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MAGNETOHYDRODYNAMIC (MHD)-CONJUGATE FREE CONVECTION FLOW FROM AN ISOTHERMAL HORIZONTAL CIRCULAR CYLINDER WITH JOULE HEATING EFFECT

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ABSTRACT

Magnetohydrodynamic(MHD)-conjugate natural convection flow along the outer surface from the lower stagnation point to the upper stagnation point and from an isothermal horizontal circular cylinder considering Joule heating effect is investigated. The developed governing equations with the associated boundary conditions for this analysis are transferred to dimensionless forms using a suitable transformation. The transformed non-dimensional governing equations are then solved using the implicit finite difference method with Keller box-scheme. Numerical results are found for different values of the Joule heating parameter, Magnetic parameter and Prandtl number. Detail results of the velocity profiles, temperature distributions, the skin friction and the rate of heat transfer are shown graphically.

Keywords: Natural Convection, Horizontal Cylinder, Magneto Hydrodynamic, Joule Heating, Conduction.

1. INTRODUCTION

Natural convection flow from a horizontal cylinder due to thermal buoyancy was analyzed by a number of researchers [1-4] under diverse surface boundary conditions (isothermal, uniform heat flux and mixed boundary conditions) using different mathematical technique. The conjugate heat transfer process (CHT) formed by the interaction between the conduction inside the solid and the convection flow along the solid surface has a significant importance in many practical application. In fact, conduction within the tube wall is significantly influenced by the convection in the surrounding fluid. Consequently, the conduction in the solid body and the convection in the fluid should have to determine simultaneously. Gdalevich and Fertman[5] studied the conjugate problems of natural convection. Miyamoto et al. [6] analysed the effects of axial heat conduction in a vertical flat plate on free convection heat transfer. Miyamoto observed that a mixed-problem study of the natural convection has to be performed for an accurate analysis of the thermo-fluid-dynamic (TFD) field if the convective heat transfer depends strongly on the thermal boundary conditions. Pozzi et al. [7] investigated the entire TFD field resulting from the coupling of natural convection along and conduction inside a heated flat plate by means of two expansions, regular series and asymptotic expansions. Moreover, Kimura and Pop [8] analysed conjugate natural convection from a horizontal circular cylinder.

MHD flow and heat transfer process are now an important research area due to its potential application in engineering and industrial fields. A considerable amount of research has been done in this field. Wilks et al. [9] studied MHD free convection about a semi-infinite vertical plate in a strong cross field. Takhar and Soundalgekar [10] investigated dissipation effects on MHD free convection flow past a semi-infinite vertical plate. Hossain [11] studied viscous and Joule heating effects on MHD free convection flow with variable plate temperature. Aldoss et al. [12] analysed MHD mixed convection from a horizontal circular cylinder. El-Amin [13] found out the combined effect of viscous dissipation and Joule heating on MHD forced convection over a non-isothermal horizontal circular cylinder embedded in a fluid saturated porous medium. He observed that both the velocity profiles and temperature profiles shifted down for increasing value of magnetic parameter and that are rise up for increasing value of Joule heating parameter.

In this paper, the MHD-conjugate free convection flow from an isothermal horizontal circular cylinder with Joule heating effect is investigated. The governing boundary layer equations are transformed into a non dimensional form and the resulting non linear partial differential equations are solved numerically using the implicit finite difference method together with the Keller box technique [15,16]. The temperature distributions, velocity profiles, skin friction coefficients and the heat transfer rates are presented graphically.

2. MATHEMATICAL ANALYSIS

Let us consider a steady natural convection flow of a viscous incompressible and electrically conducting fluid from an isothermal horizontal circular cylinder of radius a placed in a fluid of uniform temperature T_∞. The cylinder has a heated core region of temperature T_b and the normal distance from inner surface to the outer surface is b with $T_b > T_\infty$. A uniform magnetic field having strength B_0 is acting normal to the cylinder surface. The x-axis is taken along the circumference of

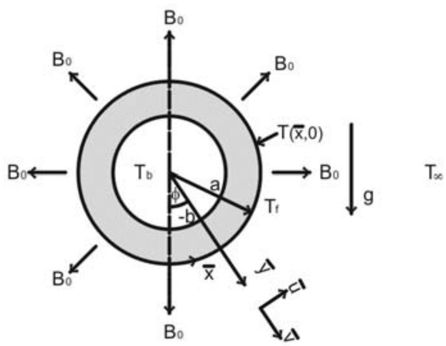

Fig. 1: Physical Model and coordinate system

the cylinder measured from the lower stagnation point and the y-axis is taken normal to the surface. It is assumed the fluid properties to be constant and the induced magnetic field is ignored. The effects of Joule heating in the flow region and conduction from inner surface to the outer surface considered in the present study. Under the balance laws of mass, momentum and energy and with the help of Boussinesq approximation for the body force term in the momentum equation, the equations governing this boundary-layer natural convection flow can be written as:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta\left(T_f - T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right) - \frac{\sigma B_0^2 \bar{u}}{\rho} \quad (2)$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T_f}{\partial \bar{y}^2} + \frac{\sigma B_0^2 \bar{u}^2}{\rho c_p} \quad (3)$$

The physical situation of the system suggests the following boundary conditions

$$\left.\begin{aligned} &\bar{u} = \bar{v} = 0,\ T_f = T(\bar{x}, 0) \\ &\partial T_f / \partial \bar{y} = \kappa_s\left(T_f - T_b\right)/ b\kappa_f \end{aligned}\right\} on\ \bar{y} = 0,\ x > 0 \quad (4)$$

$$\bar{u} \to 0,\ T_f \to T_\infty\ as\ \bar{y} \to \infty,\ \bar{x} > 0$$

The governing equations and the boundary conditions (1)-(4) can be made non-dimensional, using the Grashof number $Gr = \frac{g\beta a^3 (T_b - T_\infty)}{\nu^2}$ which is assumed large and the following non-dimensional variables:

$$\left.\begin{aligned} &x = \frac{\bar{x}}{a},\ y = \frac{\bar{y}}{a} Gr^{\frac{1}{4}},\ u = \frac{\bar{u} a}{\nu} Gr^{-\frac{1}{2}}, \\ &v = \frac{\bar{v} a}{\nu} Gr^{-\frac{1}{4}},\ \theta = \frac{T_f - T_\infty}{T_b - T_\infty} \end{aligned}\right\} \quad (5)$$

Where θ is the dimensionless temperature. The non dimensional form of the equations (1)-(3) are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + Mu = \frac{\partial^2 u}{\partial y^2} + \theta \sin x \quad (7)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ju^2 \quad (8)$$

Where $M = (\sigma a^2 B_0^{\ 2})/(\nu\,\rho\, Gr^{1/2})$ is the magnetic parameter, $J = (\sigma \nu B_0^{\ 2} Gr^{1/2})/\{\rho\, c_p \left(T_b - T_\infty\right)\}$ is the joule heating parameter and $Pr = \mu c_p / \kappa$ is the Prandtl number.

The boundary condition (4) can be written as in the following dimensionless form:

$$\left.u = v = 0,\ \theta - 1 = p\frac{\partial \theta}{\partial y}\right\} on\ \ y = 0,\ x > 0 \quad (9)$$

$$u \to 0,\ \theta \to 0\ \ as\ \ y \to \infty,\ x > 0$$

Where $p = \frac{b\kappa_f Gr^{1/4}}{a\kappa_s}$ is the conjugate conduction parameter. The present problem is governed by the magnitude of p. The values of p depends on b/a, κ_f/κ_s and Gr. The ratios b/a and κ_f/κ_s are less than one where as Gr is large for free convection. Therefore the value of p may be zero (b=0) or greater than zero. In the present investigation we have considered p =1.

To solve equation (6)-(8), subject to the boundary condition (9), we assume following transformations

$$\psi = x\, f\left(x, y\right),\ \theta = \theta(x, y) \quad (10)$$

Where ψ is the stream function usually defined as

$$u = \partial\psi / \partial y\ \ v = -\partial\psi / \partial x \quad (11)$$

Substituting (11) into the equations (6)-(9), the new forms of the dimensionless equations (7) and (8) are

$$f''' + ff'' - f'^2 - Mf' + \theta\frac{\sin x}{x} = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \quad (12)$$

$$\frac{1}{\Pr}\theta'' + f\theta' + Jx^2 f'^2 = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \quad (13)$$

In the above equations primes denote differentiation with respect to *y*. The corresponding boundary conditions take the following form

$$f = f' = 0,\ \theta - 1 = p\frac{\partial\theta}{\partial y}\ \ at\ \ y = 0,\ x > 0 \qquad (14)$$

$$f' \to 0,\ \theta \to 0\ \ as\ \ y \to \infty,\ x > 0$$

Principle physical quantities, the shearing stress and the rate of heat transfer in terms of skin friction coefficient C_f and Nusselt number Nu respectively can be written as

$$C_f\, Gr^{1/4} = x\, f''(x,0) \qquad (15)$$

$$Nu\, Gr^{-1/4} = -\theta(x,0) \qquad (16)$$

The results of the velocity profiles and temperature distributions can be calculated by the following relations respectively.

$$u = f'(x,y)\ ,\ \theta = \theta(x,y) \qquad (17)$$

3. METHOD OF SOLUTION

Equation (12) and (13) are solved numerically based on the boundary conditions as described in equation (14) using one of the most efficient and accurate methods known as implicit finite difference method with Keller box scheme [19, 20].

4. RESULTS AND DISCUSSION

The main objective of the present work is to analyze the flow of the fluid and the heat transfer processes due to the conjugate heat transfer from an isothermal horizontal circular cylinder. The Prandtl numbers are considered to be 1.63, 1.44, 1.0 and 0.733 for the simulation that correspond to Glycerin, water, steam and hydrogen, respectively. The value of the conjugate conduction parameter *p* is considered 1.0 for entire solutions.

A comparison of the local Nusselt number and the local skin friction factor obtained in the present work with M = 0.0, J = 0.0, p = 0.0 and Pr = 1.0 and obtained by Merkin [1] and Nazar et al. [14] have been shown in Tables 1 and 2 respectively. There is an excellent agreement among these three results.

The magnetic field opposes the fluid flow. As a result the peak velocity decreases with the increasing M as shown in fig. 2. Consequently, the separation of the boundary layer occurs earlier and the momentum boundary layer becomes thicker. From Fig. 3 it can be observed that the magnetic field decreases the temperature gradient and increases the temperature in the boundary layer for a particular value of y. Thus, the magnetic parameter increases the thickness of the thermal boundary layer. Temperature at the interface also varies with different M since the conduction is considered within cylinder.

The variation of the local skin friction coefficient and local rate of heat transfer with Pr =1.0 and J =0.10 for different values of M at different positions are illustrated in Fig. 4 and Fig. 5. The Magnetic force opposes the flow, as mentioned earlier, and reduces the shear stress at the wall as illustrated in Fig. 4. Moreover, the heat transfer rate also decreases as revealed in Fig. 5.

The velocity profiles, temperature distributions, local skin friction coefficients and the heat transfer rate for different values of Joule heating parameter J are presented in Fig. 6, Fig. 7, Fig. 8 and Fig. 9, respectively with Pr =1.0 and M =0.5. Increasing value of the Joule heating parameter containing magnetic field strength $\mathbf{B_0}$ increases the temperature and finally the fluid motion is accelerated as plotted in Fig.7 and Fig.6 respectively. The variation of the skin friction coefficient increases for the increasing J as depicted in Fig. 8 which is expected. The increased temperature for increasing J within the boundary layer reduced the temperature difference between the boundary layer region and the core region eventually decreases heat transfer rate as illustrated in Fig.9.

In Fig. 10 and Fig. 11 different values of Prandtl number Pr, with M = 0.5 and J = 0.10, are considered for the velocity and temperature distributions respectively. It is observed in Fig. 10 that the peak velocity decreases as well as its position moves toward the surface of the cylinder for the increasing values of Prandtl number. The overall temperature profiles shift downward with increasing Prandtl number as shown in Fig. 11. Consequently, temperature difference increases between the boundary layer region and the core region which increases the rate of heat transfer as observed in Fig. 13. This result supports the physical fact that the thermal boundary layer thickness decreases with increasing Pr. The skin friction coefficient decreases for the increasing values of Prandtl number as plotted in Fig.12.

5. TABLES AND FIGURES

Table 1: Numerical values of $-\theta'(x,0)$ for different values of x while Pr=1.0, M = 0.0, J=0.0 and p = 0.0.

$Nu\, Gr^{-1/4} = -\theta'(x,0)$			
x	Merkin [1]	Nazar et al. [14]	Present
0.0	0.4214	0.4214	0.4216
$\pi/6$	0.4161	0.4161	0.4163
$\pi/3$	0.4007	0.4005	0.4006
$\pi/2$	0.3745	0.3741	0.3741
$2\pi/3$	0.3364	0.3355	0.3355
$5\pi/6$	0.2825	0.2811	0.2811
π	0.1945	0.1916	0.1912

Table 2: Numerical values of $x\, f''(x,0)$ for different values of x while Pr = 1.0, M = 0.0, J=0.0 and p = 0.0.

$C_f\, Gr^{1/4} = x\, f''(x,0)$			
x	Merkin [1]	Nazar et al. [14]	Present
0.0	0.0000	0.0000	0.0000
$\pi/6$	0.4151	0.4148	0.4139
$\pi/3$	0.7558	0.7542	0.7528
$\pi/2$	0.9579	0.9545	0.9526
$2\pi/3$	0.9756	0.9698	0.9678
$5\pi/6$	0.7822	0.7740	0.7718
π	0.3391	0.3265	0.3239

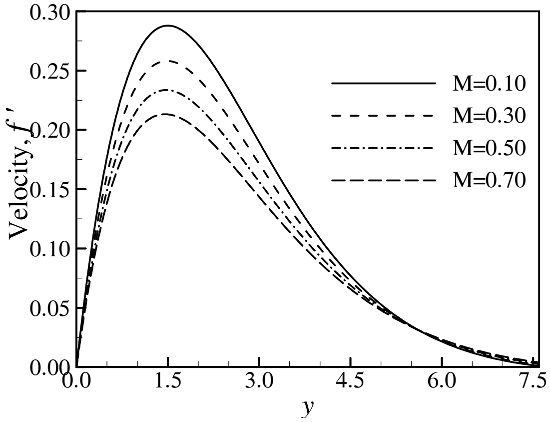

Fig 2. Variation of velocity profiles against *y* for varying of *M* with *Pr* = 1.0, *J* = 0.10 and *p* =1.0.

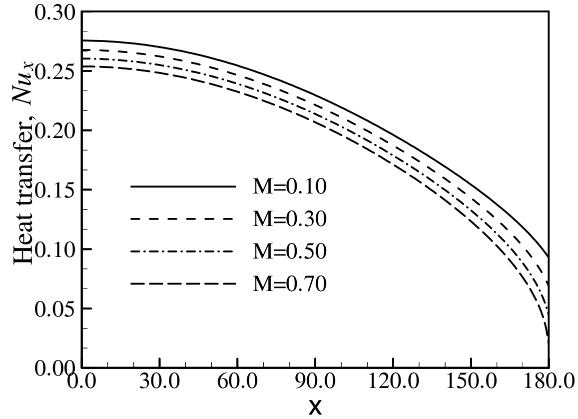

Fig 5. Variation of rate of heat transfer against *x* for varying of *M* with *Pr* = 1.0, *J* = 0.10 and *p* =1.0.

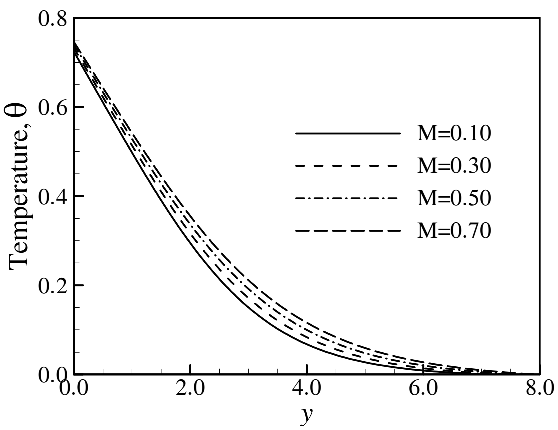

Fig 3. Variation of temperature distributions against *y* for varying of *M* with *Pr* = 1.0, *J* = 0.10 and *p* =1.0.

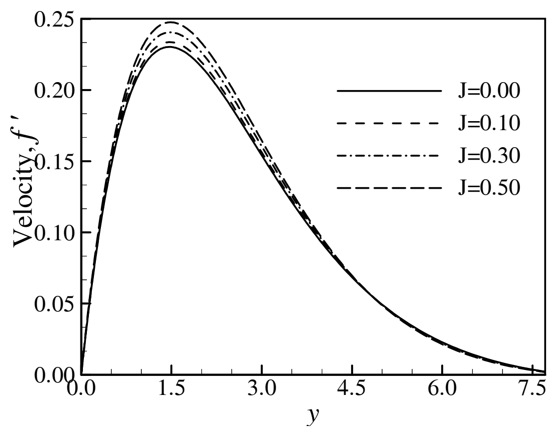

Fig 6. Variation of velocity profiles against *y* for varying of *J* with *Pr* = 1.0, *M* = 0.5 and *p* = 1.0.

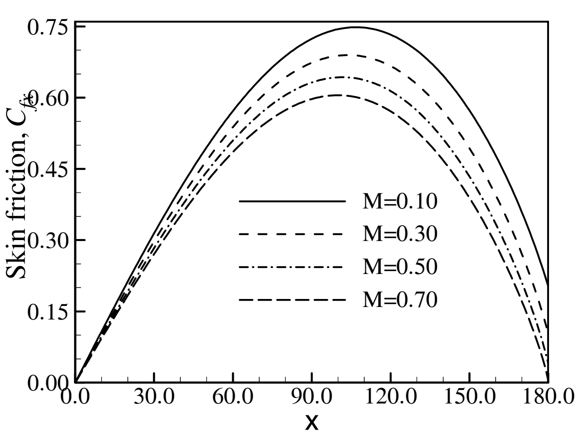

Fig 4. Variation of skin friction coefficients against *x* for varying of *M* with *Pr* = 1.0, *J* = 0.10 and *p*=1.0.

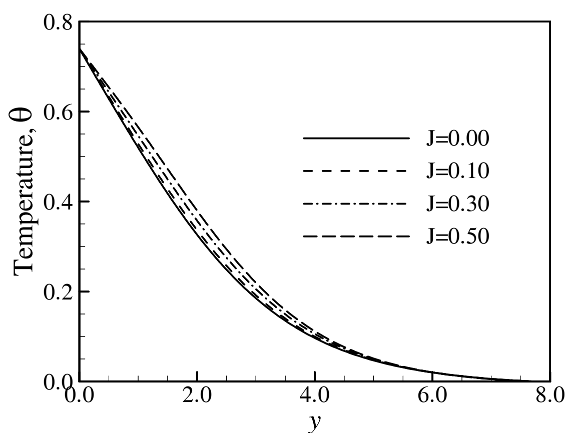

Fig 7. Variation of temperature distributions against *y* for varying of *J* with *Pr* = 1.0, *M* = 0.5 and *p* = 1.0.

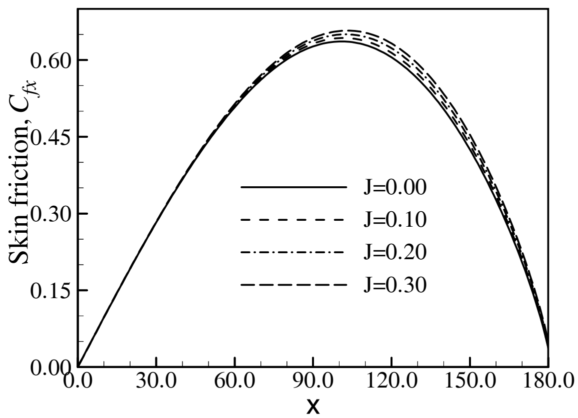

Fig 8. Variation of skin friction coefficients against *x* for varying of *J* with *Pr* = 1.0, *M* = 0.5 and *p* =1.0.

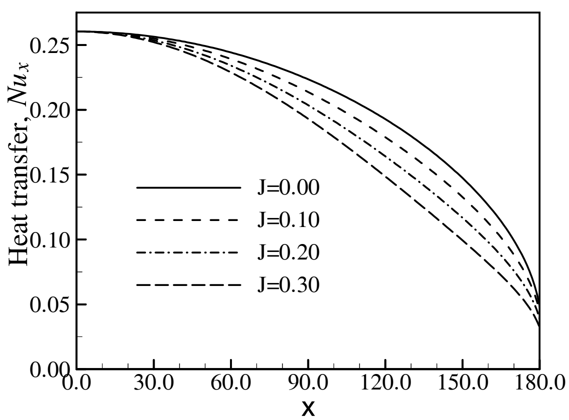

Fig 9. Variation of rate of heat transfer against *x* for varying of *J* with *Pr* = 1.0, *M* = 0.5 and *p* =1.0.

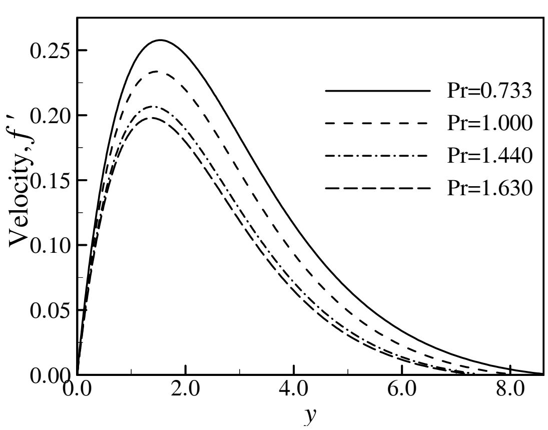

Fig 10. Variation of velocity profiles against *y* for varying of *Pr* with *J* = 0.10, *M* = 0.5 and *p* = 1.0.

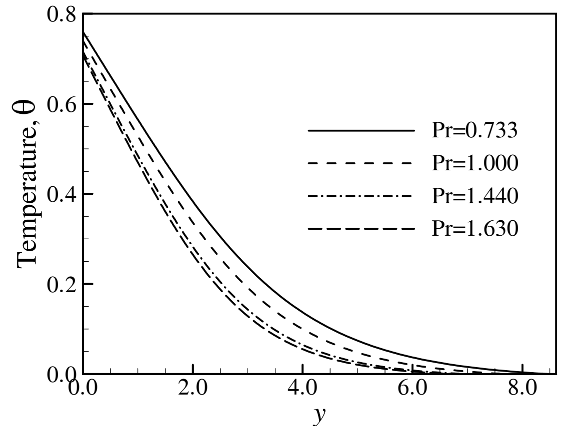

Fig 11. Variation of temperature distributions against *y* for varying of *Pr* with *J* = 0.10, *M* = 0.5 and *p* = 1.0.

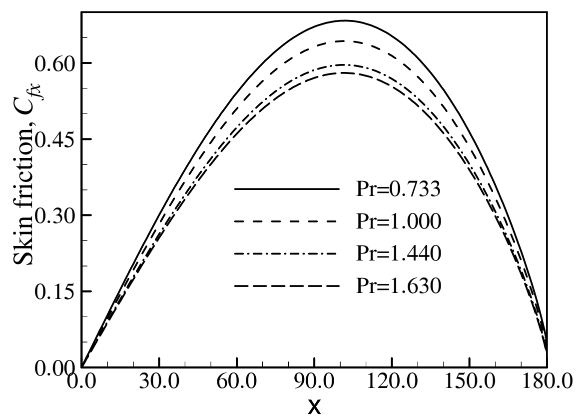

Fig 12. Variation of skin friction coefficients against *x* for varying of *Pr* with *J* = 0.10, *M* = 0.5 and *p* = 1.0.

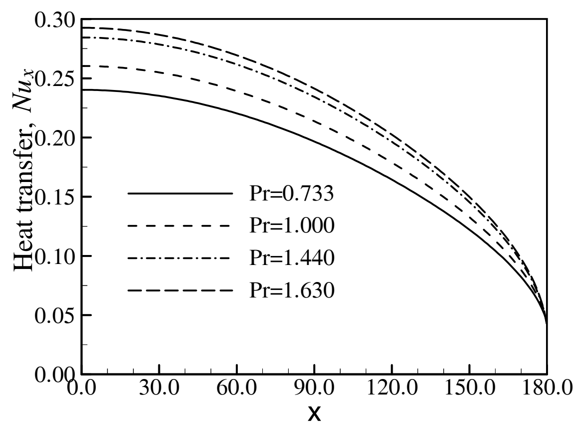

Fig 13. Variation of rate of heat transfer against *x* for varying of *Pr* with *J* = 0.10, *M* = 0.5 and *p* = 1.0.

6. CONCLUSION

A steady, two dimensional, MHD-conjugate free convection flow is studied considering joule heating phenomenon. The effects of the magnetic parameter *M*, Joule heating parameter *J* and Prandtl number *Pr* are analysed on the fluid flow with conjugate conduction parameter p = 1.0. The velocity of the fluid within the boundary layer and the skin friction coefficients along the cylinder surface decreases with increasing *M* and *Pr* while it increases with increasing *J*. The temperature distribution increases for increasing *M* and *J* whereas it decreases for increasing *Pr*. On the other hand the skin friction coefficients decreases for increasing *M* and *J* and it increases for increasing *Pr*.

7. REFERENCES

1. Merkin, J. H., 1976, "Free convection boundary layer on an isothermal horizontal cylinder", ASME, paper no.76-HT-16.
2. Merkin, J. H. and Pop, I., 1988, "A note on the free convection boundary layer on a horizontal circular cylinder with constant heat flux", Wärme und Stoffübertragung, Vol. 22, pp-79-81.
3. Kuehn, T. H. and Goldstein, R. J., 1980, "Numerical solution to the Navier-Stokes equations for laminar natural convection about a horizontal isothermal circular cylinder", Int. J. heat Mass Transfer, Vol.23, pp-971-979.
4. Wang, P., Kahawita, R. and Nguyen, T. H., 1990, "Numerical computation of natural convection flow about a horizontal cylinder Using Splines", Numerical Heat Transfer, Part-A, Vol.17, pp.191-215.
5. Gdalevich, L. B. and Fertman, V. E., 1977, "Conjugate problems of natural convection", Inzh-Fiz. Zh. 33, 539–547.
6. Miyamoto, M., Sumikawa, J., Akiyoshi, T. and Nakamura,T., 1980, "Effects of axial heat conduction in a vertical flat plate on free convection heat transfer", International Journal of Heat and Mass Transfer 23, 1545–1553.
7. Pozzi, A. and Lupo, M., 1988, "The coupling of conduction with laminar natural convection along a flat plate", International Journal of Heat and Mass Transfer 31, 1807–1814.
8. Kimura, S. and Pop, I., 1994, "Conjugate natural convection from a horizontal circular cylinder", Numerical heat transfer, Part A, Vol.25, pp.347-361.
9. Wilks, G., 1976, "Magnetohydrodynamics free convection about a semi-infinite vertical plate in a strong cross field", ZAMP, Vol.27, pp.621-631.
10. Takhar, H. S. and Soundalgekar, V. M., 1980, "Dissipation effects on MHD free convection flow past a semi-infinite vertical plate", Applied Science Research, Vol.36, pp.163-171.
11. Hossain, M. A., 1992, "Viscous and Joule heating effects on MHD-free convection flow with variable plate temperature", Int. J. heat Mass Transfer, Vol.35, pp.3485-3487.
12. Aldoss, T. K., Ali, Y. D. and Al-Nimr, M. A., 1996, "MHD mixed convection from a horizontal circular cylinder", Numerical Heat Transfer, Part A, Vol.30, pp.379-396.
13. El-Amin, M. F., 2003, "Combined effect of viscous dissipation and Joule heating on MHD forced convection over a non-isothermal horizontal cylinder embedded in a fluid saturated porous medium", Journal of Magnetism and Magnetic materials, Vol.263, pp.337-343.

8. NOMENCLATURE

Symbol	Meaning	Unit
b	Thickness of the cylinder	(cm)
B_0	Applied magnetic field	(N)
C_{fx}	Skin friction coefficient	…
c_p	Specific heat	(J/Kg.K)
f	Dimensionless stream function	…
g	Acceleration due to gravity	(cm/s^2)
J	Joule heating parameter	…
l	Length of the plate	(cm)
M	Magnetic parameter	…
Nu_x	Local Nusselt number	…
p	Conjugate conduction parameter	…
Pr	Prandtl number	…
T_b	Temperature of the inner cylinder	(K)
T_f	Temperature at the boundary layer region	(K)
T_s	Temperature of the solid of the cylinder	(K)
T_∞	Temperature of the ambient fluid	(K)
$\bar{u}, \bar{v}$	Velocity components	(cm/s)
u, v	Dimensionless velocity components	…
$\bar{x}, \bar{y}$	Cartesian coordinates	(cm)
x, y	Dimensionless Cartesian coordinates	…

Greek symbols

Symbol	Meaning	Unit
β	Co-efficient of thermal expansion	(K^{-1})
ψ	Dimensionless stream function	…
ρ	Density of the fluid inside the boundary layer	(Kg/m^3)
ν	Kinematic viscosity	(m^2/s)
μ	Viscosity of the fluid	(N.s/m^2)
θ	Dimensionless temperature	…
σ	Electrical conductivity	J/msK
K_f	Thermal conductivity of the ambient fluid	(kW/mK)
K_s	Thermal conductivity of the ambient solid	(kW/mK)

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