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NATURAL CONVECTION AND RADIATION IN CIRCULAR AND ARC CAVITY

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ABSTRACT
The radiation effect of gray surfaces on multiple steady-state solutions obtained in circular and arc-square inclined enclosure filled with air has been investigated numerically, by a finite-volume procedure. The left and right surfaces of the cavity are, respectively, heated and cooled at constant temperatures, while its horizontal walls are adiabatic. Parameters of the problem are the Rayleigh number $Ra(10^2 > Ra > 10^6)$, the inclination angle γ (0 to 90∘), aspect ratio equal to 1 and the surface emissivity $(0 < \varepsilon < 1)$.

Keywords: Constant Temperature, Circular And Arc Cavity, Nusselt Number, Radiation

1. INTRODUCTION

Heat transfer by combination of natural convection, conduction and radiation occurs when simulating building components, in particular passive heating and cooling systems, cooling of electronic components, design of solar collectors etc. Natural convection, coupled with surface radiation in a square cavity, heated from below [1], the conjugate heat transfer in an inclined square enclosure bounded by a solid wall [2], mixed convection and thermal radiation in ventilated cavities with gray surfaces [3] have been studied numerically. In the past, a great number of studies have focused on a variety of cavity configurations formed with straight walls, but the effects of curving the walls inward to form a derived circular cavity are beneficial because the fluid follows a less convoluted path. The flow and temperature field alters with the shape and orientation of the cavity.

2. PHYSICAL MODEL

The Physical diagram of the problem and the boundary conditions are shown in the fig. 1 (a) and (b). They are circular and arc cavities filled with air. Diameter of the cavity is H, where adiabatic horizontal sides are 0.57 H. The left wall is at a constant temperature T_h and it is heated. The cold wall is also at a constant temperature T_c. It is inclined at an angle γ with the horizontal axis. Laminar fluid flow and constant fluid properties are assumed here. The gravitational acceleration is considered in the normal vertical direction. The fluid outside and the horizontal walls are maintained to an ambient temperature T_a. In the present study, Rayleigh number is varied from 10^2 to 10^6; emissivities from 0 to 1; inclination angles from 0 to 90^0 .The aspect ratio and Prandtl number are 1 and 0.7 respectively throughout the problem.

3. MATHEMATICAL MODELING

It is assumed that the flow is two-dimensional, steady state, laminar and the fluid is incompressible. The thermo-physical properties of the fluid are assumed to be constant except in the buoyancy term of the momentum equation, i.e., the Boussinesq approximation. Radiation heat transfer is taken into consideration along with natural convection mode. The viscous energy dissipation term in the energy equation can be neglected because of the small flow velocities associated with free convection. Continuity Equation:

$$\frac{\delta U}{\delta X} + \frac{\delta V}{\delta Y} = 0 \tag{1}$$

X-Momentum Equation:

$$U\frac{\delta U}{\delta X} + \frac{\delta U}{\delta Y} = -\frac{\delta P}{\delta X} + \frac{1}{\text{Re}}(\frac{\delta^2 U}{\delta X^2} + \frac{\delta^2 U}{\delta Y^2}) + Ri\theta\sin(\gamma) \tag{2}$$

Y-Momentum Equation:

$$U\frac{\delta V}{\delta X} + \frac{\delta V}{\delta Y} = -\frac{\delta P}{\delta Y} + \frac{1}{\text{Re}}(\frac{\delta^2 V}{\delta X^2} + \frac{\delta^2 V}{\delta Y^2}) + Ri_y\theta\cos(\gamma) \tag{3}$$

Energy Equation:

$$U\frac{\delta\theta}{\delta X} + V\frac{\delta\theta}{\delta Y} = \frac{1}{\text{Re*Pr}}(\frac{\delta^2\theta}{\delta X^2} + \frac{\delta^2\theta}{\delta Y^2}) \quad \dots \tag{4}$$

At the surface:

$$Nr\varsigma = (\frac{\delta\theta_f}{\delta X} - k_r\frac{\delta\theta_s}{\delta X}) \tag{5}$$

where $Nr = \sigma T_\infty^4 / q''$ is the radiation number, ζ is dimensionless radiative heat flux, $\zeta = q_r/\sigma T_\infty^4$, $\delta\theta_f / \delta X$ and $\delta\theta_s / \delta X$ are the heat flux from the wall surface to the fluid on the right and to the solid on the left, respectively. The governing parameters are Ra, $Pr = \upsilon/\alpha$, k_r, ε, and $A = L/H$ and $w = l/H$.

The average convective and radiative Nusselt numbers are calculated at $X = 0$ plane as

$$Nu_c = -\int_0^1 \frac{1}{\theta} \frac{\delta\theta}{\delta X} dY \quad (6) \qquad Nu_r = -\int_0^1 \frac{1}{\theta} Nr\varsigma_s dY \quad (7)$$

The dimensionless temperature $\theta = \dfrac{T - Tc}{Th - Tc}$ (8)

4. GRID SENSITIVITY TEST

In order to obtain grid independent solution, a grid refinement study is performed for the cavity under constant temperature considering Ra=10^6, Pr = 0.71, γ = 0 as shown in Table 1.

Table 1: Grid Sensitivity Test

No. of Nodes	Nu_{av}
10	0.99587268
30	1.0433495
50	1.0516758
70	1.0532773
90	1.0531155

5. CODE VALIDATION

Figure 2(a) and 2(b) reveals the streamline and isotherm obtained in this present study have excellent agreement with those obtained by El Hassan Ridouane and Antonio Campo [4]. The table 2 shows the comparative values of Nusselt numbers for different Rayleigh numbers.

Table 2: Code Validation

Rayleigh number	Circular caviy Present study Nu	Circular cavity Paper Nu	Arc cavity Present study Nu	Arc cavity Paper Nu
10^2	1.281	1.245	1.138	1.114
10^3	1.4038	1.32	1.1758	1.175
10^5	4.8195	4.94	4.700	4.71
10^6	9.1151	9.2	9.0378	9.05

6. RESULTS AND DISCUSSION

The numerical results obtained here are governed by several variable influential parameters. Those are Rayleigh numbers varied from 10^2 to 10^6, the inclination angles ranged from 0° to 90° and the emissivity of materials changed from 0 to 1.

6.1 Effect of Rayleigh Number

From the stream line plots it can be seen that all four configurations contain a single rotating vortex. As the Rayleigh number increases, the circulation evolving from buoyancy inside the cavities gets energetic. It is seen that there are two vortices instead of a single, in the lower right and upper left corners. Vortex rotation is clockwise.

The isotherms are parallel to the gravity field at low Ra that indicates the typical conduction temperature distribution. Marked compression of the isotherms toward the boundary surfaces of the enclosure occurs to a greater extent with increasing Ra. Combined Nusselt number is a strong increasing function of Ra. It is almost horizontal up to critical Ra=10^3, then the deviation begins.

6.2 Effect of Emissivity

The flow pattern and isotherms for the surface emissivity ε = 0.0, 0.6 and 0.1 for circular cavities are shown in fig 3 and 4 for circular and arc cavity respectively. For the case with γ = 0 we present Nu_t as a function of the Rayleigh number in Fig.5. We see the combined nusselt number is an increasing function of ε.

6.3 Comparision of Heat Transfer Rate for Natural Convection and Radiation

It is seen that generally the temperature field is more brilliant and the gradients much higher for ε = 0 than ε = 1. So it can be anticipated that the convective heat transfer be decreased with surface radiation in spite of enhanced circulation.

The spatial arrangement of temperature field changes creating distance within the isotherms which grows with Ra. The fluid temperature inside the cavity goes toward homogeneity with the existence of wall's radiation.

It is seen from Figure 6(a), Hc decreases by increasing ε for a given Ra. So, radiation affects convection negatively. Hr increases quickly with ε (for a given Ra). So, surface emissivity acts positively for Hr. The Heat transfer due to radiation, and therefore, the total heat transfer, increases in a drastic manner for highly emissive walls of the cavity (Figure 6(b))

6.4 Comparision of Circular and Arc Cavity

The nusselt number for total heat transfer rate considering radiation with natural convection increases at the same course as for only natural convection. Nu_t is always higher for circular cavity in comparison with arc cavity.

For convection Nuc is 0.86% higher for Ra=10^6, γ = 0, and Nut is 1.13% higher considering radiation, ε =1. Heat transfer increases 1.74% for circular cavity in comparison of arc taking Ra=10^6, γ = 0 and ε =1.

6.5 Effect of Inclination Angles

In Figure 7(a), convective Nusselt numbers for circular cavity is presented as a function of the inclination angle, γ with Ra=10^2, 10^3, 10^5, 10^6, Nuc is an increasing function of Ra and at Ra=10^6, it goes through a maximum at 22degree for circular cavity and 9 degree for arc(fig 7(b)). Percent of radiation in total heat transfer is quasi constant for γ from 0 to 20°, thereafter they increase by 6.81% from 30 to 90° in fig 8(a). Invariance of the radiative heat transfer is observed in fig 8(b) where total heat transfer is a decreasing function of γ.

7. CONCLUSION

The surface radiation modifies temperatures on all enclosure walls; the velocity and temperature gradients are decreased. Considering radiation, the percentage of radiation heat transfer decreases gradually at low Rayleigh, increases at high Rayleigh and again decreases at higher Rayleigh number. The radiative heat flux is a strong increasing function of the surface emissivity starting at its low values; as a consequence, the convective heat flux is a strongly decreasing function of it. The combined Nusselt number Nu_t is an increasing function of Ra as well as surface emissivity, ε. Nu_t is always higher for circular cavity in comparison with arc cavity.

The convective Nusselt number Nu_c goes through a broad maximum at the inclination angle of 22degree for circular cavity and 9 degree for arc cavity at Ra=10^6 while the radiative Nusselt number Nu_r is unaffected to its fluctuation. At high inclination angle, Nu_c declines; i.e. total heat transfer rate decays. But the Radiation heat transfer in insensible to angle variation.

The percentage of radiation heat transfer increases with higher inclination angle.

8. REFERENCES

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9. NOMENCLATURE

Symbol	Meaning	Unit
Cp	specific heat capacity at constant pressure	J/kg.K
k	thermal conductivity	W/m.K
Nu	average Nusselt number; Nu= hL/k, $h\Delta T/k(\Delta T/L)$	
Gr	Grashof number, $Gr=g\beta L^3 (T_w- T_a)/\nu^2$	
Pr	Prandtl number ,$Pr = \nu/\alpha$	
Ra	Rayleigh number, Ra = Gr* Pr	
T	temperature	°K
U,V	non-dimensional x & y velocity component , $U =uH/\alpha_f$, $V = vH/\alpha_f$	
Ri	Richardson number, Gr/Re^2	
Symbols		
α	thermal diffusivity	m^2/s
γ	inclination angle	degree
ε	emissitivity	
θ	non-dimensional temperature	
υ	dynamic viscosity	kg/m.s
Subscripts		
s	surface	
h	hot	
c	cold	
f	fluid	

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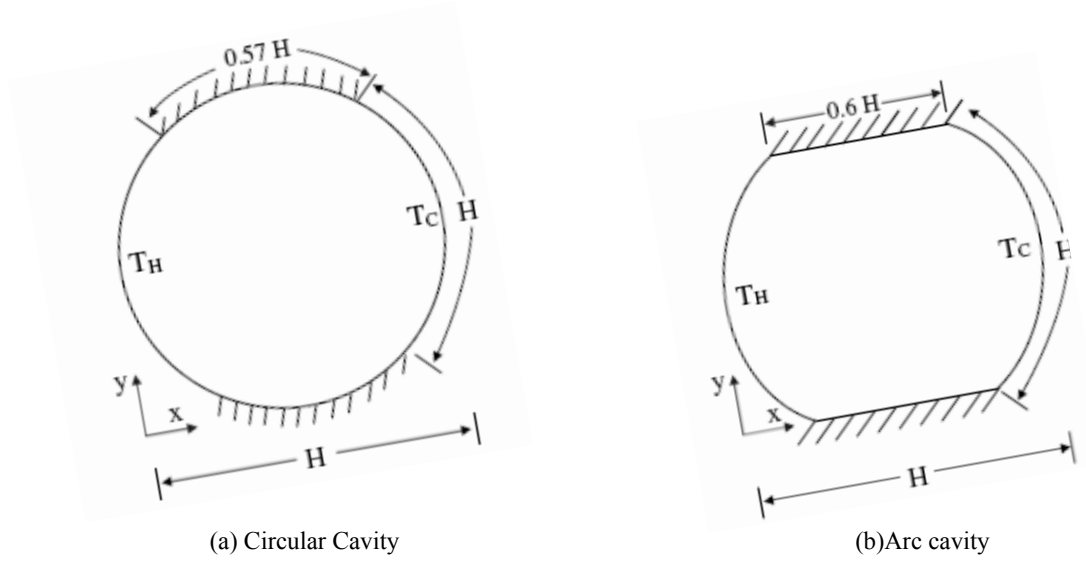Dhaka 1000, Bangladesh.
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(a) Circular Cavity   (b)Arc cavity

Fig 1. Physical Model of (a) circular (b) arc geometry

(a) present study   (b) existed paper
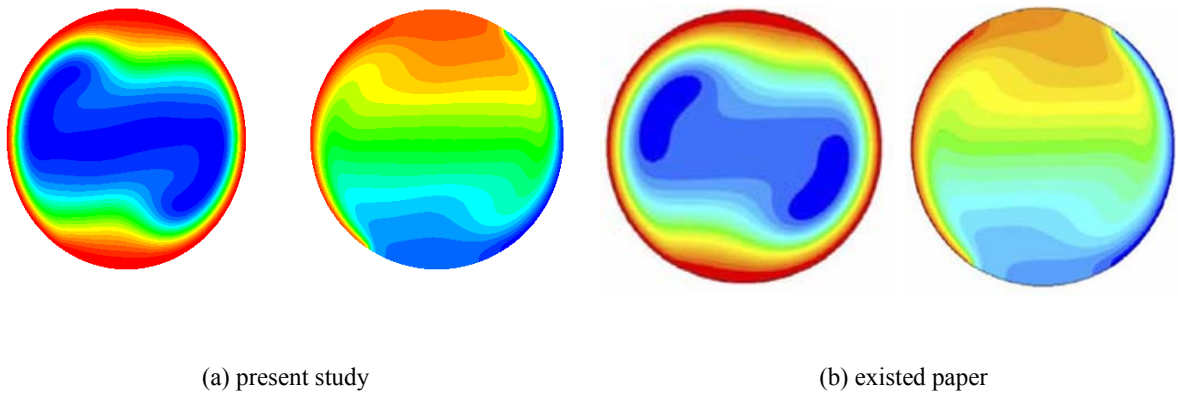
Fig 2. Code Validation; Comparison of stream lines and isotherms for high Ra=10^6 In circular cavity

ε = 1

Ra = 10³ Ra = 10⁵ Ra = 10⁶
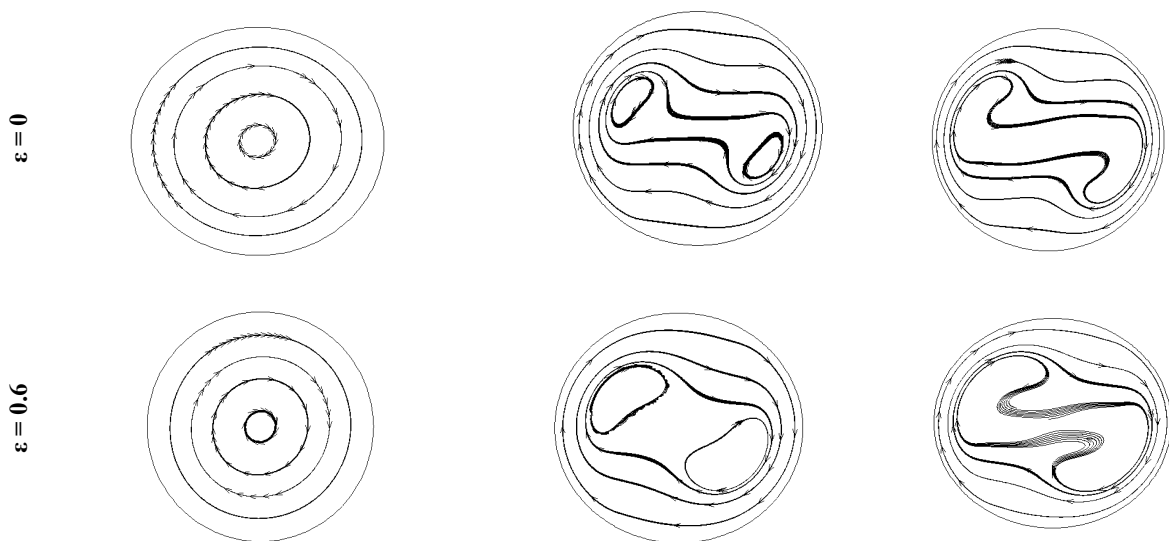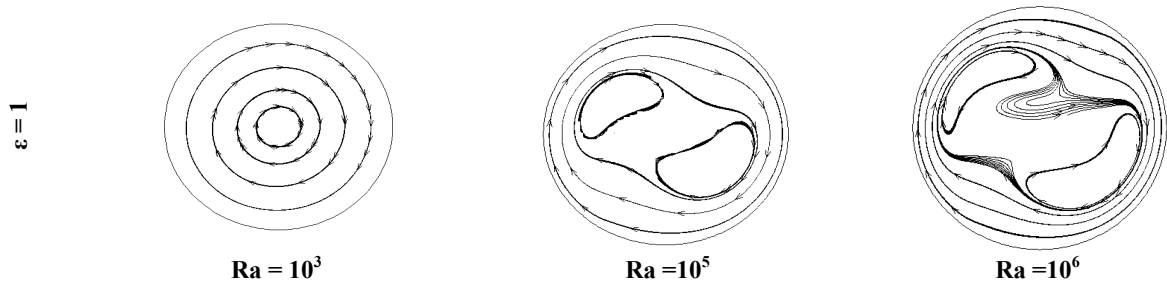
Fig 3. Streamlines Variation of Ra at γ =0° (circular cavity)

ε = 0

ε = 0.6

ε = 1

Ra = 10³ Ra = 10⁵ Ra = 10⁶

Fig 4. Isotherms Variation of Ra at γ =0° (circular cavity)

ε = 0.6 (streamlines row)

ε = 0.6 (isotherms row)

| Ra = 10³ | Ra =10⁵ | Ra =10⁶ |
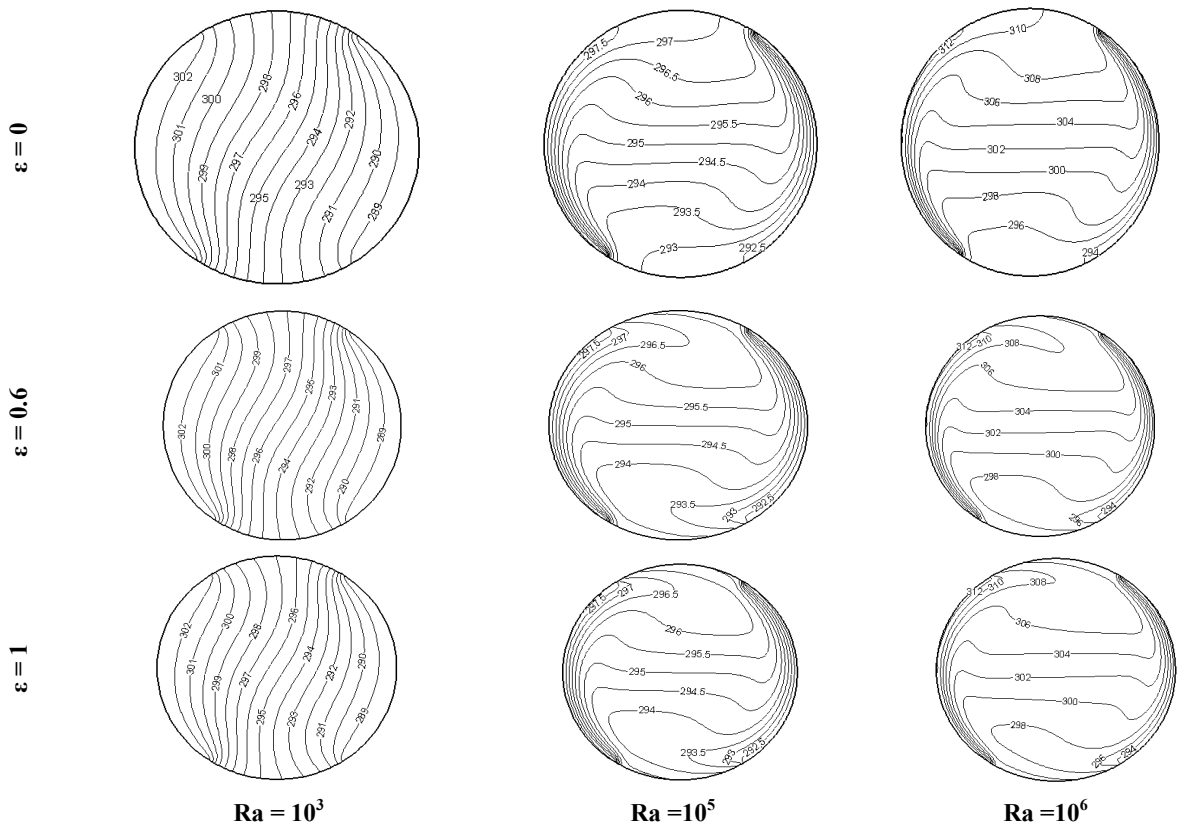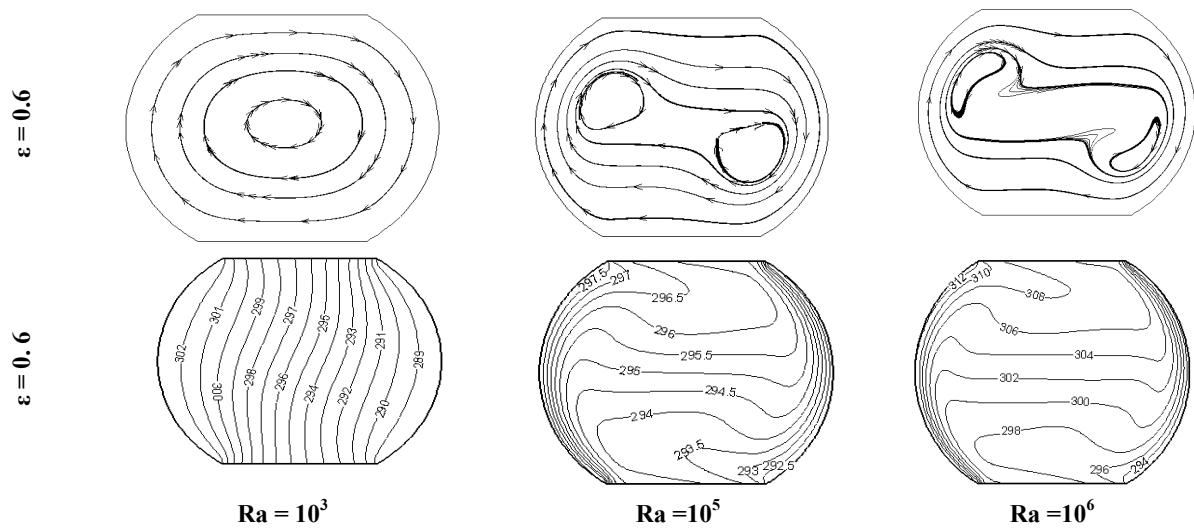
Fig 5. Streamlines and isotherms Variation of Ra at γ =0° (for arc cavity)

(a)

(b)
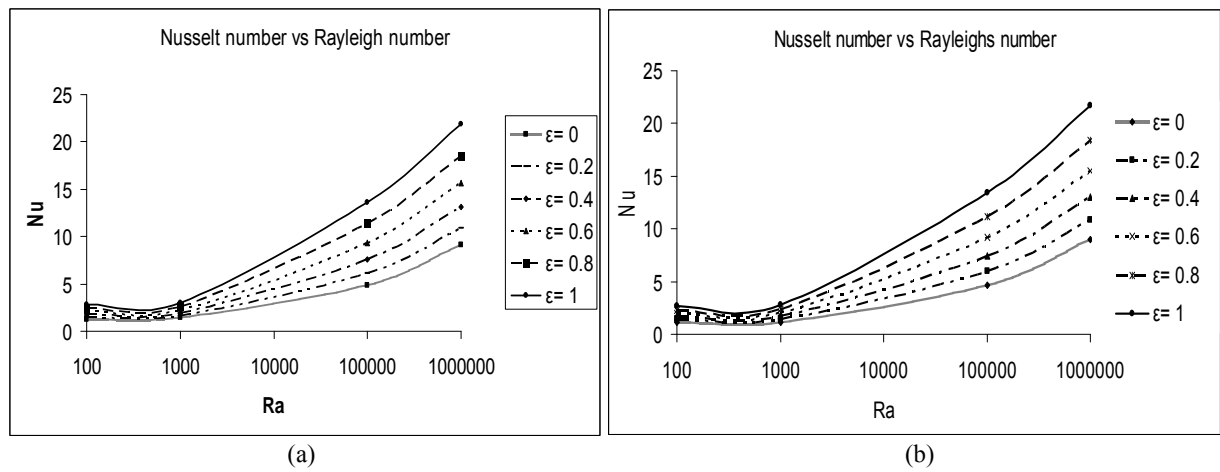
Fig 6. variation of Ra on Nusselt number at different emissivity at γ =0° (a) for circular cavity (b) for arc cavity
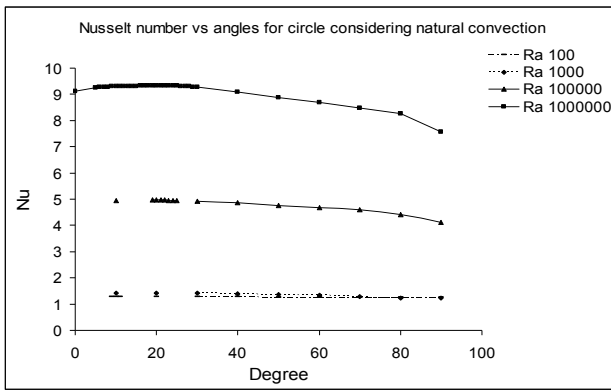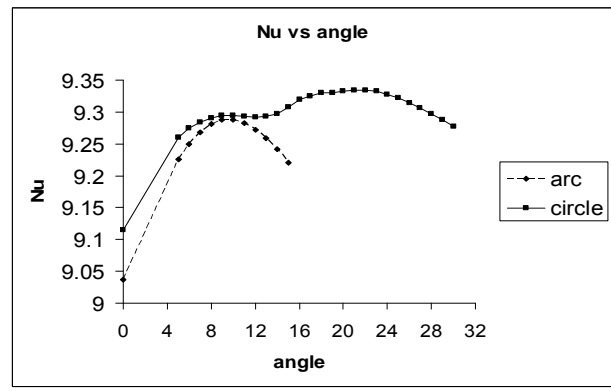

(a)

(b)
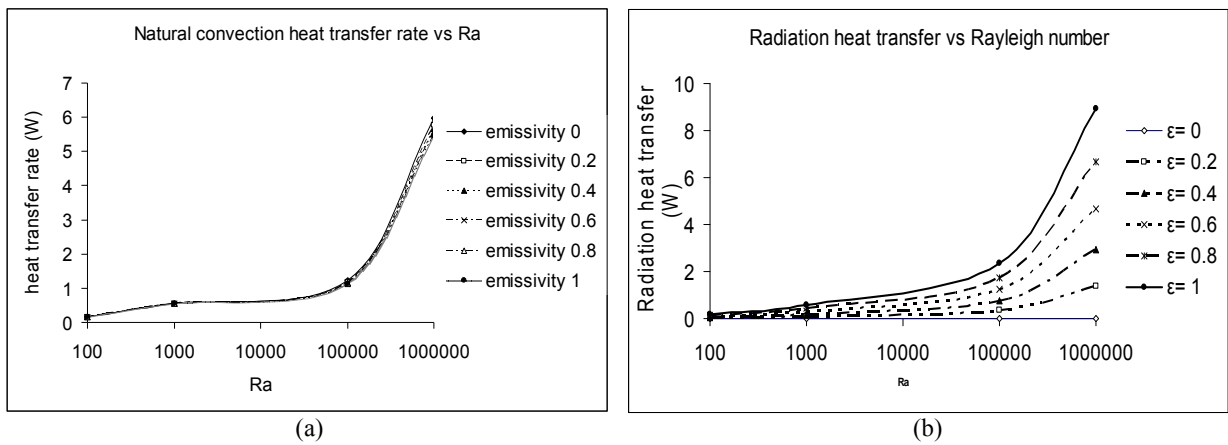
Fig 7. (a) Natural convection heat transfer rate for different emissivities in a circular cavity (inclination angle 0)
(b) Radiation heat transfer rate for different emissivities in a circular cavity (inclination angle 0)
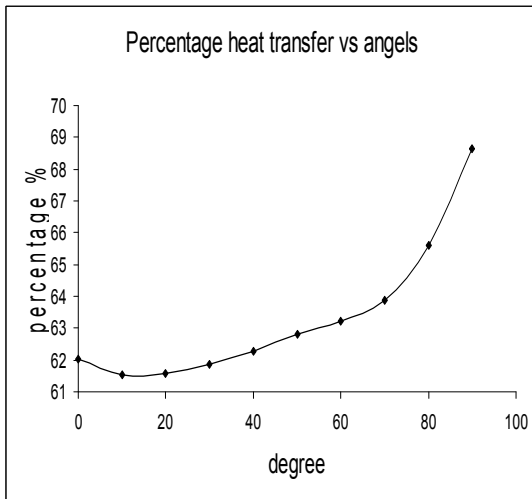
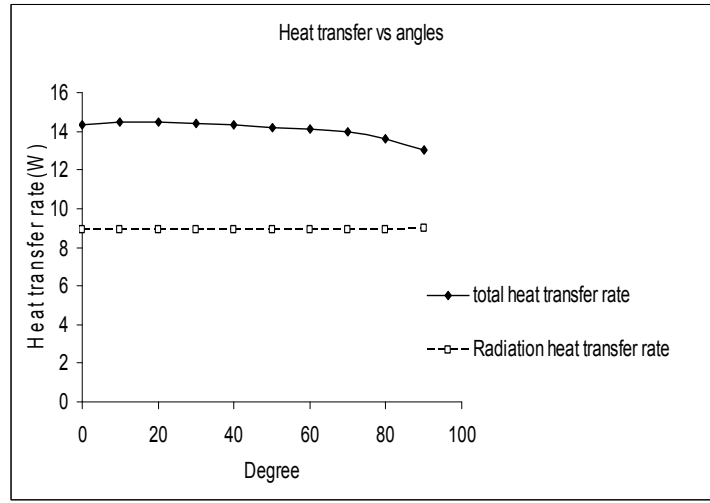(a)   (b)

Fig 8. (a) Nusselt number variation for different inclination angle in a circular cavity considering natural convection; (b) Finding of the optimum inclination angles for maximum Nusselt number for arc and circular cavity considering natural convection (Ra 10^6)

(a)   (b)

Fig 9. (a) Percentage of radiation heat transfer in the combined heat transfer rate for different γ in a circular cavity for ε=1 and Ra 10^6 (b) combined and radiation heat transfer rate for different γ in a circular cavity, taking ε=1 and Ra 10^6