 material. L_{27} orthogonal array is taken for the analysis. From the analysis, it can be concluded that the spindle speed is the most significant factor affecting the fractal dimension. With increase in spindle speed, fractal dimension also increases. An optimum cutting parameter combination is found out for maximum fractal dimension and it may be useful in computer aided process planning.

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8. MAILING ADDRESS

T K Barman
Department of Mechanical Engineering, Jadavpur
University, Kolkata- 700032, India
FAX : +91 33 2414 689
E-mail: tkbarman@gmail.com