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MIXED CONVECTION ANALYSIS IN A LID DRIVEN TRAPEZOIDAL CAVITY WITH ISOTHERMAL HEATING AT BOTTOM FOR VARIOUS ASPECT ANGLES

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ABSTRACT
Mixed convection heat transfer in a two-dimensional trapezoidal cavity has been investigated with a locally heated lower wall and moving cold top lid. The enclosure represents a practical system where the space requirement is very important factor for efficient electronic cooling system. The numerical study is conducted for several values of aspect angle of the cavity and a range of Richardson numbers with constant Reynolds and Prandtl numbers. The influence of Richardson number on average Nusselt number is investigated for various aspect angles. Results are represented in the form of isotherms and streamlines under different conditions. The solution procedure is conducted using the Galerkin finite element method.

Keywords: Mixed Convection, Richardson Number, Aspect Angle, Finite Element Method

1. INTRODUCTION
Air cooling is one of the preferred methods for cooling various electrical equipments as well as in some critical places of engineering plants where cooling is very essential for better performance. This study focuses especially on mixed convection heat transfer, which is being used for cooling different devices through worldwide. Various studies and researches are still carried out to make this cutting edge field more advanced as well as developed.

Mixed convection, in which, neither natural convection nor forced convection is dominant but both modes of convection heat transfer are in a balance condition that arises in many technological processes (Incropera, 1988). Various works have already been done on mixed convection heat transfer, where almost all of them are handled with vented cavity and very few considered enclosures having one or two walls are moving. Davis and Jones (1983) studied the pure natural convection with uniformly heated walls. Papaniclaou and Jaluria (1990, 1992, 1993 and 1994) carried out a series of numerical studies to investigate the combined forced and natural convective cooling of heat-dissipating electronic components, located in rectangular enclosure and cooled by an external through flow of air. The results indicated that flow patterns generally consisted of high-or low-velocity recirculating cells because of buoyancy forces induced by the heat source. Computations for turbulent flow in mixed convection in a cavity by k–ω model were later performed by Papaniclaou and Jaluria (1995). Numerical solutions were obtained for Re = 1000 and 2000 in the range of Gr = 5×10^7 to 5×10^8. Iwatsu et al. (1992) performed numerical studies for the flow of a viscous, thermally stratified fluid in a square container. Shaw (1993) investigated three-dimensional mixed convection heat transfer in a cavity heated from below. The influences of Reynolds number and Grashof number on the Nusselt number had been discussed extensively. Aydin and Yang (2000) numerically studied mixed convection heat transfer in a two-dimensional square cavity having an aspect ratio of 1.

The physical model has shown in Fig. 1 along with boundary conditions. It consists of base length (W), height (H) and an aspect angle (φ) between the side walls with the vertical axis. In this study, the bottom wall is heated at a uniform temperature (T_h) whereas the upper wall is lid driven maintained at a lower temperature (T_c) and constant velocity, u_o in right-ward (+x) direction and side walls are kept adiabatic. The flow is induced by the combined effect of shear force raised by moving top lid and buoyancy force by the hot bottom wall. The main objective of this study is to observe the flow situation as well as the mixed convection heat transfer characteristics due to the change of aspect angle of the enclosure.

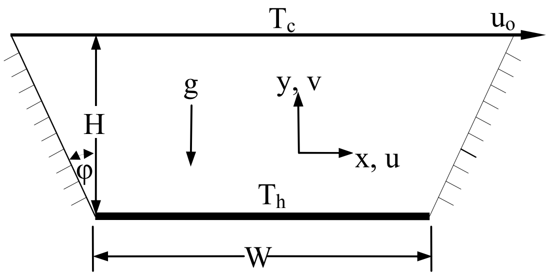

Fig 1. Schematic of the problem with the domain and boundary conditions

2. MATHEMATICAL EQUATIONS

The non-dimensional Navier-Stokes equations along with energy equation for two-dimensional, incompressible, steady flow with constant properties and considering the Boussinesq approximation in Cartesian co-ordinates can be written as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\Theta \quad (3)$$

$$U\frac{\partial \Theta}{\partial X}+V\frac{\partial \Theta}{\partial Y}=\frac{1}{Re\,Pr}\left(\frac{\partial^2 \Theta}{\partial X^2}+\frac{\partial^2 \Theta}{\partial Y^2}\right) \quad (4)$$

Equations (1-4) are normalized using the following dimensionless scales:

$$(X,Y)=\frac{(x,y)}{W},(U,V)=\frac{(u,v)}{u_o},P=\frac{p}{\rho u_o^2},\Theta=\frac{T-T_c}{T_h-T_c}$$

The Grashof number (Gr), Prandtl number (Pr), Reynolds number (Re), and the Richardson number (Ri) are given by

$$\mathrm{Gr}=\frac{g\beta(T_h-T_c)W^3}{\upsilon^2},\mathrm{Pr}=\frac{\upsilon}{\alpha},\mathrm{Re}=\frac{u_oW}{\upsilon},\mathrm{Ri}=\frac{\mathrm{Gr}}{\mathrm{Re}^2}$$

where α, β, ρ and υ are thermal diffusivity, thermal expansion coefficient, fluid density and kinematic viscosity respectively. The average Nusselt number of the heated wall is calculated as

$$Nu=\int_0^1 -\left(\frac{\partial \Theta}{\partial Y}\right)\Bigg|_{Y=0} dX \quad (5)$$

3. COMPUTATIONAL PROCEDURE

A finite element formulation based on the Galerkin method (Reddy, 1993) is employed to solve the governing equations subject to the boundary conditions for the present study. In the current investigation, triangular elements are utilized to discretize the physical domain. A variable grid-size system is generated in the present investigation to capture the rapid changes in the dependent variables. To ensure convergence of the numerical algorithm the following criteria is applied to all dependent variables over the solution domain

$$\sum\left|\phi_{ij}^{m}-\phi_{ij}^{m-1}\right|\le 10^{-5} \quad (6)$$

where ϕ represents a dependent variable U, V, P, and θ, the indexes i, j indicate a grid point, and the index m is the current iteration at the grid level. To meet the higher accuracy, a grid refinement study is performed for aspect angle, φ = 45º and Ri = 1. Table 1 shows the accuracy of the average Nusselt number which is become independent of mesh size after 5966 elements.

Table 1: Comparison of the results for various grid dimensions (Ri = 1 and φ = 45º)

Elements	*Nu*
2846	4.278
3430	4.285
4958	4.296
5966	4.305
7712	4.305
9008	4.305

4. RESULTS AND DISCUSSION

In this computation, the working fluid is chosen as air with Prandtl number, Pr = 0.71. For aspect angle φ = 0º, 15º, 30º and 45º, the computations are performed at Ri from 0.1 to 10 keeping the Reynolds number (Re) and the aspect ratio of the enclosure (H/W) fixed at 100 and 1, respectively.

4.1 Effect of Richardson Number on the Flow and Heat Transfer

Streamlines for φ = 0º shown in Fig. 2 indicate that one large vortex is created rotating in the clockwise direction. It shows that heat is carried out along with the motion of the moving upper wall and the intensity of the flow is increased at the middle region of the cavity with the help of increased buoyancy forces for higher Richardson number. Flow starts from the upper moving wall to the right and it goes downward until the lower region is reached. As soon as it reaches the lower region, it starts to move in the left direction and turns to the upper region as it reaches the left corner of the bottom wall. The flow along the upper and the bottom walls is different for different Richardson number, especially at the lower right corner. And for Ri = 1 and 5, the streamlines are almost same. The difference is that the small vortex at the lower right end increases with the increase in Richardson number which helps the more heat transfer rapidly. Also it is observed that the center of the main large vortex is shifted towards the center of the cavity.

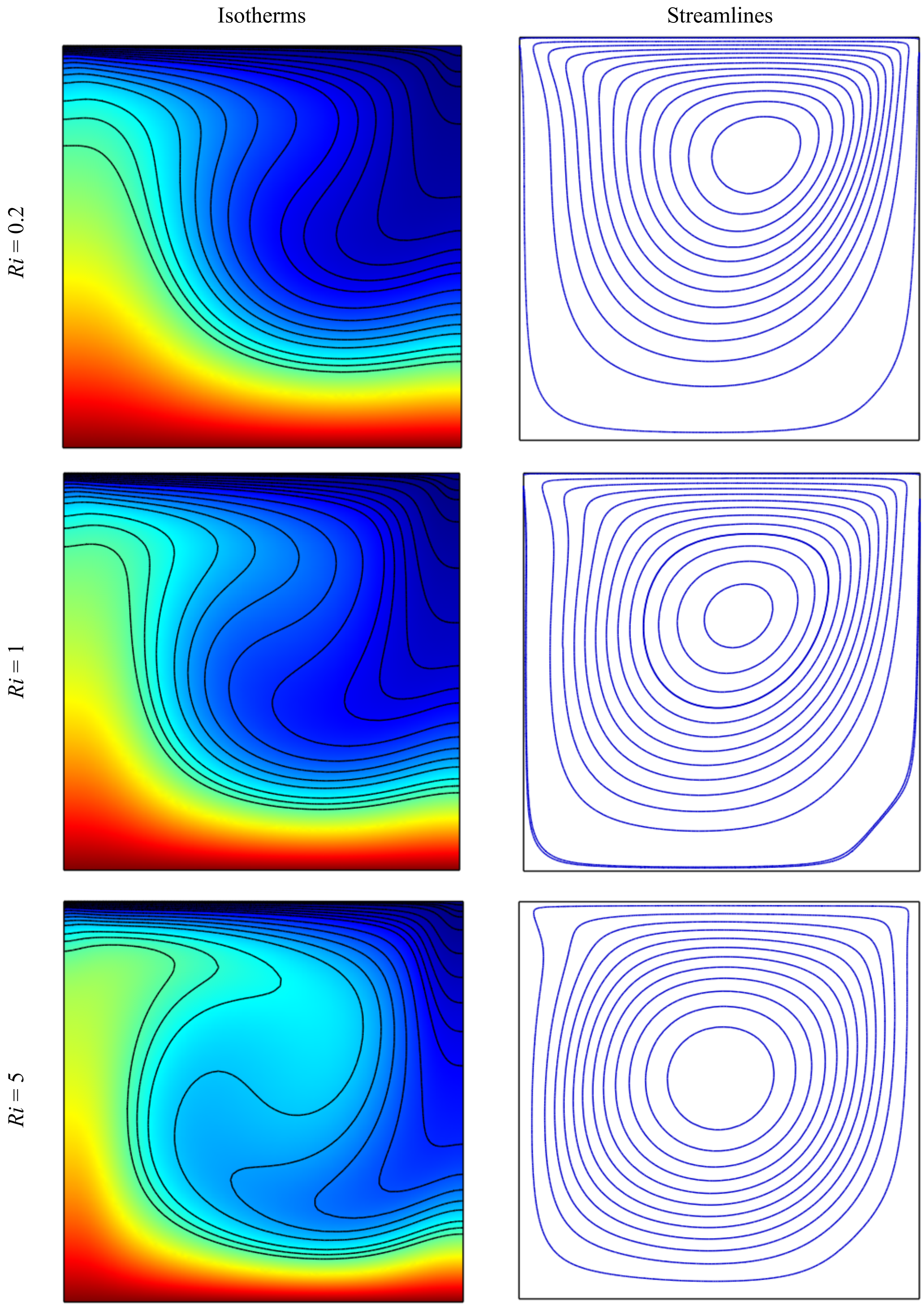

Fig 2. Effect of Richardson number on isotherm and streamline patterns for $\varphi = 0^{o}$.

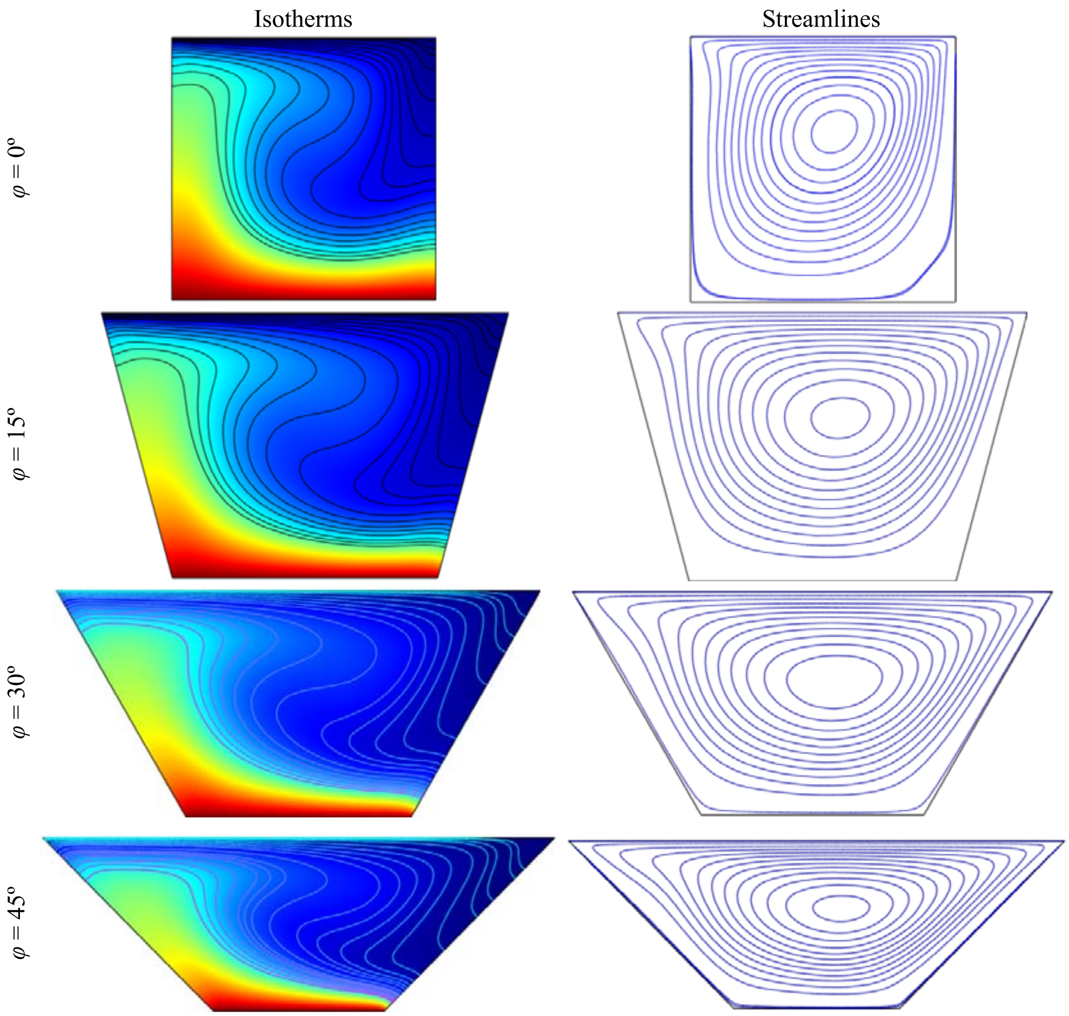

Fig 3. Effect of aspect angle on isotherm and streamline patterns for Ri = 1.

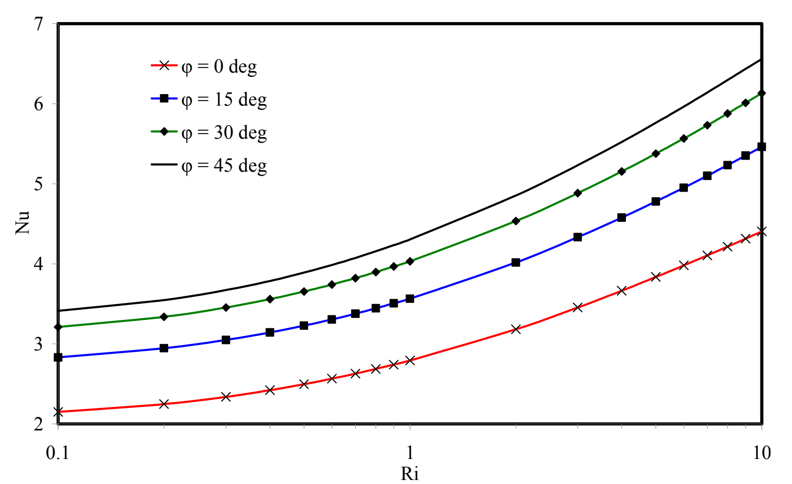

Fig 4. Variation of average Nusselt number as a function of Richardson number for different aspect angles.

Some significant changes in the isotherms are also observed for the different values of the Richardson number (Ri) in Fig. 2. The isotherms also reveal the similar scenario of the heat transfer characteristic within the cavity. The isotherms get stronger near the hot wall with the increasing value of the Richardson number (Ri). The isotherms are at first more uniform in nature. With the increase of Richardson number (Ri), the non-linearity of the isotherms increases at a higher value resulting the higher plume formation and a distorted plume is formed indicating better convective heat transfer for Ri = 5. When Ri = 1, the buoyancy and shear effects are in comparable level and with the further increase of Ri leads to dominant natural convection over mixed convection. However, for lower Ri, where dominant forced convection existed, shows less heat transfer as evident in Fig. 4.

4.2 Effect of Aspect Angle on the Flow and Heat Transfer

The streamlines and isotherms are shown for the aspect angle, φ = 0º, 15º, 30º and 45º in Fig. 3. From the observation of these figures, it is implied that the patterns of the streamline and isotherm are almost same in all four configurations but resulting heat transfer enhancement with the increase in aspect angle is different as appeared in Fig. 4.With the increase of aspect angle, average Nusselt number increases to a great extent.

5. CONCLUSIONS

In this investigation, the results of a numerical study of shear and buoyancy induced flow and heat transfer in a two-dimensional trapezoidal cavity with localized heating from below and cooling from the moving upper wall are presented. The main parameters of interest are mixed convection parameters, Richardson number (Ri) and the aspect angle (φ). With the increasing value of Richardson number in the mixed convection regime shows that the behavior of progressively dominant natural convection, which has been analyzed in this study. It has also been shown that for a particular aspect angle of the cavity, the value of average Nusselt number increases with the increase of Richardson number. Again it is found that the maximum value of the average Nusselt number can be attained at the maximum value of the aspect angle (φ = 45º).

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