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EVALUATION OF PROCESS CAPABILITY USING FUZZY INFERENCE SYSTEM

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ABSTRACT

In many industrial instances product quality depends on a multitude of dependent characteristics and as a consequence, attention on capability indices shifts from univariate domain to multivariate domain. In this research fuzzy inference system is used to determine the process capability index. Fuzzy sets can represent imprecise quantities as well as linguistic terms. Fuzzy inference system (FIS) is a method, based on the fuzzy theory, which maps the input values to the output values. The mapping mechanism is based on some set of rules, a list of if-then statements. In this research Mamdani fuzzy inference system is used to derive the overall output process capability when subjected to six crisp input and one output. This paper deals with a novel approach to evaluating process capability based on readily available information using fuzzy inference system.

Keywords: Process Capability, Fuzzy Inference System, Design of Experiment.

1. INTRODUCTION

In manufacturing industry, Process capability analysis is designed to monitor the proportion of items which are expected to fall outside the engineering specifications in order to prevent an excessive production of nonconforming output. Capability analysis is typically performed by evaluating capability indices which relate the allowable spread of the process, defined by the engineering specifications, to the natural spread of the process. Capability of a process may not depend on only one variable. The process depends on the combined effect of different factors which are known as multivariate process. The process potential index, one of process capability indices depends upon the process variability. To determine the value of process capability index the traditional method is not suitable. Because it can not show the combined effect of different factors affect the process capability. For measuring the capability of this type of process, fuzzy inference system is a good tool. Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. The mapping then provides a basis from which decisions can be made, or patterns discerned. The expert based fuzzy inference system makes it easy to measure the process capability. The membership function and the range of process specification help to produce the inference system.

2. LITERATURE REVIEW

Process Capability Indices (PCIs) have generated extensive attention in statistical and quality control publications in recent times. Juran [1] introduced the first process capability index is a simple relative number comparing the value of the required process capability. Capability index

$$Cp= \frac{USL-LSL}{6\sigma} \tag{1}$$

Where USL and LSL are upper and lower specification limit respectively and σ is the process standard deviation.

Kane [2] proposed capability index C_{pk} that reacts to deviation of the measurement process average mean from check standard nominal value G.Taguchi and T.C. Hsiang [3] designed a loss function as a new approach to production process quality improvement.Yeh and Bhattacharya [4] proposed use of process capability indices based on the ratios of expected proportion NC to actual observed or estimated proportion NC. Flaig [5-8] strongly supports the use of 'fractional conforming' [=1-proportion NC] as a basis for PCIs which will be suitable for any uni modal distribution of X, using Camp-Meidell inequality. Singpurwalla [9] uses the even broader Chebyshev inequality, applicable for any distribution of X.

Clements [10] suggested that 6σ be replaced by the length of the interval between the upper and lower 0.135 percentage points of the distribution of X (this 6σ' for a normal N (distribution). Wright [11] suggested PCIs sensitive to skewness. Bai and Choi [12] have constructed PCI for use with possibly skewed distribution of X, based on a 'Weighted variance' (WV) approach. When the capability of the process depends on many variables, then the calculation of multivariate indices needed to be took place. In this way, Taam, Subbaiah and Liddy [13] generated the first multivariate capability index for the bivariate case. Pal [14] proposed the index. Bothe [15] proposed a method in order to compute the multivariate C_{pk} index. Wang and Cheng [16] proposed multivariate equivalents for Cp, $C_{pk,}$ C_{pm} and C_{pmk} based on PCA (Principal Component Analysis) decomposition. Wang and Du [17] proposed the same indices and one extension to the multivariate case.

Shahriari et al. [18] and Hubel proposed a process capability multivariate vector in order to evaluate the process performance. The multivariate characteristics of the processes that specify geometric dimensions have led to much research and new capability indices for analyzing the multivariate processes. Nuehard [19] proposed a method for calculating capability indices for multivariate processes in which the variance is adjusted for correlation by multiplying it by a factor, and then the adjusted variance is used to calculate the indices. Hubele et al [20] discussed the disadvantage of using univariate capability index and the advantage of the bivariate process capability vector. They considered the bivariate normal distribution and analyzed the process for its capability.

For non-normal processes, Rogowski [21] proposed to transfer the data into normal and then to use the Ordinary Cp of the normally distributed process. The type of transformation is obtained by goodness-of-fit tests.

Geometric Dimensioning and Tolerancing (GD&T) is the engineering standard that provides a unified terminology and approach to describe both the geometry of the features and the associated tolerance of the product. GD&T contains different types of tolerances such as form, orientation, profile, position, and run-out. Gruner [22] explained how to calculate a PCR for a specified variable tolerance (tolerance varies as the actual size of the feature varies) using GD&T. Methods for generating the PCR for a spherical or circular tolerance zone and concentricity was studied by Davis[23] et al. Sakar and Pal[24].

Process capability measures that have been introduced in the past years can be generally classified into three categories: process capability measures for coordinate tolerance specifications, process capability measures for geometric tolerance specifications, and process capability measures for multivariate processes.

The above methods didn't use fuzzy logic to find out the process capability index Cp. In this paper some factors which has significant impact on process capability is considered to calculate Cp using fuzzy inference system.

3. OBJECTIVE OF THE STUDY

The objective of the present work is to compute the process capability of a soap production process. Six input variables which has effect on the process capability of a soap production and one output variable that is the process capability are considered. The input variables are mixing temperature, coolant temperature, flow rate, moisture content, urea content and free fatty acid content.

4. METHODOLOGY

The work has been carried out in a soap factory to do this job. Six factors have been identified which affect the process capability of soap production by doing the Design of Experiment (DoE). Those are: mixing temperature, coolant temperature, flow rate of coolant, percentage of fatty acid, percentage of urea, and the moisture content. The combined effect of these six factors of this process is shown in figure 1.

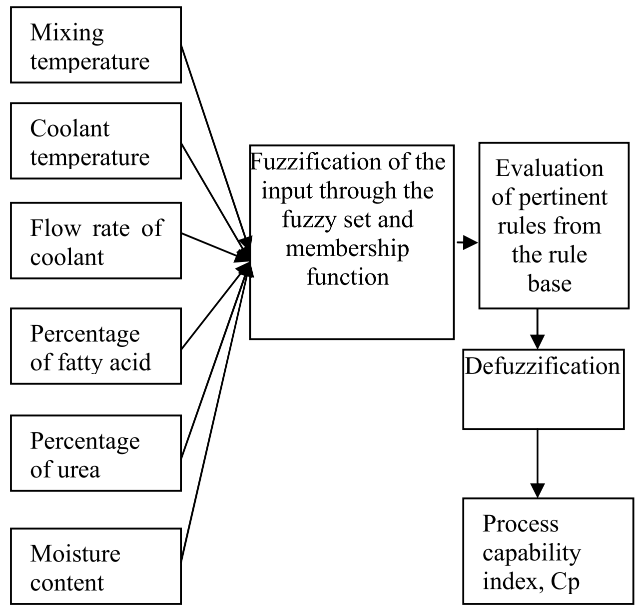

Fig 1. Fuzzy Model of proposed system

4.1 Membership Function

Membership function is the mathematical function which defines the degree of an element's membership in a fuzzy set. The Fuzzy Logic Toolbox of MATLAB includes 11 built-in membership function types. These functions are built from several basic functions:

- Piecewise linear functions,
- The Gaussian distribution function,
- The sigmoid curve and
- Quadratic and cubic polynomial curve.

4.2 Input Fuzzification

The step is to take the inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions. Here the input variables are mixing temperature, coolant temperature and flow rate of coolant, moisture content, urea content and free fatty acid content.

The membership function and the linguistic variables of input and output variables are entered in Matlab Fuzzy Toolbox' Membership Function Editor. Gaussian membership function is used for each linguistic variable. Figure 3 represents the membership function and linguistic variables for one input mixing temperature. Similarly membership function and linguistic variables are entered for other variables.

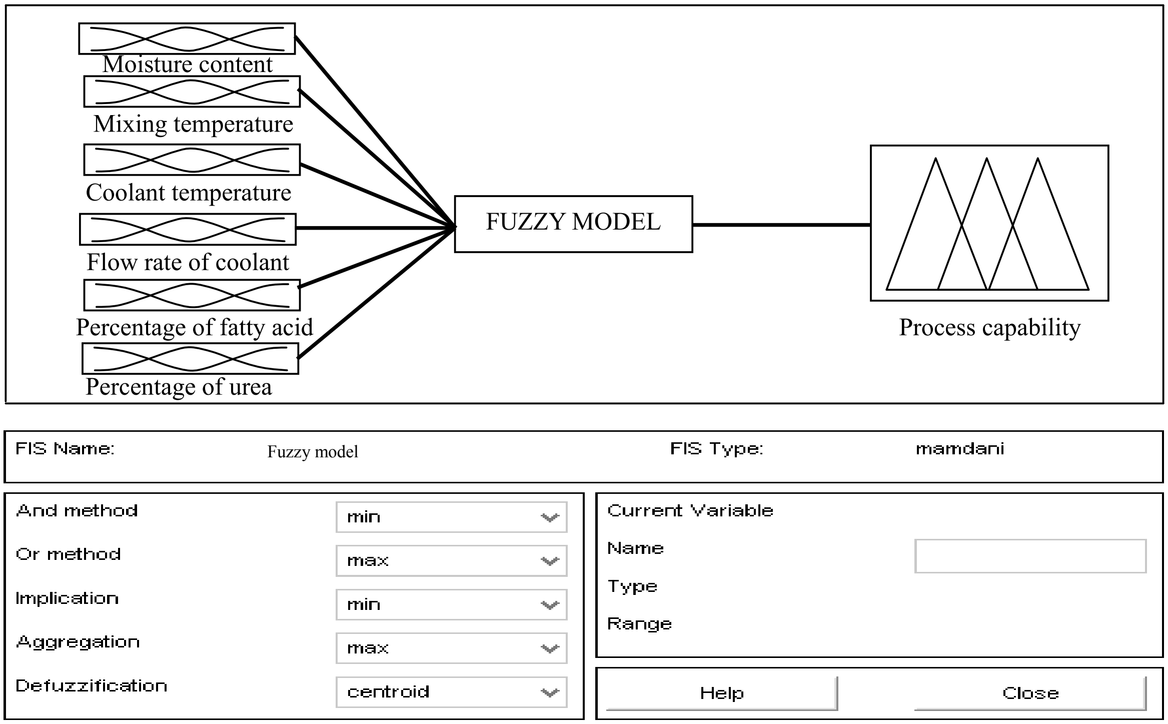

Fig 2. FIS model for Process capability analysis

Table 1: The variation of input linguistic variables

Input variable	Low	Medium	High
Mixing temperature	[17.00,60]	[21.2,150]	[10.6,225]
Coolant temperature	[2.12,2.5]	[6.37,5]	[4.25,20]
Flow rate of coolant	[318.5,750]	[424.7,2000]	[318.5,3250]
Percentage of fatty acid	[1.7,6]	[1.06,12.5]	[2.12,20]
Percentage of urea	[5.3,17.5]	[4.25,40]	[5.31,62.5]
Moisture content	[1.7,4]	[1.50,11.5]	[1.66,19]

Table 2: The variation of output linguistic variables

Output variable	Totally incapable	Semi incapable	Almost incapable	Semi capable	Almost capable	Capable	Highly capable
Process capability	[0.064, 0.15]	[0.074,0.425]	[0.053,0.675]	[0.042,0.9]	[0.042,1.1]	[0.11,1.45]	[0.06,1.85]

Different input variables and corresponding values of low, medium and high range in terms of standard deviation and mean are shown in table1. Output variable and different ranges are shown in table 2.

The membership function and the linguistic variables of input and output variables are entered in Matlab Fuzzy Toolbox' Membership Function Editor. Gaussian membership function is used for each linguistic variable. Figure 3 represents the membership function and linguistic variables for one input; mixing temperature. Similarly membership function and linguistic variables are entered for other variables.

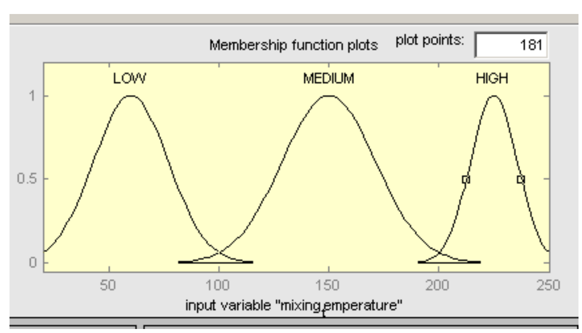

Fig 3. Membership function and linguistic variables for mixing temperature

4.3 Antecedent Matching

Once the inputs have been fuzzified, the degree to which each part of the antecedent has been satisfied for each rule is known. If the antecedent of a given rule has more than one part, the fuzzy operator is applied to obtain one number that represents the result of the antecedent for that rule. This number will then be applied to the output function. The input to the fuzzy operator is two or more membership values from fuzzified input variables. The output is a single truth value. Here AND operation has been used.

4.4 Rule Fulfillment

After the membership functions are determined, the rules are designated and written in Matlab Fuzzy Toolbox for evaluation. And after the needed data are entered, inputs are solved according to change of input positions for solving of fuzzified systems. Here an example of rule is given:

If (mixing _temperature is Low) and (coolant_ temperature is Medium) and (flow_rate is Low)and (urea _content is Low) and (moisture _content is Low) and (free fatty acid _content is High) then (process is semi incapable)

4.5 Consequent Aggregation

Since decisions are based on all the rules in a fuzzy inference system, the rules have been combined in some manner in order to make a decision. Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set.

4.6 Output Defuzzification

Taking fuzzy sets as input, defuzzification outputs a crisp value, which is suitable for analysis and control. There are usually two types of fuzzy inference systems differing in defuzzification part, namely Mamdani-type and Sugeno-type. Here Mamdani-type has been used.

Mamdani-type inference expects the output membership functions to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification. In this work, the weight of each factors have been taken as one.

5. RESULT AND DISCUSSION

The data are taken from a soap manufacturing industry. The measurements are done for 30 data and the sample size for each time is 4. Then the result is analyzed on the basis the data has been performed.

5.1 Goodness of Fit Test

To check whether the taken data fit the normal distribution, Kolmogorov-Smirnov goodness of fit test For hypothesis testing, the null hypothesis is

H_0 : Data fit normal distribution.

H : Data don't fit normal distribution.

According to the basic of Kolmogorov-Smirnov goodness of fit test, D is calculated. If the calculated value of D is less than that of critical value than the null hypothesis is accepted.

For 95.5% confidence level D (upper tail percentage point) is 0.895. The formula for calculating modified D is: The maximum value of D from the 9^{th} column ($\sqrt{n}$ - 0 .01 +.85/ $\sqrt{n}$) Here 'n 'is the no of samples:

Table 3: Value of Upper tail Percentage Point (D)

Factors	Modified value of D
Mixing temperature	0.8865
Coolant temperature	0.6645
Flow rate	0.862

As rejection region starts from 0.895 so it can be said that the data for above factors are fit for normal distribution. At the same way for other three factors, it can be said that the data are normally distributed.

5.2 The Rules Editor

Total 25 rules are developed and the Matlab Fuzzy Toolbox Rule Editor is used to develop these rules.

5.3 Surface Viewer

The results which have been obtained from Matlab Fuzzy Toolbox are shown below in figure 4, 5 and 6. These figures show the effects of the inputs to the output. Figure 4 shows the how process capability is varied with the input variables mixing temperature and coolant temperature. In figure 4, the step wise changes of the variables are shown. If the mixing temperature is taken as 150, it is seen from the surface viewer that the process is semi capable. If the coolant temperature is taken as 0, it will also show semi capable. Some time it is seen that the actual results don't match with the surface viewer. This is because of the combined effect. Figure 5 represents the process capability is varied with flow rate and mixing temperature. Moreover figure 6 shows the change of process capability with changing of input; moisture content and mixing temperature. The variation of process capability against the input variables depends on the developed rules.

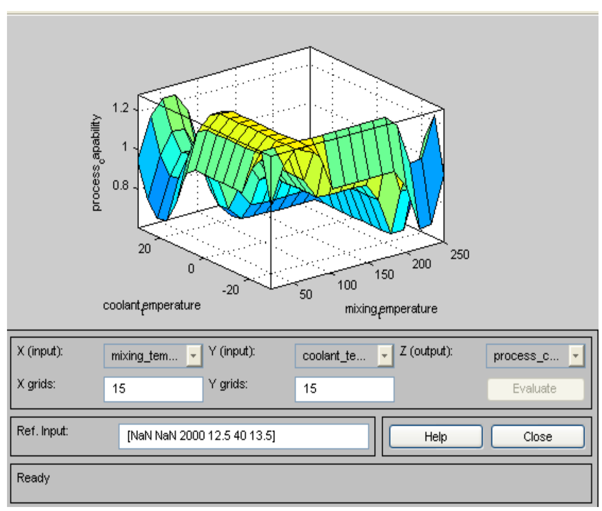

Fig 4. Surface Analysis between mixing temperature and coolant temperature with process capability

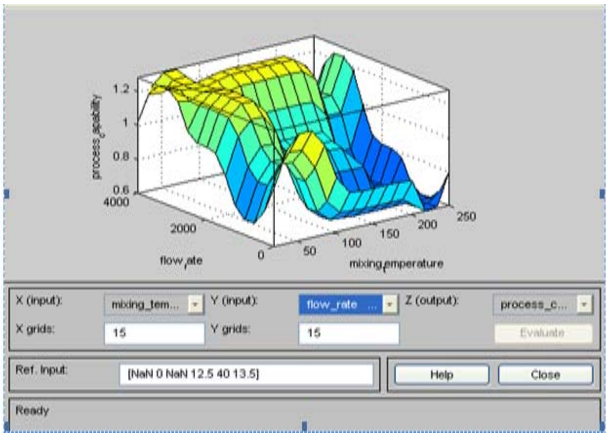

Fig 5. Surface Analysis between flow rate and mixing temperature with process capability

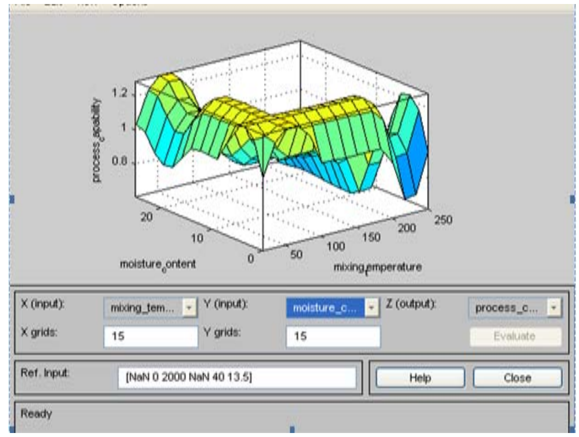

Fig 6. Surface analysis between moisture content and mixing temperature with process capability

5.4 Case Study

A case study is done to show the effect of the factors on process capability. The inputs for the six factors are feed in a rule viewer of MATLAB fuzzy inference system.

The rule viewer is shown in figure 7. Different output (process capability) can be measured in response to changing input parameters. The values of input variables and measured output (process capability) are shown in table4.

Fig 7. Rule Viewer for case study

Table 4: Values of different parameters for case study

SL. No	Factors	Value
1.	Mixing temperature(^{0}c)	222
2.	Coolant temperature(^{0}c)	-10
3.	Flow rate of coolant(lt/s)	2500
4.	Moisture content (%)	12.5
5.	Urea content (%)	40
6.	Free fatty acid content (%)	13.8
Output variable	Process capability	0.608

The rule viewer of MATLAB fuzzy inference system is used to feed the inputs and to get the output; that is the process capability.

6. CONCLUSION

In this paper, the process capability is obtained by considering moisture content, mixing temperature, flow rate of coolant, urea content, free fatty acid content and moisture content using the fuzzy logic is important as it gives the optimal result. The manufacturing companies should use the optimal result which reduce the total cost and increase the overall profit. Fuzzy logic helps to find these optimal results if the company's expert provided data are available. In this research, using data of a soap manufacturing company the final result is obtained by which process capability is determined. Here, six input variables are considered to find the process capability. For each input and output variable Gaussian membership functions are considered to design the model.

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8. NOMENCLETURE

Symbol	Meaning
FIS	Fuzzy Inference System
MF	Membership Function
DoE	Design of Experiment
PCI	Process Capability Index

9. MAILING ADDRESS

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