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NATURAL CONVECTION FLOW ALONG THE WAVY CONE IN CASE OF UNIFORM SURFACE HEAT FLUX WHERE VISCOSITY IS INVERSLY PROPORTIONAL TO TEMPERATURE

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ABSTRACT
The effect of temperature dependent viscosity μ(T), on steady two dimensional natural convection flow along a vertical wavy cone with uniform surface heat flux has been investigated. Viscosity is considered to be inversely proportional to temperature. Using the appropriate variables the basic equations are transformed to non-dimensional boundary layer equations and then solved numerically employing implicit finite difference method. The effects viscosity variation parameter on the velocity profile, temperature profile, velocity vector field, skin friction, average Nusselt number, streamlines and isotherm have been discussed. The results have been shown graphically by utilizing the visualizing software Techplot. The present numerical result shows excellent agreement with the published results when the effect of temperature dependent viscosity was passed over.

Keywords: Natural convection, wavy cone, viscosity variation parameter

1. INTRODUCTION
Wavy surfaces are encountered in several heat transfer devices such as flat plate solar collectors and flat plate condensers in refrigerators. Larger scale surface non-uniformities are encountered, for example, in cavity wall insulating systems and grain storage containers, room heater etc. If the surface is wavy, the flow is disturbed by the surface and this alters the rate of heat transfer.

The only papers to date that study the effects of such non-uniformities on the vertical convective boundary layer flow of a Newtonian fluid are those of Yao [1], Moulic and Yao [2, 3]. Natural convection over a vertical wavy cone and frustum of a cone has been studied by Pop and Na [4, 5]. Cheng [6] have investigated natural convection heat and mass transfer near a vertical wavy cone with constant wall temperature and concentration in a porous medium. Hossain et al. [7, 8, 9] have studied the problem of natural convection flow along a vertical wavy cone and wavy surface with uniform surface temperature in presence of temperature dependent viscosity and thermal conductivity. Wang and Chen [10], have studied mixed convection boundary layer flow on inclined wavy plates including the magnetic field effect. Yao [11] has studied natural convection along a vertical complex wavy surface. Molla et al. [12] have studied natural convection flow along a vertical complex wavy surface with uniform heat flux.

In all of the above mentioned studies except Hossain et al. [7, 8, 9], the authors considered that the viscosity of the fluids are constant in the flow regime. But the physical properties may change significantly with temperature. For instance, the viscosity of water decreases about 240% when the temperature increases from 10^{o}C to 50^{o}C. Ling and Dybbs [13] have considered the viscosity to vary inversely to a linear function of temperature. On the other hand, Chrraudeau [14] has proposed a formula assuming the viscosity of the fluid to be proportional to a linear function of temperature. Hossain et al. [8, 15] investigated the natural convection flow past a permeable wedge and wavy cone for fluid having temperature dependent viscosity. In many application of practical importance, the surface temperature is non-uniform.

The case of uniform surface heat flux, which is often approximated in practical applications, has great importance in engineering applications. Very few of the aforementioned authors have studied natural convection flow for a surface which exhibits the uniform surface heat flux.

In the present study, the natural convection boundary layer flow along a vertical wavy cone with uniform heat flux has been considered. In addition the viscosity of the fluid is taken to be inversely proportional to the temperature. The formula proposed by Ling and Dybbs [13] is used to define the relationship between viscosity and temperature. The current problem is solved numerically by using Straightforward Finite Difference method (SFFD), reported by Yao [1, 2, 11]. Solutions are obtained for

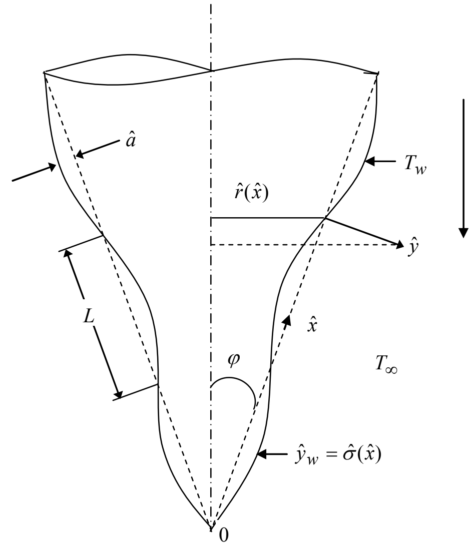


Figure 1: Physical model and the coordinate system

the fluid having Prandtl number Pr = 7.0 (water) and with the different values viscosity variation parameter.

2. FORMULATION OF THE PROBLEM

The boundary layer analysis outlined below allows the shape of the wavy surface, $\hat{\sigma}(\hat{x})$ to be arbitrary, but our detailed numerical work will assume that the surface exhibits sinusoidal deformations. Thus the wavy surface of the cone is described by

$$\hat{y}_w = \sigma(\hat{x}) = \hat{a}\sin\left(\pi\,\hat{x}/L\right) \tag{1}$$

where $2L$ is the fundamental wavelength associated with wavy surface and $\hat{a}$ is the amplitude of the waviness.

The physical model of the problem and the two-dimensional coordinate system are shown in Figure 1, where φ is the half angle of the flat surface of the cone and $\hat{r}(\hat{x})$ is the local radius of the flat surface of the cone which is defined by

$$\hat{r} = \hat{x}\sin\varphi \tag{2}$$

Under the Boussinesq approximation, we consider the flow to be governed by the following equations:

$$\frac{\partial(\hat{r}\,\hat{u})}{\partial\hat{x}} + \frac{\partial(\hat{r}\,\hat{v})}{\partial\hat{y}} = 0 \tag{3}$$

$$\hat{u}\frac{\partial\hat{u}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{u}}{\partial\hat{y}} = -\frac{1}{\rho}\frac{\partial\hat{p}}{\partial\hat{x}} + \frac{1}{\rho}\overline{\nabla}.(\mu\overline{\nabla}\hat{u}) + g\beta\left(T - T_\infty\right)\cos\varphi \tag{4}$$

$$\hat{u}\frac{\partial\hat{v}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{v}}{\partial\hat{y}} = -\frac{1}{\rho}\frac{\partial\hat{p}}{\partial\hat{y}} + \frac{1}{\rho}\overline{\nabla}.(\mu\overline{\nabla}\hat{v}) + g\beta\left(T - T_\infty\right)\sin\varphi \tag{5}$$

$$\hat{u}\frac{\partial T}{\partial\hat{x}} + \hat{v}\frac{\partial T}{\partial\hat{y}} = \frac{k}{\rho C_p}\overline{\nabla}^2 T \tag{6}$$

where $(\hat{x},\hat{y})$ are the dimensional coordinates and $(\hat{u},\hat{v})$ are the velocity components parallel to $(\hat{x},\hat{y})$. Also C_p is the specific heat at constant pressure and μ is the temperature dependent viscosity of the fluid which is defined as a linear function of the temperature.

$$\mu = \mu_\infty\left[1/1 + \gamma\left(T - T_\infty\right)\right] \tag{7}$$

where μ_∞ is the viscosity of ambient fluid outside the boundary layer and γ is a constant.
The boundary condition for the present problem is

$$\hat{u} = 0,\ \hat{v} = 0,\ q_w = -k(\hat{n}.\overline{\nabla}\hat{T})\ \text{ at }\ \hat{y} = \hat{y}_w = \sigma(\hat{x}) \tag{8a}$$

$$\hat{u} = 0,\quad T = T_\infty\ \text{ as }\ \hat{y} \to \infty \tag{8b}$$

where q_w is the uniform heat flux and $\hat{n}$ is the unit vector normal to the wavy surface. Now the following non-dimensional variables are introduced to obtain a set of non-dimensional governing equation:

$$x = \frac{\hat{x}}{L},\quad y = \frac{\hat{y} - \sigma(\hat{x})}{L}Gr^{1/5},\quad r = \frac{\hat{r}}{L},\quad a = \frac{\hat{a}}{L},$$

$$\sigma(x) = \frac{\sigma(\hat{x})}{L}, \sigma_x = \frac{d\hat{\sigma}}{d\hat{x}} = \frac{d\sigma}{dx},\ p = \frac{L^2}{\rho\,\nu_\infty{}^2}Gr^{-4/5}\hat{p},$$

$$u = \frac{\rho L}{\mu_\infty}Gr^{-2/5}\hat{u},\quad v = \frac{\rho L}{\mu_\infty}Gr^{-1/5}\left(\hat{v} - \sigma_x\hat{u}\right),$$

$$\theta = \frac{T - T_\infty}{\left(q_w L/k\right)}Gr^{1/5},\quad Gr = \frac{g\beta\,q_w\cos\varphi}{k\nu_\infty{}^2}L^4 \tag{9}$$

where θ is the dimensionless temperature function and $\nu_\infty = \mu_\infty/\rho$ is the kinematic viscosity. Here the new coordinate system $(x,\ y)$ are not orthogonal, but a regular rectangular computational grid can be easily fitted in the transformed coordinate. On introducing the above dimensionless dependent and independent variables into the equations (3)-(6) the following dimensionless form of the governing equations are obtained at leading order in the Grashof number, Gr:

$$\frac{\partial(r\,u)}{\partial x} + \frac{\partial\left(r\,v\right)}{\partial y} = 0 \tag{10}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma_x Gr^{1/5}\frac{\partial p}{\partial y} - \frac{\varepsilon}{\left(1+\varepsilon\,\theta\right)^2}\left(1+\sigma_x{}^2\right)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} + \frac{\left(1+\sigma_x{}^2\right)}{\left(1+\varepsilon\,\theta\right)}\frac{\partial^2 u}{\partial y^2} + \theta \tag{11}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\sigma_{xx}u^2=-Gr^{1/5}\frac{\partial p}{\partial y}+$$

$$\frac{\sigma_x\left(1+\sigma_x{}^2\right)}{(1+\varepsilon\,\theta)}\frac{\partial^2 u}{\partial y^2}-\frac{\varepsilon\sigma_x\left(1+\sigma_x{}^2\right)}{(1+\varepsilon\,\theta)^2}\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}+\theta\tan\varphi \quad (12)$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\left(1+\sigma_x{}^2\right)\frac{\partial^2\theta}{\partial y^2} \quad (13)$$

where $\Pr=\dfrac{\mu_\infty C_p}{k}$, $\mu=\mu_\infty\left[1/(1+\varepsilon\,\theta)\right]$ and

$$\varepsilon=\gamma\frac{q_w L}{k}Gr^{-1/5} \quad (14)$$

Here ε is a parameter which controls the value of γ and hence the temperature dependent viscosity μ as it is defined by equation (7) and (14).

It can easily be seen that the convection induced by the wavy surface is described by equations (10)-(13). Equation (12) represents that the pressure gradient along the x direction is in the order of $Gr^{-1/5}$. In the present problem this pressure gradient is zero because, no externally induced free stream exists. The elimination of $\partial p/\partial y$ from equations (11) and (12) leads to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\left(1+\sigma_x{}^2\right)}{(1+\varepsilon\,\theta)}\frac{\partial^2 u}{\partial y^2}-\frac{\varepsilon\left(1+\sigma_x{}^2\right)}{(1+\varepsilon\,\theta)^2}\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}$$

$$-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x{}^2}u^2+\frac{\left(1+\sigma_x\tan\varphi\right)}{1+\sigma_x{}^2}\theta \quad (15)$$

The corresponding boundary conditions for the present problem then turn into

$$u=0,\ v=0,\quad \partial\theta/\partial y=-1\Big/\sqrt{1+\sigma_x^2}\ \text{ at } y=0$$
$$u=0,\quad \theta=0\ \text{ as } y\to\infty \quad (16)$$

3. NUMERICAL METHODS

Investigating the present problem we have employed the straightforward finite difference method, which is described below. Firstly we introduce the following transformations to reduce the governing equation to a convenient form:

$$X=x,\ Y=y\Big/\left\{(5x)^{1/5}\right\},\ R=r,\ U(X,Y)=u\Big/\left\{(5x)^{3/5}\right\},$$
$$V(X,Y)=(5x)^{1/5}v,\ \Theta(X,Y)=\theta\Big/\left\{(5x)^{1/5}\right\} \quad (17)$$

Introducing the transformations given in equation (17) into the equations (10), (15) and (13) we have,

$$8U+(5X)\frac{\partial U}{\partial X}-Y\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial Y}=0 \quad (18)$$

$$(5X)U\frac{\partial U}{\partial X}+(V-YU)\cdot\frac{\partial U}{\partial Y}+\left\{3+\frac{\sigma_x\sigma_{xx}}{1+\sigma_x{}^2}(5X)\right\}U^2=$$
$$\frac{\left(1+\sigma_x{}^2\right)}{\left\{1+\varepsilon(5X)^{1/5}\Theta\right\}}\frac{\partial^2 U}{\partial Y^2}+\frac{\left(1+\sigma_x\tan\varphi\right)}{1+\sigma_x{}^2}\Theta-$$
$$\frac{\varepsilon\left(1+\sigma_x{}^2\right)}{\left\{1+\varepsilon(5X)^{1/5}\Theta\right\}^2}(5X)^{1/5}\frac{\partial\Theta}{\partial Y}\frac{\partial U}{\partial Y} \quad (19)$$

$$5XU\frac{\partial\Theta}{\partial X}+(V-YU)\frac{\partial\Theta}{\partial Y}+U\Theta=\frac{1}{\Pr}\left(1+\sigma_x{}^2\right)\frac{\partial^2\Theta}{\partial Y^2} \quad (20)$$

The boundary conditions now take the following form:

$$U=0,\quad V=0,\quad \frac{\partial\Theta}{\partial Y}=-1\Big/\sqrt{1+\sigma_x^2}\ \text{ at } Y=0$$
$$U=0,\quad \Theta=0\ \text{ as } Y\to\infty \quad (21)$$

Solutions of the non-dimensional partial differential system given by (18)-(20) and subject to the boundary conditions (21) are obtained by using the straightforward finite difference method developed by L.S. Yao [1, 2, 11]. However, once we know the values of the function U, V and Θ and their derivatives, it is important to calculate the values of the average Nusselt number, Nu_m from the following relation which is obtained by using the set of transformations:

$$Nu_m(5/Gr)^{1/5}=\frac{X^{1/5}\int_0^X\sqrt{1+\sigma_x^2}\,dX}{\int_0^X\sqrt{1+\sigma_x^2}\,X^{1/5}\,\Theta(X,0)\,dX} \quad (22)$$

Also the skin friction coefficients is defined as

$$C_{f_x}(Gr)^{1/5}\Big/2(5X)^{2/5}=\frac{\sqrt{1+\sigma_x^2}}{\left\{1+\varepsilon(5X)^{1/5}\Theta\right\}}\frac{\partial U}{\partial Y}\Bigg]_{Y=0} \quad (23)$$

The stream function for the wavy cone is defined as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y}\quad\text{and}\quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x} \quad (24)$$

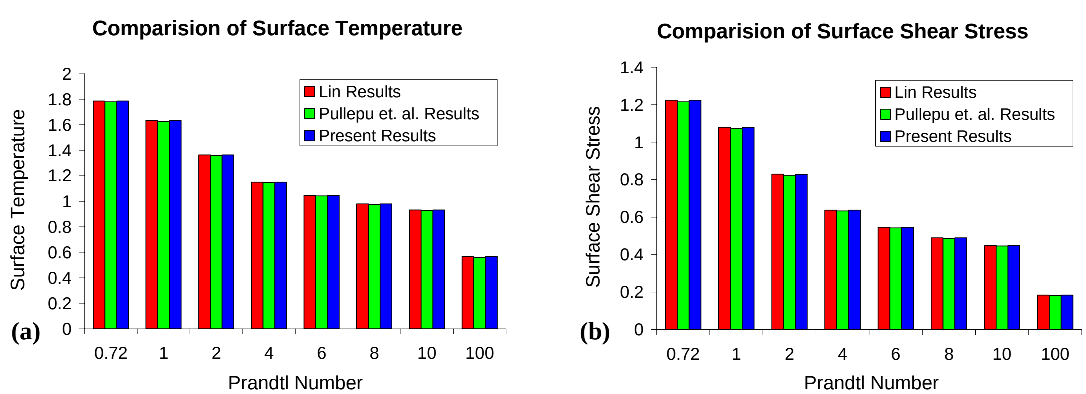
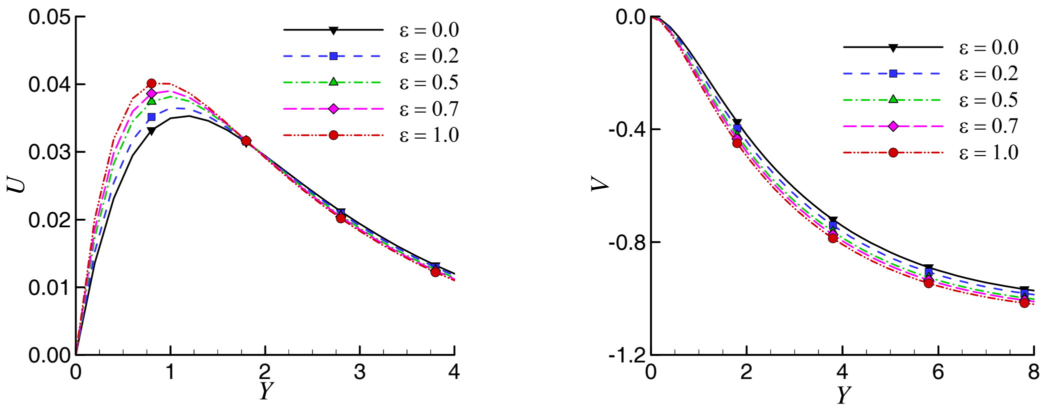


Figure 2: Comparison of present results on a) Surface temperature and b) Surface shear stress with Lin and Pullepu et. al. results



(a) **(b)**

Figure 3: (a) Tangential velocity distribution and (b) Normal velocity distribution at $X = 1.0$ for Prandtl number Pr = 7.0, a = 0.3 and φ = 30^{o}.

For calculating the stream function ψ, we have integrated the fluid velocity over the whole boundary layer, which may be defined as

$$\psi = \int_0^Y R\left(5X\right)^{3/5} U \, dY \text{ , where } R = X \sin\varphi \qquad (25)$$

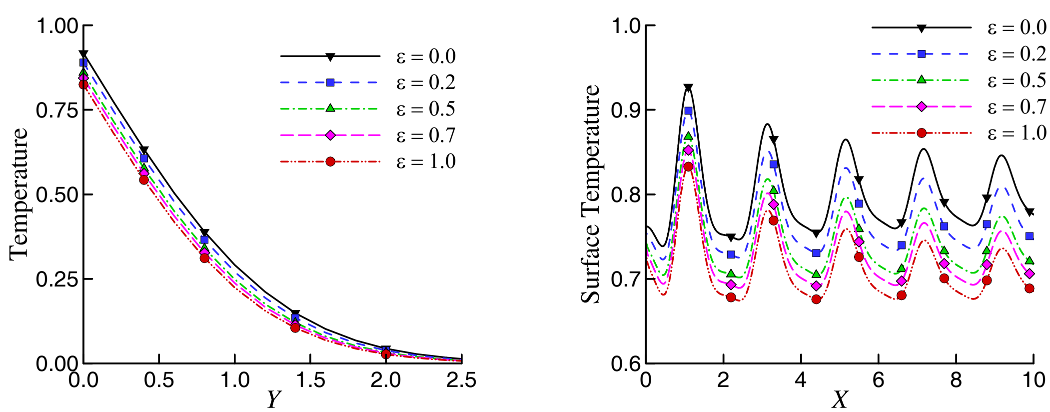
4. RESULTS AND DISCUSSION

In this paper, the effect of temperature dependent viscosity on a steady two-dimensional natural convection laminar flow of viscous incompressible fluid along a vertical wavy cone has been investigated by using very efficient finite difference method. It is seen that the solutions are affected by the viscosity variation parameter as well as the amplitude of the cone. Here we have focused our attention on the effect of ε on the average Nusselt number $Nu_m(5/Gr)^{1/5}$, skin friction C_{fx} as well as velocity and temperature distribution. We also show the graphical representation of velocity vectors, stream lines and isotherms of the flow field.

In order to validate the present numerical results, the skin friction coefficient and the surface temperature have been compared with those of Lin [16] and Pullepu et. al. [17]. The present comparison is done for the flat vertical cone with uniform surface heat flux case. Lin [16] has studied the free convection from a vertical cone with uniform surface heat flux case. On the other hand, Pullepu et al. [17] have studied unsteady laminar free convection from a vertical cone with uniform heat flux case. The comparative studies are illustrated graphically in fig. 2 which shows that the present results have excellent agreement with those results when the effect of viscosity variation parameter was passed over.

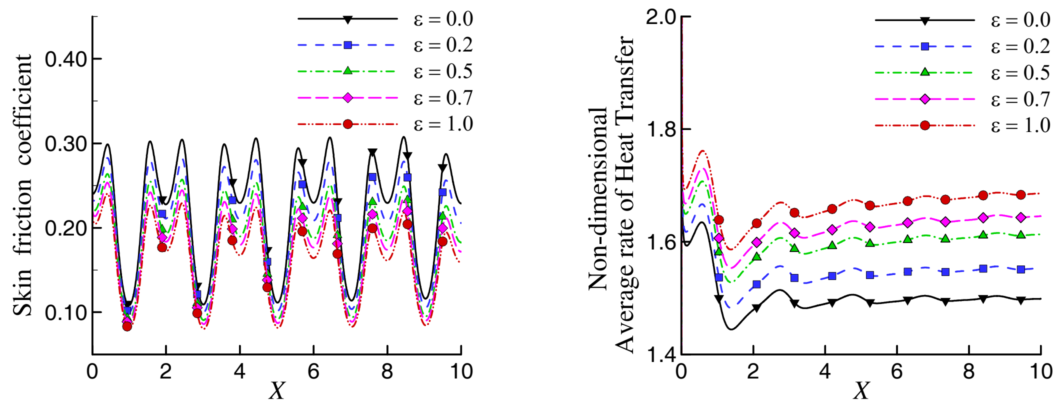
The numerical results are presented for the different values of viscosity variation parameter ε for a suitable fluid having Prandtl number Pr = 7.0 (water). To examine the effect of ε we also considered that a = 0.3 and φ=30^{o} remain constant. Figure 3 (a), (b) represents the non-dimensional tangential and normal velocity distribution for different values of ε at a fixed point X = 1.0. It is found that the increasing value of ε increase the tangential velocity inside the boundary layer slightly. The thickness of the boundary layer remains same as ε increases. Figure 3(b) shows that the normal velocity decreases slightly when ε increases.

Fluid temperature distribution at a fixed point X = 1.0 and surface temperature distribution for different values of ε are shown in fig 4(a) and 4(b) respectively.



(a) **(b)**

Figure 4: (a) Fluid temperature distribution at $X = 1.0$ and (b) Surface Temperature distribution for Pr = 7.0, a = 0.3 and φ = 30^{o}.



(a) **(b)**

Figure 5: (a) Skin friction coefficient and (b) Average rate of heat transfer for Pr = 7.0, $a = 0.3$ and $\varphi = 30^o$

From the figure it is evident that the temperature distribution inside the boundary layer at any fixed point decreases slightly when ε increases. The surface temperature decreases significantly due to the increasing value of ε and the surface temperature is found to fluctuate along the wavy surface.

The effect of ε on the surface shear stress in terms of skin friction coefficient and on the average rate of heat transfer in terms of average Nusselt number are given in the fig 5(a) and 5(b) respectively. The skin friction decreases faintly with the increase of viscosity variation parameter. While the average rate of heat transfer increases significantly for higher value of ε.

Figure 6(a)-(c) show the isotherm for a wavy cone, while the viscosity variation parameter ε is taken as 0.0, 0.5 and 1.0 respectively. The figures indicate that the increases of ε affect the isotherm and leads to the thinner thermal boundary layer.

4. CONCLUSIONS

The effect of viscosity variation parameter ε, on the natural convection boundary layer flow along a vertical wavy surface with uniform heat flux, has been studied numerically. New variables transform the complex geometry into a simple shape where a very efficient straightforward finite difference (SFFD) method was used to solve the non-dimensional boundary layer equations. From the present investigation the result can be summarized as follows:

- The skin friction decreases within the computational domain for increasing value of the viscosity variation parameter ε.
- The average rate of heat transfer enhance significantly with the increases of ε.
- Tangential velocity increase slightly with the increasing value of viscosity variation parameter ε.
- It was found that the temperature inside the boundary layer at any fixed point decreases slightly when ε increases.
- One important finding is that, the increases of ε affect the isotherm and leads to the thinner thermal boundary layer.

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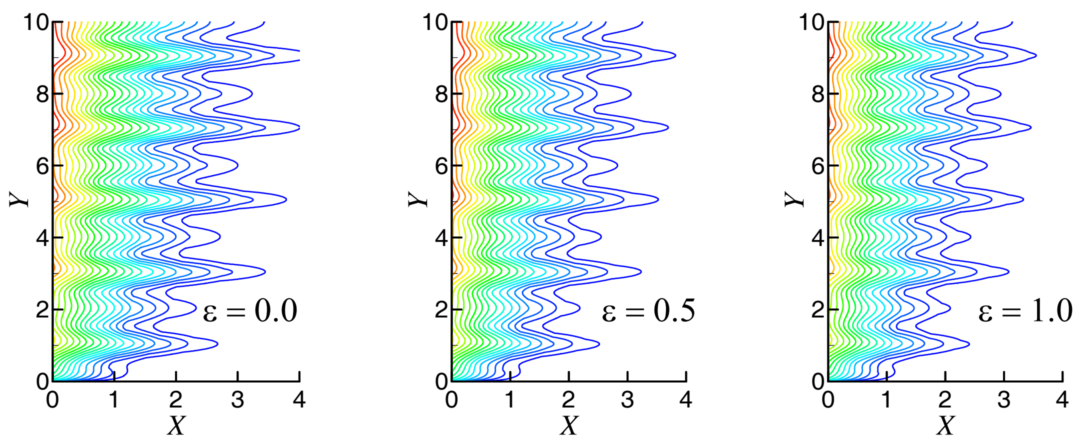

(a) **(b)** **(c)**

Figure 6: Isotherm for different values of ε for a wavy cone with $a = 0.3$, $\varphi = 30^o$ and Pr = 7.0.

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6. NOMENCLATURE

Symbol	Meaning	Units
a	Amplitude wavelength ratio	(- -)
$\hat{a}$	Amplitude of the wavy cone	(m)
Cp	Specific heat at constant pressure	$(m^2 \cdot s^{-2} \cdot K^{-1})$
C_f	Skin friction coefficient	(- -)
g	Acceleration due to gravity	$(m.s^{-2})$
Gr	Grashof number	(- -)
k	Thermal conductivity	$(m.kg.s^{-3}K^{-1})$
L	Half of the fundamental wavelength	(m)
$\hat{n}$	Unit vector normal to the wavy surface	(- -)
Nu_m	Average Nusselt number	(- -)
p	Dimensionless pressure function	(- -)
Pr	Prandtl number	(- -)
q_w	Uniform heat flux at the surface	$(kg \cdot s^{-3})$
$\hat{r}(\hat{x})$	Local radius of the of the cone	(m)
r, R	Dimensionless radius of the cone	(- -)
T	Temperature in the boundary layer	(K)
$(\hat{u}, \hat{v})$	Velocity component along $\hat{x}$ and $\hat{y}$	$(m.s^{-1})$
Greek symbols		
β	Volumetric coefficient of thermal expansion	(K^{-1})
ε	Viscosity variation parameter	(- -)
θ, Θ	Dimensionless temperature function	(- -)
μ	Viscosity of the fluid	$(m^{-1}kg.s^{-1})$
μ_∞	Dynamic viscosity of the ambient fluid	$(m^{-1}kg.s^{-1})$
ν_∞	Reference kinematic viscosity	$(m^2.s^{-1})$
ρ	Density of the fluid	$(m^{-3}kg)$
$\sigma(x)$	Non-dimensional surface profile function	(- -)
$\hat{\sigma}$	Surface profile function	(m)
τ_w	Shearing stress	$(m^{-1}.kg.s^{-2})$
φ	The half angle of the cone	$(\ ^0)$
ψ	Stream function	(- -)
Subscript		
w	Wall conditions	
∞	Ambient temperature	
m	Average condition	
x	Differentiation with respect to x	